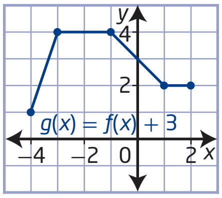
Answers

Chapter 1 Function Transformations

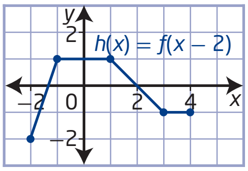
1.1 Horizontal and Vertical Translations, pages 12 to 15

1. a) $h = 0, k = 5$ **b)** $h = 0, k = -4$ **c)** $h = -1, k = 0$
d) $h = 7, k = -3$ **e)** $h = -2, k = 4$

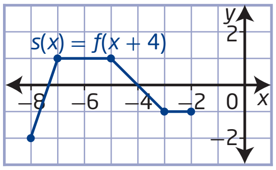
2. a) A′(−4, 1), B′(−3, 4), C′(−1, 4), D′(1, 2), E′(2, 2)



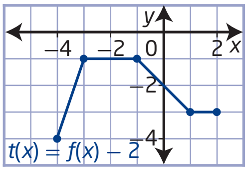
b) A′(−2, −2), B′(−1, 1), C′(1, 1), D′(3, −1), E′(4, −1)



c) A′(−8, −2), B′(−7, 1), C′(−5, 1), D′(−3, −1), E′(−2, −1)

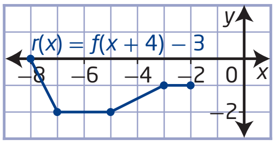


d) A′(−4, −4), B′(−3, −1), C′(−1, −1), D′(1, −3), E′(2, −3)



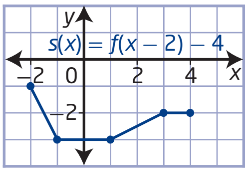
3. a) $(x, y) \rightarrow (x - 10, y)$ **b)** $(x, y) \rightarrow (x, y - 6)$
c) $(x, y) \rightarrow (x + 7, y + 4)$ **d)** $(x, y) \rightarrow (x + 1, y + 3)$

4. a)



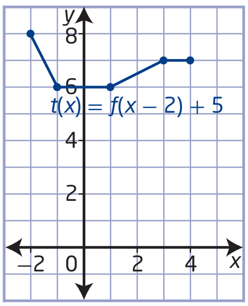
a vertical translation of 3 units down and a horizontal translation of 4 units left; $(x, y) \rightarrow (x - 4, y - 3)$

b)



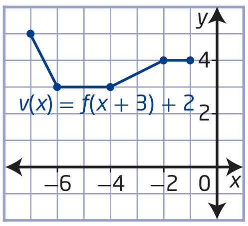
a vertical translation of 4 units down and a horizontal translation of 2 units right; $(x, y) \rightarrow (x + 2, y - 4)$

c)



a vertical translation of 5 units up and a horizontal translation of 2 units right; $(x, y) \rightarrow (x + 2, y + 5)$

d)



a vertical translation of 2 units up and a horizontal translation of 3 units left; $(x, y) \rightarrow (x - 3, y + 2)$

5. a) $h = -5, k = 4; y - 4 = f(x + 5)$
b) $h = 8, k = 6; y - 6 = f(x - 8)$
c) $h = 10, k = -8; y + 8 = f(x - 10)$
d) $h = -7, k = -12; y + 12 = f(x + 7)$

6. It has been translated 3 units up.

7. It has been translated 1 unit right.

8.

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
horizontal	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
horizontal	$y = f(x - 3)$	$(x, y) \rightarrow (x + 3, y)$
vertical	$y = f(x) - 6$	$(x, y) \rightarrow (x, y - 6)$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x, y) \rightarrow (x - 4, y - 9)$
horizontal and vertical	$y = f(x - 4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$
horizontal and vertical	$y = f(x + 2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$(x, y) \rightarrow (x + h, y + k)$

9. a) $y = (x + 4)^2 + 5$ **b)** $\{x \mid x \in R\}, \{y \mid y \geq 5, y \in R\}$
c) To determine the image function's domain and range, add the horizontal and vertical translations to the domain and range of the base function. Since the domain is the set of real numbers, nothing changes, but the range does change.

10. a) $g(x) = |x - 9| + 5$
b) The new graph is a vertical and horizontal translation of the original by 5 units up and 9 units right.
c) Example: (0, 0), (1, 1), (2, 2) → (9, 5), (10, 6), (11, 7)
d) Example: (0, 0), (1, 1), (2, 2) → (9, 5), (10, 6), (11, 7)
e) The coordinates of the image points from parts c) and d) are the same. The order that the translations are made does not matter.

11. a) $y = f(x - 3)$ **b)** $y + 5 = f(x - 6)$

12. a) Example: It takes her 2 h to cycle to the lake, 25 km away. She rests at the lake for 2 h and then returns home in 3 h.
b) This translation shows what would happen if she left the house at a later time.
c) $y = f(x - 3)$

13. a) Example: Translated 8 units right.
b) Example: $y = f(x - 8)$, $y = f(x - 4) + 3.5$, $y = f(x + 4) + 3.5$

14. a) Example: A repeating X by using two linear equations $y = \pm x$.
b) Example: $y = f(x - 3)$. The translation is horizontal by 3 units right.

15. a) The transformed function starts with a higher number of trout in 1970. $y = f(t) + 2$
b) The transformed function starts in 1974 instead of 1971. $y = f(t - 3)$

16. The first case, $n = f(A) + 10$, represents the number of gallons he needs for a given area plus 10 more gallons. The second case, $n = f(A + 10)$, represents how many gallons he needs to cover an area A less 10 units of area.

17. a) $y = (x - 7)(x - 1)$ or $y = (x - 4)^2 - 9$
b) Horizontal translation of 4 units right and vertical translation of 9 units down.
c) y-intercept 7

18. a) The original function is 4 units lower.

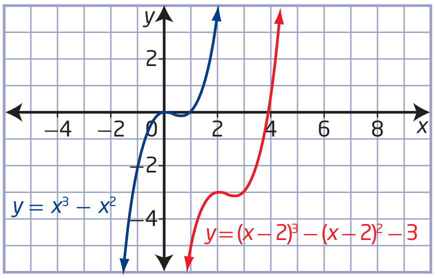
b) The original function is 2 units to the right.

c) The original function is 3 units lower and 5 units left.

d) The original function is 4 units higher and 3 units right.

19. a) The new graph will be translated 2 units right and 3 units down.

b)



C1 a) $y = f(x) \rightarrow y = f(x - h) \rightarrow y = f(x - h) + k$. Looking at the problem in small steps, it is easy to see that it does not matter which way the translations are done since they do not affect the other translation.

b) The domain is shifted by h and the range is shifted by k.

C2 a) $f(x) = (x + 1)^2$; horizontal translation of 1 unit left

b) $g(x) = (x - 2)^2 - 1$; horizontal translation of 2 units right and 1 unit down

C3 The roots are 2 and 9.

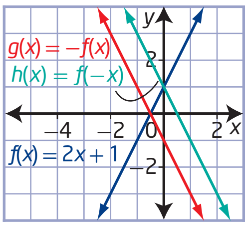
C4 The 4 can be taken as h or k in this problem. If it is h then it is -4, which makes it in the left direction.

1.2 Reflections and Stretches, pages 28 to 31

1. a)

x	$f(x) = 2x + 1$	$g(x) = -f(x)$	$h(x) = f(-x)$
−4	−7	7	9
−2	−3	3	5
0	1	−1	1
2	5	−5	−3
4	9	−9	−7

b)



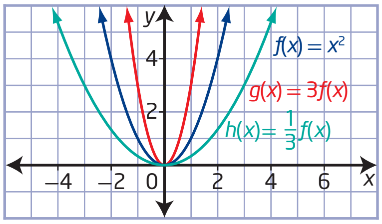
c) The y-coordinates of $g(x)$ have changed sign. The invariant point is $(-0.5, 0)$. The x-coordinates of $h(x)$ have changed sign. The invariant point is $(0, 1)$.

d) The graph of $g(x)$ is the reflection of the graph of $f(x)$ in the x-axis, while the graph of $h(x)$ is the reflection of the graph of $f(x)$ in the y-axis.

2. a)

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

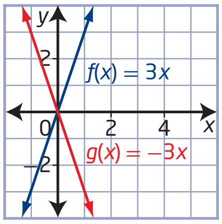
b)



c) The y-coordinates of $g(x)$ are three times larger. The invariant point is $(0, 0)$. The y-coordinates of $h(x)$ are three times smaller. The invariant point is $(0, 0)$.

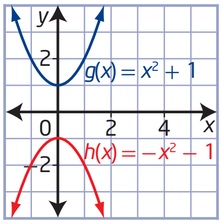
d) The graph of $g(x)$ is a vertical stretch by a factor of 3 of the graph of $f(x)$, while the graph of $h(x)$ is a vertical stretch by a factor of $\frac{1}{3}$ of the graph of $f(x)$.

3. a)



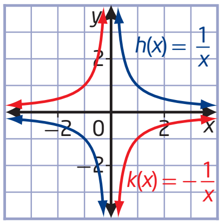
$g(x) = -3x$
$f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

b)



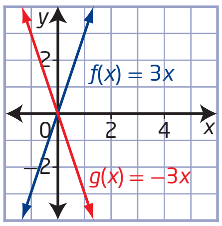
$h(x) = -x^2 - 1$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$
$h(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \leq -1, y \in R\}$

c)



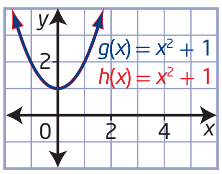
$k(x) = -\frac{1}{x}$
$h(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$

4. a)



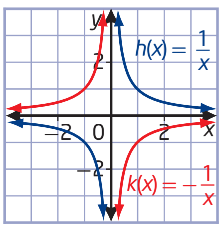
$g(x) = -3x$
$f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

b)



$h(x) = x^2 + 1$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$
$h(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$

c)



$k(x) = -\frac{1}{x}$
$h(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$

5. a) The graph of $y = 4f(x)$ is a vertical stretch by a factor of 4 of the graph of $y = f(x)$. $(x, y) \rightarrow (x, 4y)$

b) The graph of $y = f(3x)$ is a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of $y = f(x)$. $(x, y) \rightarrow \left(\frac{x}{3}, y\right)$

c) The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$. $(x, y) \rightarrow (x, -y)$

d) The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$. $(x, y) \rightarrow (-x, y)$

6. a) domain $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, range $\{y \mid -8 \leq y \leq 8, y \in \mathrm{R}\}$

b) The vertical stretch affects the range by increasing it by the stretch factor of 2.

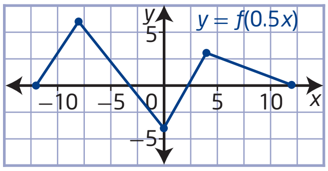
7. a) The graph of $g(x)$ is a vertical stretch by a factor of 4 of the graph of $f(x)$. $y = 4f(x)$

b) The graph of $g(x)$ is a reflection in the x-axis of the graph of $f(x)$. $y = -f(x)$

c) The graph of $g(x)$ is a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of $f(x)$. $y = f(3x)$

d) The graph of $g(x)$ is a reflection in the y-axis of the graph of $f(x)$. $y = f(-x)$

8.



9. a) horizontally stretched by a factor of $\frac{1}{4}$

b) horizontally stretched by a factor of 4

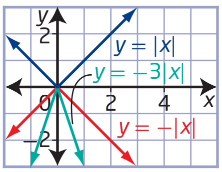
c) vertically stretched by a factor of $\frac{1}{2}$

d) vertically stretched by a factor of 4

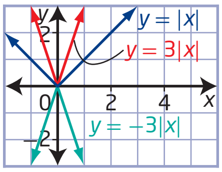
e) horizontally stretched by a factor of $\frac{1}{3}$ and reflected in the y-axis

f) vertically stretched by a factor of 3 and reflected in the x-axis

10. a)

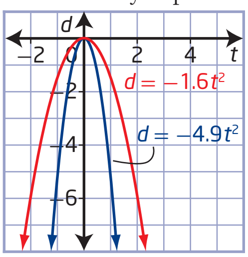


b)



c) They are both incorrect. It does not matter in which order you proceed.

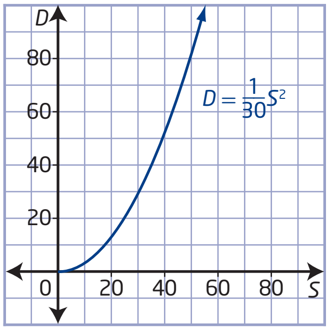
11. a)



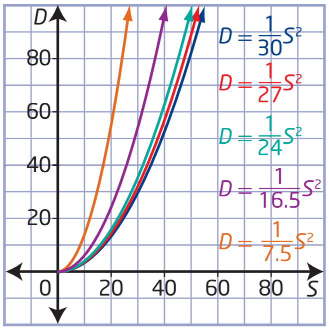
b) Both the functions are reflections of the base function in the t-axis. The object falling on Earth is stretched vertically more than the object falling on the moon.

12. Example: When the graph of $y = f(x)$ is transformed to the graph of $y = f(bx)$, it undergoes a horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$ and only the x-coordinates are affected. When the graph of $y = f(x)$ is transformed to the graph of $y = af(x)$, it undergoes a vertical stretch about the x-axis by a factor of $|a|$ and only the y-coordinates are affected.

13. a)



b) As the drag factor decreases, the length of the skid mark increases for the same speed.

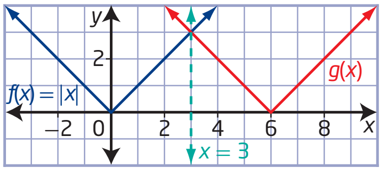


14. a) $x = -4$, $x = 3$ **b)** $x = 4$, $x = -3$

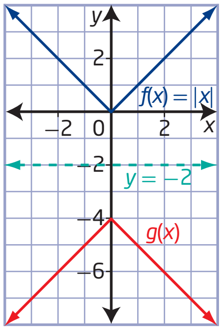
c) $x = -8$, $x = 6$ **d)** $x = -2$, $x = 1.5$

15. a) I **b)** III **c)** IV **d)** IV

16. a)



b)



C1 Example: When the input values for $g(x)$ are b times the input values for $f(x)$, the scale factor must be $\frac{1}{b}$ for the same output values. $g(x) = f\left(\frac{1}{b}(bx)\right) = f(x)$

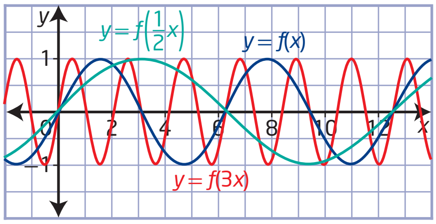
C2 Examples:

a) a vertical stretch or a reflection in the x-axis

b) a horizontal stretch or a reflection in the y-axis

C3

$f(x)$	$g(x)$	Transformation
(5, 6)	(5, −6)	reflection in the x-axis
(4, 8)	(−4, 8)	reflection in the y-axis
(2, 3)	(2, 12)	vertical stretch by a factor of 4
(4, −12)	(2, −6)	horizontal stretch by a factor of $\frac{1}{2}$ and vertical stretch by a factor of $\frac{1}{2}$

C4 

C5 a) $t_n = 4n - 14$ **b)** $t_n = -4n + 14$

c) They are reflections of each other in the x-axis.

1.3 Combining Transformations, pages 38 to 43

1. a) $y = -f\left(\frac{1}{2}x\right)$ or $y = -\frac{1}{4}x^2$

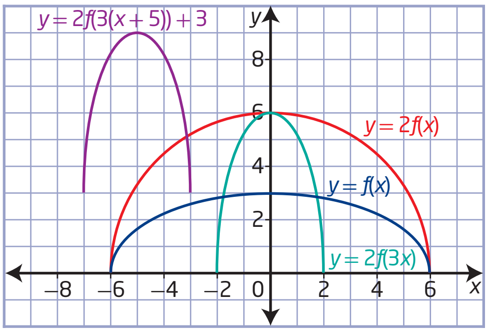
b) $y = \frac{1}{4}f(-4x)$ or $y = 4x^2$

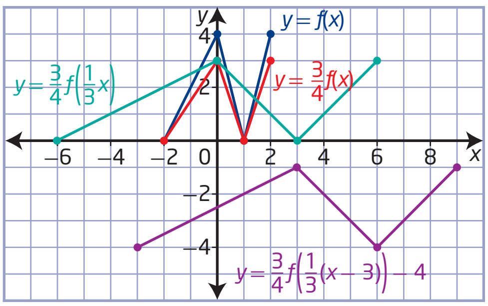
2. The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the y-axis by a factor of $\frac{1}{4}$. It is vertically stretched about the x-axis by a factor of 3. It is reflected in the x-axis, and then translated 4 units right and 10 units down.

3.

Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$	none	none	none	4	5
$y + 5 = 2f(3x)$	none	2	$\frac{1}{3}$	−5	none
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	none	$\frac{1}{2}$	2	none	4
$y + 2 = -3f(2(x + 2))$	x-axis	3	$\frac{1}{2}$	−2	−2

4. a) $y = f(-(x + 2)) - 2$ **b)** $y = f(2(x + 1)) - 4$

5. a) 

b) 

6. a) $(-8, 12)$ **b)** $(-4, 72)$ **c)** $(-6, -32)$

d) $(9, -32)$ **e)** $(-12, -9)$

7. a) vertical stretch by a factor of 2 and translation of 3 units right and 4 units up; $(x, y) \rightarrow (x + 3, 2y + 4)$

b) horizontal stretch by a factor of $\frac{1}{3}$, reflection in the x-axis, and translation of 2 units down; $(x, y) \rightarrow \left(\frac{1}{3}x, -y - 2\right)$

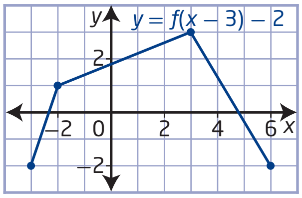
c) reflection in the y-axis, reflection in the x-axis, vertical stretch by a factor of $\frac{1}{4}$, and translation of 2 units left; $(x, y) \rightarrow \left(-x - 2, -\frac{1}{4}y\right)$

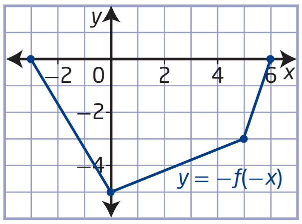
d) horizontal stretch by a factor of $\frac{1}{4}$, reflection in the x-axis, and translation of 2 units right and 3 units up; $(x, y) \rightarrow \left(\frac{1}{4}x + 2, -y + 3\right)$

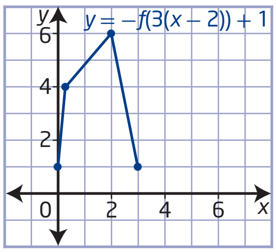
e) reflection in the y-axis, horizontal stretch by a factor of $\frac{4}{3}$, reflection in the x-axis, and vertical stretch by a factor of $\frac{2}{3}$; $(x, y) \rightarrow \left(-\frac{4}{3}x, -\frac{2}{3}y\right)$

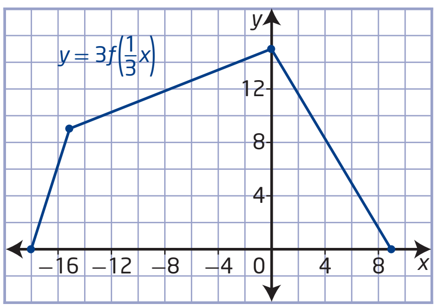
f) reflection in the y-axis, horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{3}$, and translation of 6 units right and 2 units up; $(x, y) \rightarrow \left(-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right)$

8. a) $y + 5 = -3f(x + 4)$ **b)** $y - 2 = -\frac{3}{4}f(-3(x - 6))$

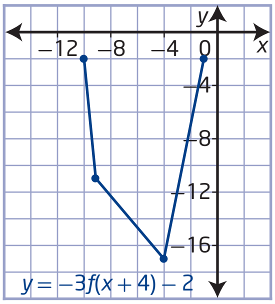
9. a) 

b) 

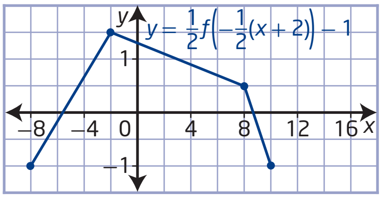
c) 

d) 

e)



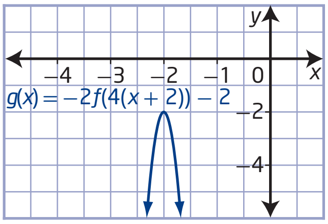
f)



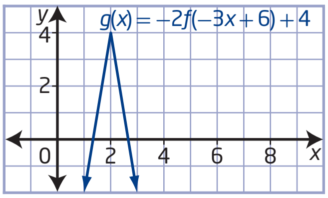
10. a) $y = -3f(x - 8) + 10$ **b)** $y = -2f(x - 3) + 2$

c) $y = -\frac{1}{2}f(-2(x + 4)) + 7$

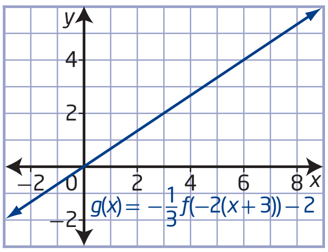
11. a)



b)



c)



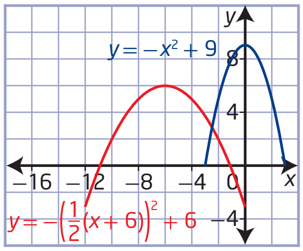
12. a) A′(−11, −2), B′(−7, 6), C′(−3, 4), D′(−1, 5), E′(3, −2)

b) $y = -f\left(\frac{1}{2}(x + 3)\right) + 4$

13. a) The graphs are in two locations because the transformations performed to obtain Graph 2 do not match those in $y = |2x - 6| + 2$. Gil forgot to factor out the coefficient of the x-term, 2, from −6. The horizontal translation should have been 3 units right, not 6 units.

b) He should have rewritten the function as $y = |2(x - 3)| + 2$.

14. a)



b) $y = -\left(\frac{1}{2}(x + 6)\right)^2 + 6$

15. a) $(-a, 0)$, $(0, -b)$ **b)** $(2a, 0)$, $(0, 2b)$

c) and **d)** There is not enough information to determine the locations of the new intercepts. When a transformation involves translations, the locations of the new intercepts will vary with different base functions.

16. a) $A = -2x^3 + 18x$ **b)** $A = -\frac{1}{8}x^3 + 18x$

c) For (2, 5), the area of the rectangle in part a) is 20 square units.
$A = -2x^3 + 18x$
$A = -2(2)^3 + 18(2)$
$A = 20$

For (8, 5), the area of the rectangle in part b) is 80 square units.
$A = -\frac{1}{8}x^3 + 18x$
$A = -\frac{1}{8}(8)^3 + 18(8)$
$A = 80$

17. $y = 36(x - 2)^2 + 6(x - 2) - 2$

18. Example: vertical stretches and horizontal stretches followed by reflections

C1 **Step 1** They are reflections in the axes.
1: $y = x + 3$, 2: $y = -x - 3$, 3: $y = x - 3$
Step 2 They are vertical translations coupled with reflections. 1: $y = x^2 + 1$, 2: $y = x^2 - 1$, 3: $y = -x^2$, 4: $y = -x^2 - 1$

C2 a) The cost of making $b + 12$ bracelets, and it is a horizontal translation.

b) The cost of making b bracelets plus 12 more dollars, and it is a vertical translation.

c) Triple the cost of making b bracelets, and it is a vertical stretch.

d) The cost of making $\frac{b}{2}$ bracelets, and it is a horizontal stretch.

C3 $y = 2(x - 3)^2 + 1$; a vertical stretch by a factor of 2 and a translation of 3 units right and 1 unit up

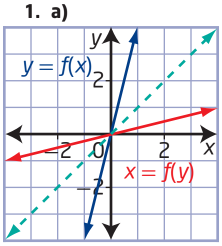
C4 a) H is repeated; J is transposed; K is repeated and transposed

b) H is in retrograde; J is inverted; K is in retrograde and inverted

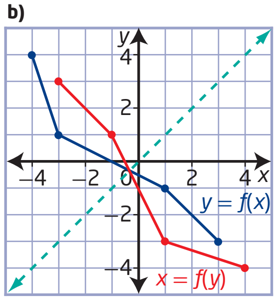
c) H is inverted, repeated, and transposed; J is in retrograde inversion and repeated; K is in retrograde and transposed

1.4 Inverse of a Relation, pages 51 to 55

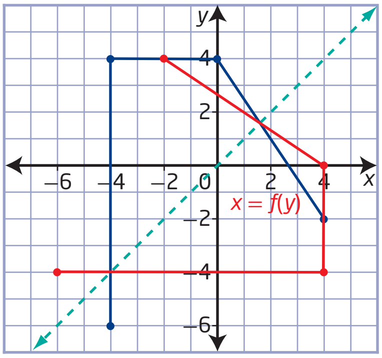
1. a)



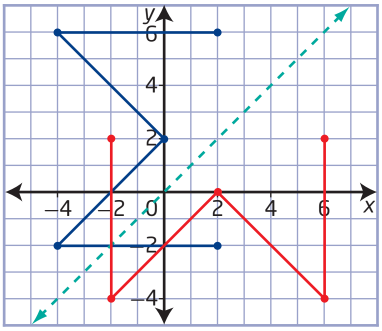
b)



2. a)



b)



3. a) The graph is a function but the inverse will be a relation.

b) The graph and its inverse are functions.

c) The graph and its inverse are relations.

4. Examples:

a) $\{x \mid x \geq 0, x \in R\}$ or $\{x \mid x \leq 0, x \in R\}$

b) $\{x \mid x \geq -2, x \in R\}$ or $\{x \mid x \leq -2, x \in R\}$

c) $\{x \mid x \geq 4, x \in R\}$ or $\{x \mid x \leq 4, x \in R\}$

d) $\{x \mid x \geq -4, x \in R\}$ or $\{x \mid x \leq -4, x \in R\}$

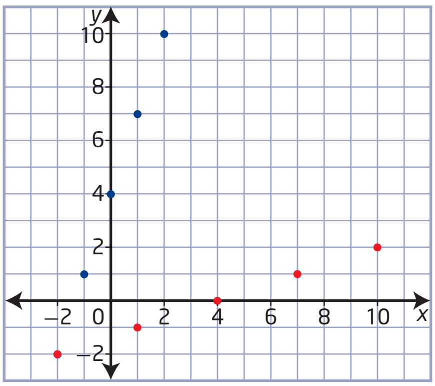
5. a) $f^{-1}(x) = \frac{1}{7}x$ **b)** $f^{-1}(x) = -\frac{1}{3}(x - 4)$

c) $f^{-1}(x) = 3x - 4$ **d)** $f^{-1}(x) = 3x + 15$

e) $f^{-1}(x) = -\frac{1}{2}(x - 5)$ **f)** $f^{-1}(x) = 2x - 6$

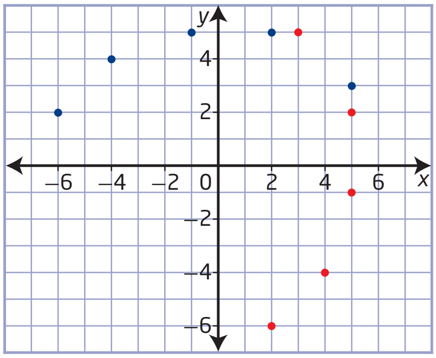
6. a) E **b)** C **c)** B **d)** A **e)** D

7. a)



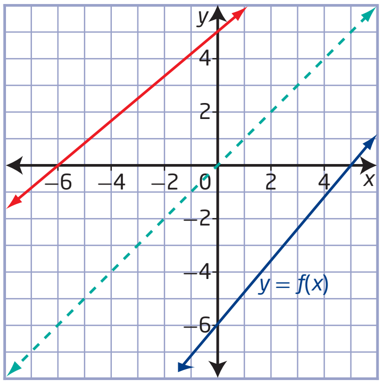
function: domain $\{-2, -1, 0, 1, 2\}$, range $\{-2, 1, 4, 7, 10\}$
inverse: domain $\{-2, 1, 4, 7, 10\}$, range $\{-2, -1, 0, 1, 2\}$

b)



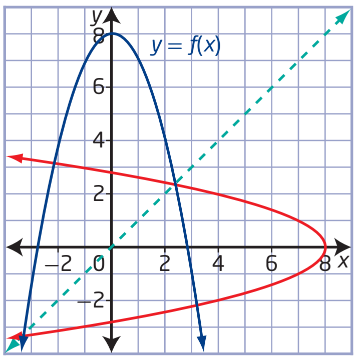
function: domain $\{-6, -4, -1, 2, 5\}$, range $\{2, 3, 4, 5\}$
inverse: domain $\{2, 3, 4, 5\}$, range $\{-6, -4, -1, 2, 5\}$

8. a)



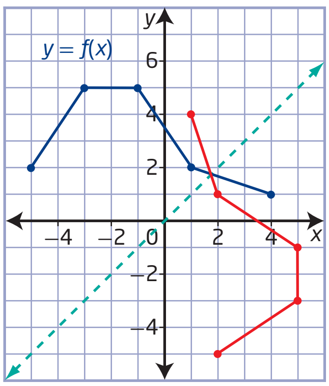
The inverse is a function; it passes the vertical line test.

b)



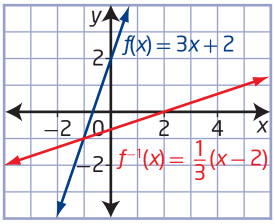
The inverse is not a function; it does not pass the vertical line test.

c)



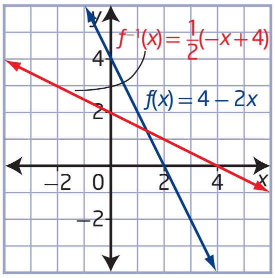
The inverse is not a function; it does not pass the vertical line test.

9. a) $f^{-1}(x) = \frac{1}{3}(x - 2)$



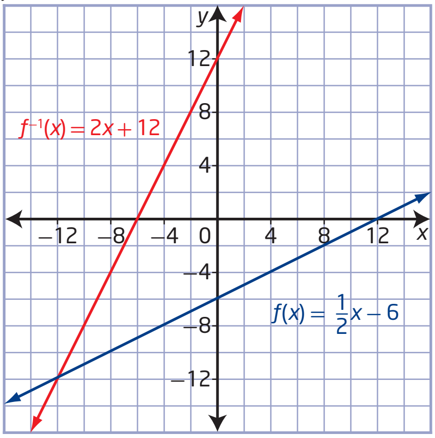
$f(x)$:
domain $\{x \mid x \in R\}$,
range $\{y \mid y \in R\}$
$f^{-1}(x)$:
domain $\{x \mid x \in R\}$,
range $\{y \mid y \in R\}$

b) $f^{-1}(x) = \frac{1}{2}(-x + 4)$



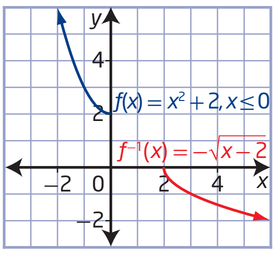
$f(x)$:
domain $\{x \mid x \in R\}$,
range $\{y \mid y \in R\}$
$f^{-1}(x)$:
domain $\{x \mid x \in R\}$,
range $\{y \mid y \in R\}$

c) $f^{-1}(x) = 2x + 12$



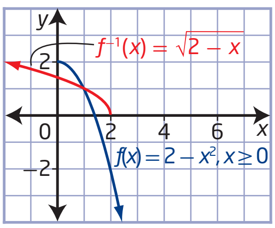
$f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$f^{-1}(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

d) $f^{-1}(x) = -\sqrt{x - 2}$



$f(x)$: domain
$\{x \mid x \leq 0, x \in R\}$,
range
$\{y \mid y \geq 2, y \in R\}$
$f^{-1}(x)$: domain
$\{x \mid x \geq 2, x \in R\}$,
range
$\{y \mid y \leq 0, y \in R\}$

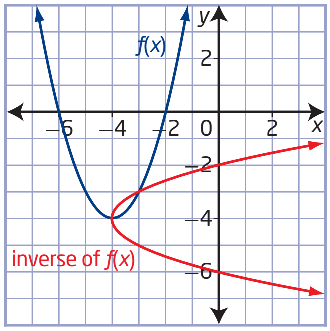
e) $f^{-1}(x) = \sqrt{2 - x}$



$f(x)$: domain
$\{x \mid x \geq 0, x \in R\}$,
range
$\{y \mid y \leq 2, y \in R\}$
$f^{-1}(x)$: domain
$\{x \mid x \leq 2, x \in R\}$,
range
$\{y \mid y \geq 0, y \in R\}$

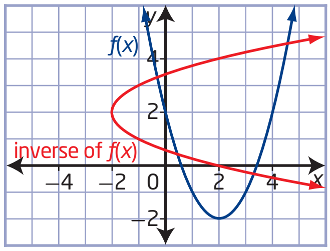
10. a) i) $f(x) = (x + 4)^2 - 4$, inverse of $f(x) = \pm\sqrt{x + 4} - 4$

ii)



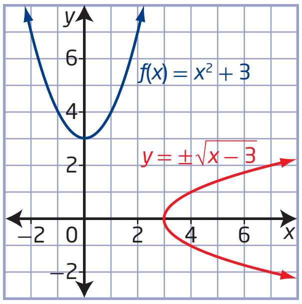
b) i) $y = (x - 2)^2 - 2$, $y = \pm\sqrt{x + 2} + 2$

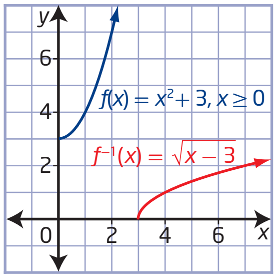
ii)



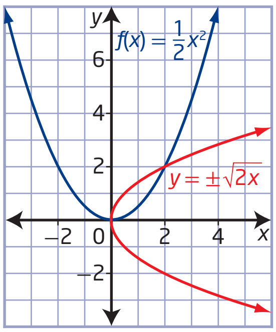
11. Yes, the graphs are reflections of each other in the line $y = x$.

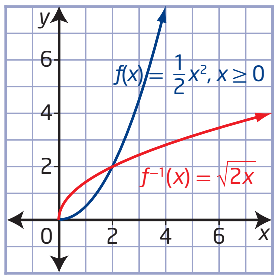
12. a) $y = \pm\sqrt{x - 3}$ restricted domain $\{x \mid x \geq 0, x \in R\}$



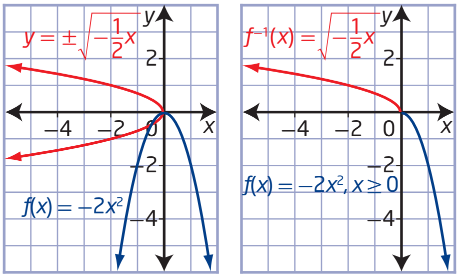


b) $y = \pm\sqrt{2x}$ restricted domain $\{x \mid x \geq 0, x \in R\}$

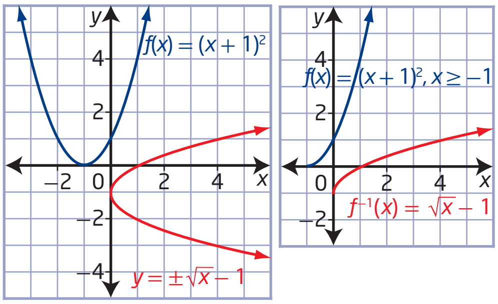




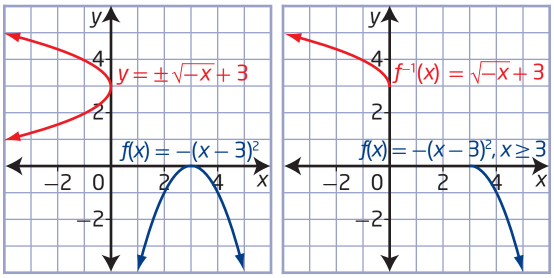
c) $y = \pm\sqrt{-\frac{1}{2}x}$ restricted domain $\{x \mid x \geq 0, x \in R\}$



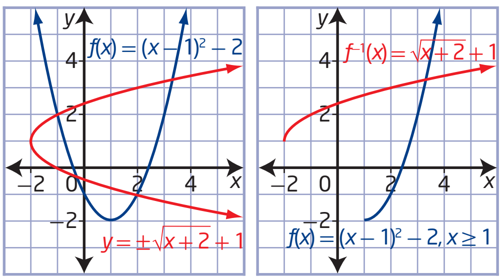
d) $y = \pm\sqrt{x} - 1$ restricted domain $\{x \mid x \geq -1, x \in R\}$



e) $y = \pm\sqrt{-x} + 3$ restricted domain $\{x \mid x \geq 3, x \in R\}$



f) $y = \pm\sqrt{x + 2} + 1$ restricted domain $\{x \mid x \geq 1, x \in R\}$

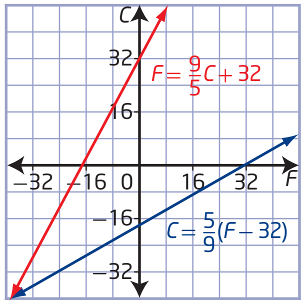


13. a) inverses **b)** inverses **c)** not inverses
d) inverses **e)** not inverses

14. Examples:
a) $x \geq 0$ or $x \leq 0$ **b)** $x \geq 0$ or $x \leq 0$
c) $x \geq 3$ or $x \leq 3$ **d)** $x \geq -2$ or $x \leq -2$

15. a) $\frac{3}{2}$ **b)** 0 **c)** $\frac{5}{2}$ **d)** $\frac{1}{2}$

16. a) approximately 32.22 °C
b) $y = \frac{9}{5}x + 32$; x represents temperatures in degrees Celsius and y represents temperatures in degrees Fahrenheit
c) 89.6 °F
d)

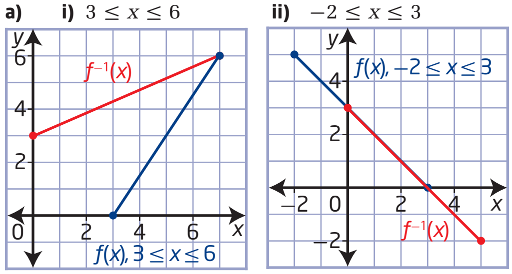


The temperature is the same in both scales (−40 °C = −40 °F).

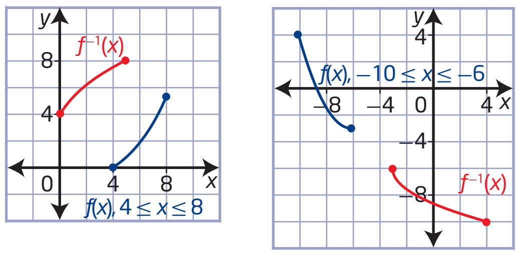
17. a) male height = 171.02 cm, female height = 166.44 cm
b) i) male femur = 52.75 cm
ii) female femur = 49.04 cm

18. a) 5
b) $y = 2.55x + 36.5$; y is finger circumference and x is ring size
c) 51.8 mm, 54.35 mm, 59.45 mm

19. Examples:
a) i) $3 \leq x \leq 6$ **ii)** $-2 \leq x \leq 3$



b) i) $4 \leq x \leq 8$ **ii)** $-10 \leq x \leq -6$

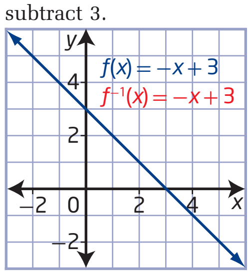


20. a) 17 **b)** $\sqrt{3}$ **c)** 10

21. a) (6, 10) **b)** (8, 23) **c)** (−8, −9)

C1 a) Subtract 12 and divide by 6.
b) Add 1, take the positive and negative square root, subtract 3.

C2 a)



b) Example: The graph of the original linear function is perpendicular to $y = x$, thus after a reflection the graph of the inverse is the same.

c) They are perpendicular to the line.

C3 Example: If the original function passes the vertical line test, then it is a function. If the original function passes the horizontal line test, then the inverse is a function.

C4 **Step 1**

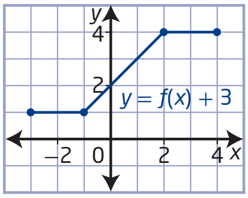
$f(x)$: (1, 2), (4, 3), (−8, −1), and $\left(a, \dfrac{a+5}{3}\right)$;

$g(x)$: (2, 1), (3, 4), (−1, −8), and $\left(\dfrac{a+5}{3}, a\right)$

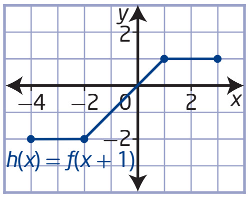
The output values for $g(x)$ are the same as the input values for $f(x)$.
Example: Since the functions are inverses of each other, giving one of them a value and then taking the inverse will always return the initial value. A good way to determine if functions are inverses is to see if this effect takes place.
Step 2 The order in which you apply the functions does not change the final result.
Step 4 The statement is saying that if you have a function that when given a outputs b and another that when given b outputs a, then the functions are inverses of each other.

Chapter 1 Review, pages 56 to 57

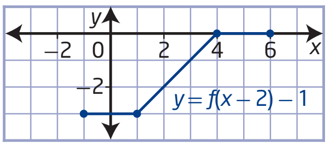
1. a)



b)



c)



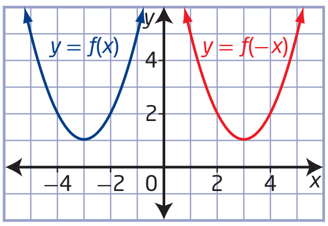
2. Translation of 4 units left and 5 units down: $y + 5 = |x + 4|$

3. range $\{y \mid 2 \le y \le 9, y \in \mathrm{R}\}$

4. No, it should be $(a + 5, b - 4)$.

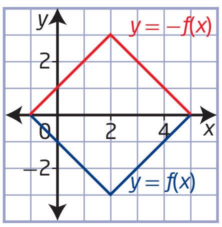
5. a) x-axis, (3, −5) **b)** y-axis, (−3, 5)

6. a)



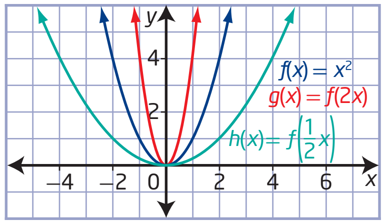
$f(-x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \ge 1, y \in \mathrm{R}\}$
(0, 10)

b)



$-f(x)$: domain $\{x \mid -1 \le x \le 5, x \in \mathrm{R}\}$, range $\{y \mid 0 \le y \le 3, y \in \mathrm{R}\}$
(5, 0), (−1, 0)

7. a)

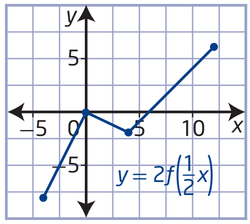


b) If the coefficient is greater than 1, then the function moves closer to the y-axis. The opposite is true for when the coefficient is between 0 and 1.

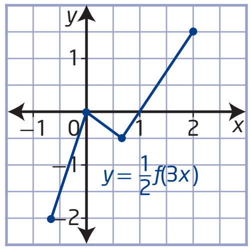
8. a) In this case, it could be either. It could be a vertical stretch by a factor of $\frac{1}{2}$ or a horizontal stretch by a factor of $\sqrt{2}$.

b) Example: $g(x) = \frac{1}{2}f(x)$

9. a)



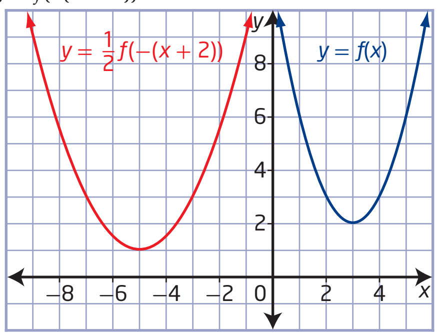
b)



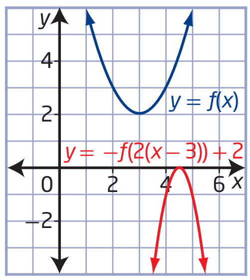
10. They are both horizontal stretches by a factor of $\frac{1}{4}$. The difference is in the horizontal translation, the first being 1 unit left and the second being $\frac{1}{4}$ unit left.

11. $g(x) = f(2(x - 5)) - 2$

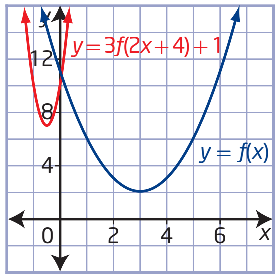
12. a)



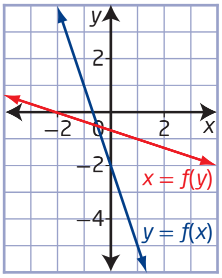
b)



c)



13. a)



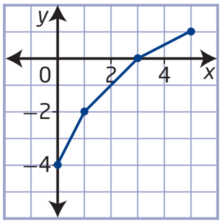
b) $y = x$, $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

c) $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$f(y)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

14.

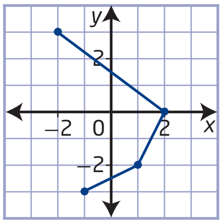
$y = f(x)$		$y = f^{-1}(x)$	
x	y	x	y
−3	7	7	−3
2	4	4	2
10	−12	−12	10

15. a)



The relation and its inverse are functions.

b)



The relation is a function. The inverse is not a function.

16. $y = \sqrt{x - 1} + 3$, restricted domain $\{x \mid x \geq 3, x \in \mathrm{R}\}$

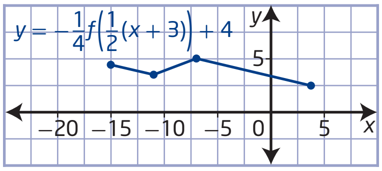
17. a) not inverses **b)** inverses

Chapter 1 Practice Test, pages 58 to 59

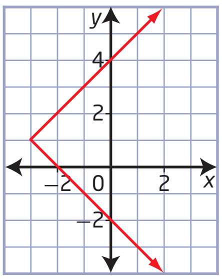
1. D **2.** D **3.** B **4.** B **5.** B **6.** C **7.** C

8. domain $\{x \mid -5 \leq x \leq 2, x \in \mathrm{R}\}$

9.



10. a)



b) To transform it point by point, switch the position of the x- and the y-coordinate.

c) $(-1, -1)$

11. $y = \frac{1}{5}(x - 2)$

12. $y = 3f\left(-\frac{1}{2}(x - 2)\right)$

13. a) It is a translation of 2 units left and 7 units down.

b) $g(x) = |x + 2| - 7$ **c)** $(-2, -7)$

d) No. Invariant points are points that remain unchanged after a transformation.

14. a) $f(x) = x^2$

b) $g(x) = \frac{1}{4}f(x)$; a vertical stretch by a factor of $\frac{1}{4}$

c) $g(x) = f\left(\frac{1}{2}x\right)$; a horizontal stretch by a factor of 2

d) $\frac{1}{4}f(x) = \frac{1}{4}x^2$; $f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$

15. a) Using the horizontal line test, if a horizontal line passes through the function more than once the inverse is not a function.

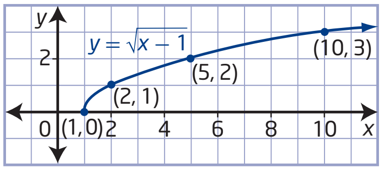
b) $y = \pm\sqrt{-x - 5} - 3$

c) Example: restricted domain $\{x \mid x \geq -3, x \in \mathrm{R}\}$

Chapter 2 Radical Functions

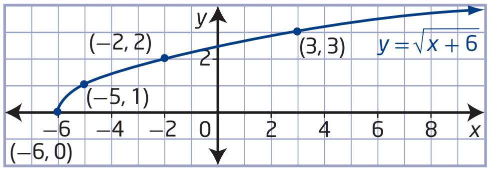
2.1 Radical Functions and Transformations, pages 72 to 77

1. a)



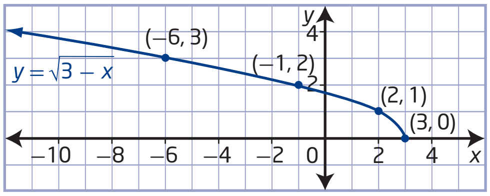
domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b)



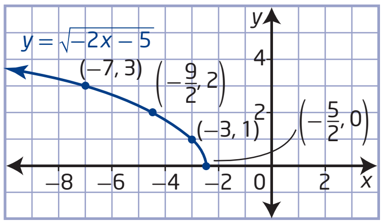
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)



domain $\{x \mid x \leq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d)



domain $\left\{x \mid x \leq -\frac{5}{2}, x \in \mathrm{R}\right\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

2. a) $a = 7 \rightarrow$ vertical stretch by a factor of 7
$h = 9 \rightarrow$ horizontal translation 9 units right
domain $\{x \mid x \geq 9, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) $b = -1 \rightarrow$ reflected in y-axis
$k = 8 \rightarrow$ vertical translation up 8 units
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 8, y \in \mathrm{R}\}$

c) $a = -1 \rightarrow$ reflected in x-axis
$b = \frac{1}{5} \rightarrow$ horizontal stretch factor of 5
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

d) $a = \frac{1}{3} \rightarrow$ vertical stretch factor of $\frac{1}{3}$
$h = -6 \rightarrow$ horizontal translation 6 units left
$k = -4 \rightarrow$ vertical translation 4 units down
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$

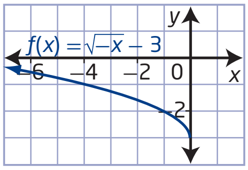
3. a) B **b)** A **c)** D **d)** C

4. a) $y = 4\sqrt{x + 6}$ **b)** $y = \sqrt{8x} - 5$

c) $y = \sqrt{-(x - 4)} + 11$ or $y = \sqrt{-x + 4} + 11$

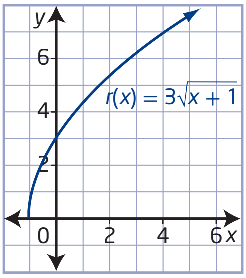
d) $y = -0.25\sqrt{0.1x}$ or $y = -\frac{1}{4}\sqrt{\frac{1}{10}x}$

5. a)



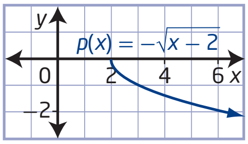
domain $\{x \mid x \le 0, x \in \mathrm{R}\}$, range $\{y \mid y \ge -3, y \in \mathrm{R}\}$

b)



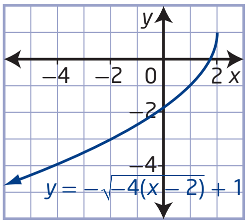
domain $\{x \mid x \ge -1, x \in \mathrm{R}\}$, range $\{y \mid y \ge 0, y \in \mathrm{R}\}$

c)



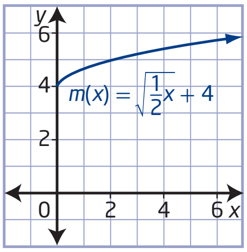
domain $\{x \mid x \ge 2, x \in \mathrm{R}\}$, range $\{y \mid y \le 0, y \in \mathrm{R}\}$

d)



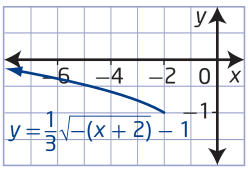
domain $\{x \mid x \le 2, x \in \mathrm{R}\}$, range $\{y \mid y \le 1, y \in \mathrm{R}\}$

e)



domain $\{x \mid x \ge 0, x \in \mathrm{R}\}$, range $\{y \mid y \ge 4, y \in \mathrm{R}\}$

f)



domain $\{x \mid x \le -2, x \in \mathrm{R}\}$ range $\{y \mid y \ge -1, y \in \mathrm{R}\}$

6. a) $a = \frac{1}{4} \rightarrow$ vertical stretch factor of $\frac{1}{4}$

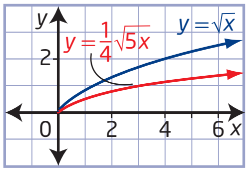
$b = 5 \rightarrow$ horizontal stretch factor of $\frac{1}{5}$

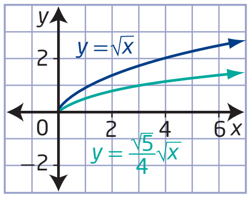
b) $y = \frac{\sqrt{5}}{4}\sqrt{x}$, $y = \sqrt{\frac{5}{16}x}$

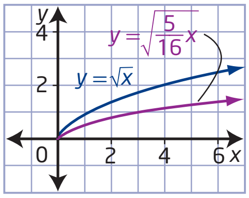
c) $a = \frac{\sqrt{5}}{4} \rightarrow$ vertical stretch factor of $\frac{\sqrt{5}}{4}$

$b = \frac{5}{16} \rightarrow$ horizontal stretch factor of $\frac{16}{5}$

d)





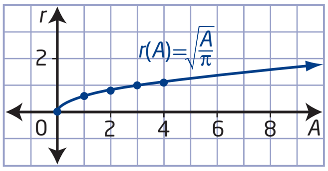


All graphs are the same.

7. a) $r(A) = \sqrt{\frac{A}{\pi}}$

b)

A	r
0	0
1	0.6
2	0.8
3	1.0
4	1.1



8. a) $b = 1.50 \rightarrow$ horizontal stretch factor of $\frac{1}{1.50}$ or $\frac{2}{3}$

b) $d \approx 1.22\sqrt{h}$ Example: I prefer the original function because the values are exact.

c) approximately 5.5 miles

9. a) domain $\{x \mid x \ge 0, x \in \mathrm{R}\}$, range $\{y \mid y \ge -13, y \in \mathrm{R}\}$

b) $h = 0 \rightarrow$ no horizontal translation

$k = 13 \rightarrow$ vertical translation down 13 units

10. a) $y = -\sqrt{x + 3} + 4$ **b)** $y = \frac{1}{2}\sqrt{x + 5} - 3$

c) $y = 2\sqrt{-(x - 5)} - 1$ or $y = 2\sqrt{-x + 5} - 1$

d) $y = -4\sqrt{-(x - 4)} + 5$ or $y = -4\sqrt{-x + 4} + 5$

11. Examples:

a) $y - 1 = \sqrt{x - 6}$ or $y = \sqrt{x - 6} + 1$

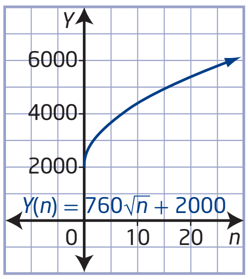
b) $y = -\sqrt{x + 7} - 9$ **c)** $y = 2\sqrt{-x + 4} - 3$

d) $y = -\sqrt{-(x + 5)} + 8$

12. a) $a = 760 \rightarrow$ vertical stretch factor of 760

$k = 2000 \rightarrow$ vertical translation up 2000

b)

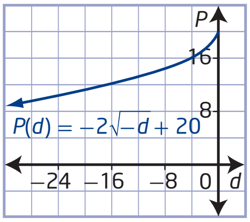


c) domain $\{n \mid n \ge 0, n \in \mathrm{R}\}$ range $\{Y \mid Y \ge 2000, Y \in \mathrm{R}\}$

d) The minimum yield is 2000 kg/hectare. Example: The domain and range imply that the more nitrogen added, the greater the yield without end. This is not realistic.

13. a) domain $\{d \mid -100 \le d \le 0, d \in \mathrm{R}\}$ range $\{P \mid 0 \le P \le 20, P \in \mathrm{R}\}$ The domain is negative indicating days remaining, and the maximum value of P is 20 million.

b) $a = -2 \rightarrow$ reflected in d-axis, vertical stretch factor of 2; $b = -1 \rightarrow$ reflected in P-axis; $k = 20 \rightarrow$ vertical translation up 20 units.

c) 

Since d is negative, then d represents the number of days remaining before release and the function has a maximum of 20 million pre-orders.

d) 9.05 million or 9 045 549 pre-orders.

14. a) Polling errors reduce as the election approaches.

b) $y = 0.49\sqrt{-x}$ There are no translations since the graph starts on the origin. The graph is reflected in the y-axis then $b = -1$. Develop the equation by using the point $(-150, 6)$ and substituting in the equation $y = a\sqrt{x}$, solving for a, then $a = 0.49$.

c) $a = 0.49 \rightarrow$ vertical stretch factor of 0.49
$b = -1 \rightarrow$ reflected in the y-axis

15. $y \approx 2.07\sqrt{-x}$

16. Examples

a) $y = -2\sqrt{x - 2} + 5$ **b)** $y = \frac{2}{3}\sqrt{3 - x} - 2$

17. a) China, India, and USA (The larger the country the more unfair the "one nation – one vote" system becomes.) Tuvalu, Nauru, Vatican City (The smaller the nation the more unfair the "one person – one vote" system becomes.)

b)

Nation	Percentage
China	18.6%
India	17.1%
US	4.5%
Canada	0.48%
Tuvalu	0.000 151%
Nauru	0.000 137%
Vatican City	0.000 014%

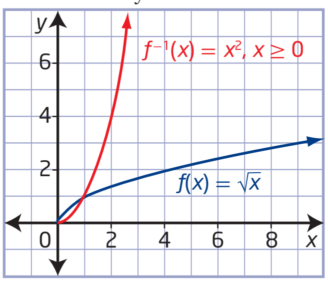
d)

Nation	Percentage
China	4.82%
India	4.62%
US	2.36%
Canada	0.77%
Tuvalu	0.014%
Nauru	0.013%
Vatican City	0.004%

c) $V(x) = \frac{1}{1000}\sqrt{x}$

e) The Penrose system gives larger nations votes based on population but also provides an opportunity for smaller nations to provide influence.

18. Answers will vary.

19. a) 

The positive domain of the inverse is the same as the range of the original function.

b) **i)** $g^{-1}(x) = x^2 + 5, x \leq 0$
ii) $h^{-1}(x) = -(x - 3)^2, x \geq 3$
iii) $j^{-1}(x) = \frac{1}{2}(x + 6)^2 + \frac{7}{2}, x \geq -6$

20. Vertical stretch by a factor of $\frac{16}{25}$. Horizontal stretch by a factor of $\frac{7}{72}$. Reflect in both the x and y axes. Horizontal translation of 3 units left. Vertical translation of 4 units down.

C1 The parameters b and h affect the domain. For example, $y = \sqrt{x}$ has domain $x \geq 0$ but $y = \sqrt{2(x - 3)}$ has domain $x \geq 3$. The parameters a and k affect the range. For example, $y = \sqrt{x}$ has range $y \geq 0$ but $y = \sqrt{x} - 4$ has range $y \geq -4$.

C2 Yes. For example, $y = \sqrt{9x}$ can be simplified to $y = 3\sqrt{x}$.

C3 The processes are similar because the parameters a, b, h, and k have the same effect on radical functions and quadratic functions. The processes are different because the base functions are different: one is the shape of a parabola and the other is the shape of half of a parabola.

C4 **Step 1** $\sqrt{2}$; **Step 2** $\sqrt{3}$

Step 4

Triangle Number, n	Length of Hypotenuse, L
First	$\sqrt{2} = 1.414...$
Second	$\sqrt{3} = 1.732...$
Third	$\sqrt{4} = 2$

Step 5 $L = \sqrt{n + 1}$ Yes, the equation involves a horizontal translation of 1 unit left.

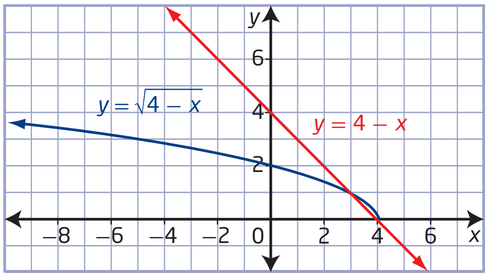
2.2 Square Root of a Function, pages 86 to 89

1.

$f(x)$	$\sqrt{f(x)}$
36	6
0.09	0.3
1	1
−9	undefined
2.56	1.6
0	0

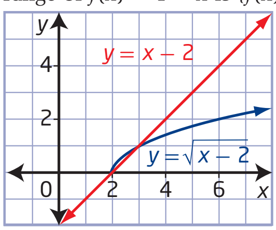
2. a) (4, 3.46) **b)** (−2, 0.63) **c)** does not exist
d) (0.09, 1) **e)** (−5, 0) **f)** $(m, \sqrt{n})$

3. a) C **b)** D **c)** A **d)** B

4. a) 

b) When $4 - x < 0$ then $\sqrt{4 - x}$ is undefined; when $0 < 4 - x < 1$ then $\sqrt{4 - x} > 4 - x$; when $4 - x > 1$ then $4 - x > \sqrt{4 - x}$; $4 - x = \sqrt{4 - x}$ when $y = 0$ and $y = 1$

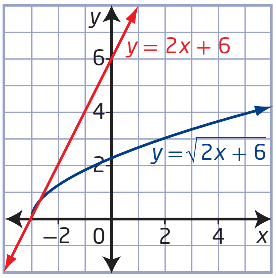
c) The function $f(x) = \sqrt{4 - x}$ is undefined when $4 - x < 0$, therefore the domain is $\{x \mid x \leq 4, x \in \mathrm{R}\}$ whereas the function $f(x) = 4 - x$ has a domain of $\{x \mid x \in \mathrm{R}\}$. Since $\sqrt{f(x)}$ is undefined when $f(x) < 0$, the range of $\sqrt{f(x)}$ is $\{f(x) \mid f(x) \geq 0, f(x) \in \mathrm{R}\}$, whereas the range of $f(x) = 4 - x$ is $\{f(x) \mid f(x) \in \mathrm{R}\}$.

5. a) 

For $y = x - 2$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for $y = \sqrt{x - 2}$, domain $\{x \mid x \geq 2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

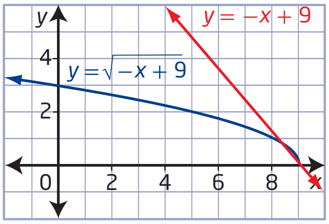
The domains differ since $\sqrt{x - 2}$ is undefined when $x < 2$. The range of $y = \sqrt{x - 2}$ is $y \geq 0$, when $x - 2 \geq 0$.

b)



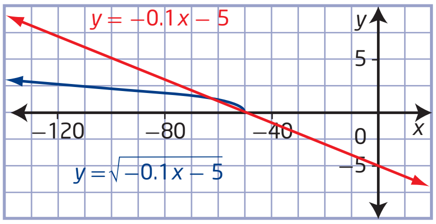
For $y = 2x + 6$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$. For $y = \sqrt{2x + 6}$, domain $\{x \mid x \geq -3, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y = \sqrt{2x + 6}$ is undefined when $2x + 6 < 0$, therefore $x \geq -3$ and $y \geq 0$.

c)



For $y = -x + 9$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y = \sqrt{-x + 9}$, domain $\{x \mid x \leq 9, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y = \sqrt{-x + 9}$ is undefined when $-x + 9 < 0$, therefore $x \leq 9$ and $y \geq 0$.

d)



For $y = -0.1x - 5$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y = \sqrt{-0.1x - 5}$, domain $\{x \mid x \leq -50, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y = \sqrt{-0.1x - 5}$ is undefined when $-0.1x - 5 < 0$, therefore $x \leq -50$ and $y \geq 0$.

6. a) For $y = x^2 - 9$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -9, y \in R\}$. For $y = \sqrt{x^2 - 9}$, domain $\{x \mid x \leq -3 \text{ and } x \geq 3, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y = \sqrt{x^2 - 9}$ is undefined when $x^2 - 9 < 0$, therefore $x \leq -3$ and $x \geq 3$ and $y \geq 0$.

b) For $y = 2 - x^2$, domain $\{x \mid x \in R\}$, range $\{y \mid y \leq 2, y \in R\}$. For $y = \sqrt{2 - x^2}$, domain $\{x \mid -\sqrt{2} \leq x \leq \sqrt{2}, x \in R\}$, range $\{y \mid 0 \leq y \leq \sqrt{2}, y \in R\}$. $y = \sqrt{2 - x^2}$ is undefined when $2 - x^2 < 0$, therefore $x \leq \sqrt{2}$ and $x \geq -\sqrt{2}$ and $0 \leq y \leq \sqrt{2}$.

c) For $y = x^2 + 6$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 6, y \in R\}$. For $y = \sqrt{x^2 + 6}$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq \sqrt{6}, y \in R\}$. $y = \sqrt{x^2 + 6}$ is undefined when $x^2 + 6 < 0$, therefore $x \in R$ and $y \geq \sqrt{6}$.

d) For $y = 0.5x^2 + 3$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 3, y \in R\}$. For $y = \sqrt{0.5x^2 + 3}$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq \sqrt{3}, y \in R\}$. $y = \sqrt{0.5x^2 + 3}$ is undefined when $0.5x^2 + 3 < 0$, therefore $x \in R$ and $y \geq \sqrt{3}$.

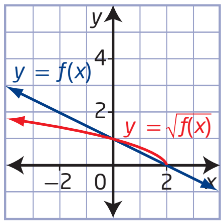
7. a) Since $y = \sqrt{x^2 - 25}$ is undefined when $x^2 - 25 < 0$, the domain changes from $\{x \mid x \in R\}$ to $\{x \mid x \leq -5 \text{ and } x \geq 5, x \in R\}$ and the range changes from $\{y \mid y \geq -25, y \in R\}$ to $\{y \mid y \geq 0, y \in R\}$.

b) Since $y = \sqrt{x^2 + 3}$ is undefined when $x^2 + 3 < 0$, the range changes from $\{y \mid y \geq 3, y \in R\}$ to $\{y \mid y \geq \sqrt{3}, y \in R\}$.

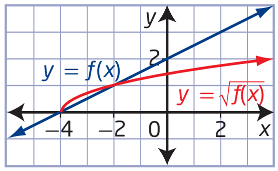
c) Since $y = \sqrt{32 - 2x^2}$ is undefined when $32 - 2x^2 < 0$, the domain changes from $\{x \mid x \in R\}$ to $\{x \mid -4 \leq x \leq 4, x \in R\}$ and the range changes from $\{y \mid y \leq 32, y \in R\}$ to $\{y \mid 0 \leq y \leq \sqrt{32}, y \in R\}$ or $\{y \mid 0 \leq y \leq 4\sqrt{2}, y \in R\}$.

d) Since $y = \sqrt{5x^2 + 50}$ is undefined when $5x^2 + 50 < 0$, the range changes from $\{y \mid y \geq 50, y \in R\}$ to $\{y \mid y \geq \sqrt{50}, y \in R\}$ or $\{y \mid y \geq 5\sqrt{2}, y \in R\}$.

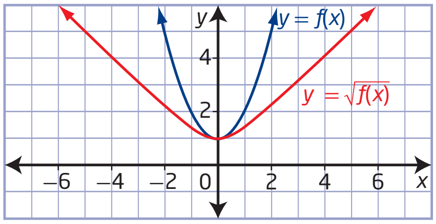
8. a)



b)

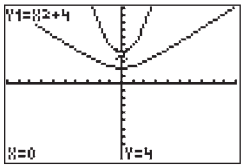


c)



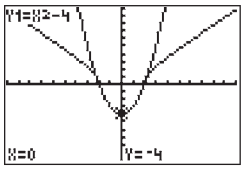
9. a) and **b)**

i)



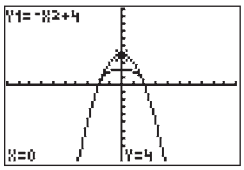
For $y = x^2 + 4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 4, y \in R\}$

ii)



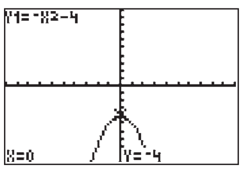
For $y = x^2 - 4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -4, y \in R\}$

iii)



For $y = -x^2 + 4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \leq 4, y \in R\}$

iv)



For $y = -x^2 - 4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \leq -4, y \in R\}$.

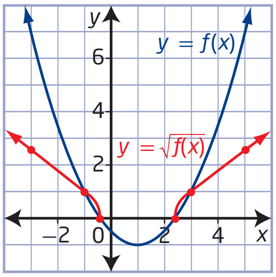
c) The graph of $y = \sqrt{j(x)}$ does not exist because all of the points on the graph $y = j(x)$ are below the x-axis. Since all values of $j(x) < 0$, then $\sqrt{j(x)}$ is undefined and produces no graph in the real number system.

d) The domains of the square root of a function are the same as the domains of the function when the value of the function ≥ 0. The domains of the square root of a function do not exist when the value of the function < 0. The ranges of the square root of a function are the square root of the range of the original function, except when the value of the function < 0 then the range is undefined.

10. a) For $y = x^2 - 4$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$; for $y = \sqrt{x^2 - 4}$, domain $\{x \mid x \leq -2 \text{ and } x \geq 2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

b) The value of y in the interval $(-2, 2)$ is negative therefore the domain of $y = \sqrt{x^2 - 4}$ is undefined and has no values in the interval $(-2, 2)$.

11. a)



I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.

b) For $y = f(x)$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -1, y \in \mathrm{R}\}$; for $y = \sqrt{f(x)}$, domain $\{x \mid x \leq -0.4 \text{ and } x \geq 2.4, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

The domain of $y = \sqrt{f(x)}$, consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$, and the range of $y = \sqrt{f(x)}$, consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.

12. a) $d = \sqrt{h^2 + 12756h}$

b) domain $\{h \mid h \geq 0, h \in \mathrm{R}\}$, range $\{d \mid d \geq 0, d \in \mathrm{R}\}$

c) Find the point of intersection between the graph of the function and $h = 800$. The distance will be expressed as the d value of the ordered pair (h, d). In this case, d is approximately equal to 3293.

d) Yes, if h could be any real number then the domain is $\{h \mid h \leq -12\,756 \text{ or } h \geq 0, h \in \mathrm{R}\}$ and the range would remain the same- since all square root values must be greater than or equal to 0.

13. a) No, since $\sqrt{a}$, $a < 0$ is undefined, then $y = \sqrt{f(x)}$ will be undefined when $f(x) < 0$, but $f(x)$ represents values of the range not the domain as Chris stated.

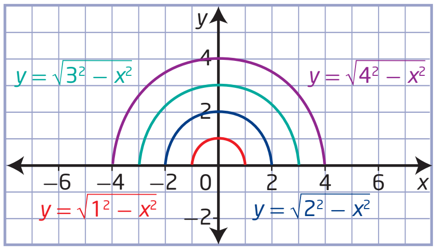
b) If the range consists of negative values, then you know that the graph represents $y = f(x)$ and not $y = \sqrt{f(x)}$.

14. a) $v = \sqrt{3.24 - h^2}$

b) domain $\{h \mid 0 \leq h \leq 1.8, h \in \mathrm{R}\}$, range $\{v \mid 0 \leq v \leq 1.8, v \in \mathrm{R}\}$ since both h and v represent distances.

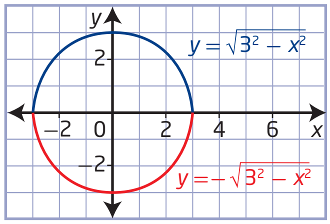
c) approximately 1.61 m

15. Step 1



Step 2 The parameter a determines the minimum value of the domain $(-a)$ and the maximum value of the domain (a); therefore the domain is $\{x \mid -a \leq x \leq a, x \in \mathrm{R}\}$. The parameter a also determines the maximum value of the range, where the minimum value of the range is 0; therefore the range is $\{y \mid 0 \leq y \leq a, y \in \mathrm{R}\}$.

Step 3 Example: $y = \sqrt{3^2 - x^2}$ the reflection of the graph in the x-axis is the equation $y = -\sqrt{3^2 - x^2}$.

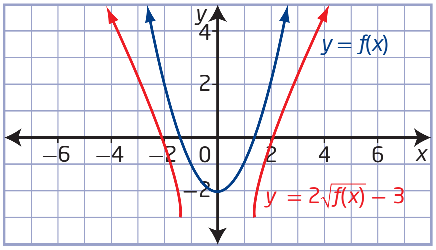


The graph forms a circle.

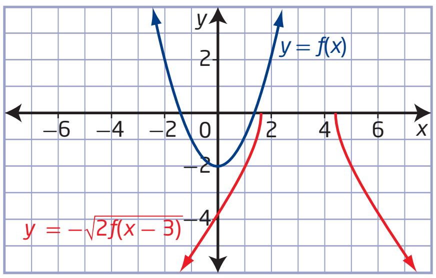
16. a) $(-27, 4\sqrt{3})$ **b)** $(-6, 12 - 2\sqrt{3})$

c) $(26, 6 - 4\sqrt{3})$

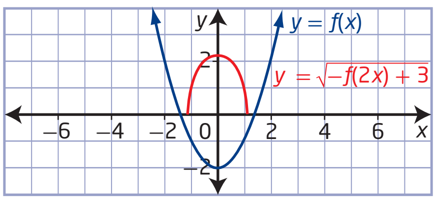
17. a)



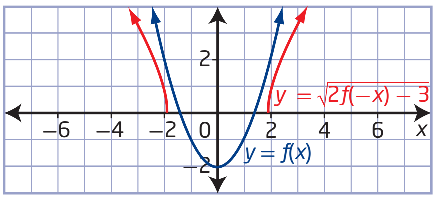
b)



c)



d)



18. Example: Sketch the graph in the following order:

1) $y = 2f(x)$ — Stretch vertically by a factor of 2.
2) $y = 2f(x - 3)$ — Translate horizontally 3 units right.
3) $y = \sqrt{2f(x - 3)}$ — Plot invariant points and sketch a smooth curve above the x-axis.
4) $y = -\sqrt{2f(x - 3)}$ — Reflect $y = \sqrt{2f(x - 3)}$ in the x-axis.

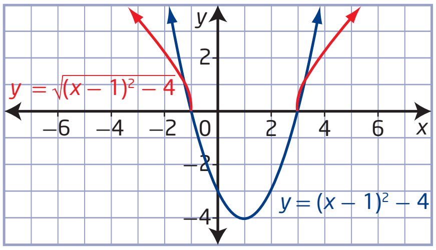
19. a) $r = \sqrt{\dfrac{A}{6\pi}}$ **b)** $r = \sqrt{\dfrac{A}{\pi(1 + \sqrt{37})}}$

C1 Example: Choose 4 to 5 key points on the graph of $y = f(x)$. Transform the points $(x, y) \rightarrow (x, \sqrt{y})$. Plot the new points and smooth out the graph. If you cannot get an idea of the general shape of the graph, choose more points to graph.

C2 The graph of $y = 16 - 4x$ is a linear function spanning from quadrant II to quadrant IV with an x-intercept of 4 and a y-intercept of 16. The graph of $y = \sqrt{16 - 4x}$ only exists when the graph of $y = 16 - 4x$ is on or above the x-axis. The y-intercept is at $\sqrt{16} = 4$ while the x-intercept stays the same. x-values for $x \leq 4$ are the same for both functions and the y-values for $y = \sqrt{16 - 4x}$ are the square root of y values for $y = 16 - 4x$.

C3 No, it is not possible, because the graph of $y = f(x)$ may exist when $y < 0$ but the graph of $y = \sqrt{f(x)}$ does not exist when $y < 0$.

C4 a)

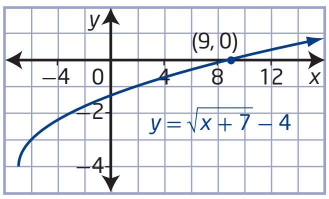


b) The graph of $y = (x - 1)^2 - 4$ is a quadratic function with a vertex of $(1, -4)$, y-intercept of -3, and x-intercepts of -1 and 3. It is above the x-axis when $x > 3$ and $x < -1$.
The graph of $y = \sqrt{(x - 1)^2 - 4}$ has the same x-intercepts but no y-intercept. The graph only exists when $x > 3$ and $x < -1$.

2.3 Solving Radical Equations Graphically, pages 96 to 98

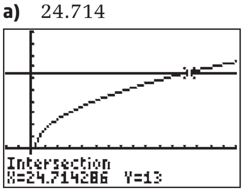
1. a) B **b)** A **c)** D **d)** C

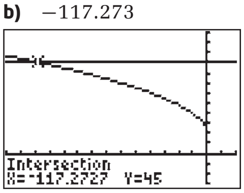
2. a) $x = 9$ **b)**



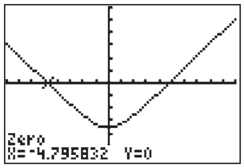
c) The roots of the equation are the same as the x-intercept on the graph.

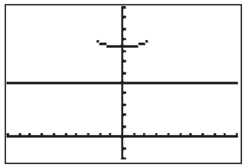
3. a) 24.714 **b)** -117.273



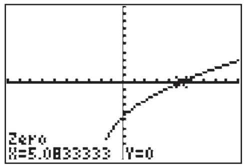


c) ± 4.796 **d)** no solution

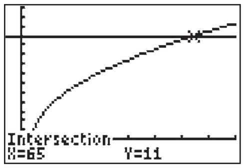


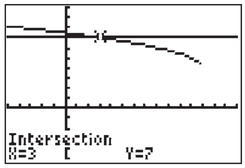


4. a) $x = 5.083$ **b)**

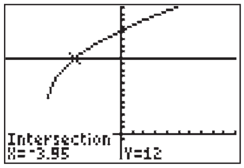


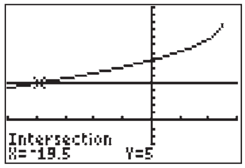
5. a) $x = 65$, $x \geq \dfrac{9}{2}$ **b)** $x = 3$, $x \leq 12$





c) $x = -3.95$, $x \geq -6.4$ **d)** $x = -19.5$, $x \leq 12.5$





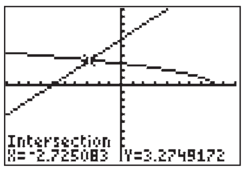
6. a) $x = \dfrac{7}{2}$, $x = -1$

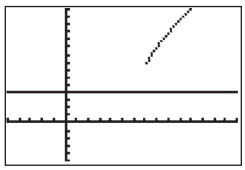
b) $x = 8$, $x = -2$, $x \leq -\dfrac{\sqrt{14}}{2}$ or $x \geq \dfrac{\sqrt{14}}{2}$

c) $x = 1.8$, $x = -1$, $-\dfrac{\sqrt{13}}{2} \leq x \leq \dfrac{\sqrt{13}}{2}$

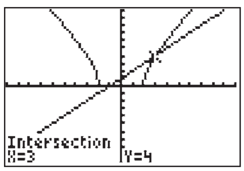
d) $x = 0$, $x = 2$, $\dfrac{-3\sqrt{2}}{2} \leq x \leq \dfrac{3\sqrt{2}}{2}$

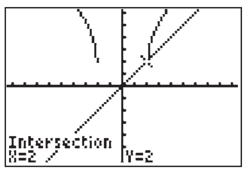
7. a) $x \approx -2.725$, $x \leq 8$ **b)** no real roots, $x \geq 7$



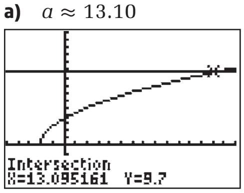


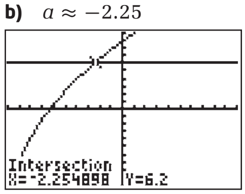
c) $x = 3$, $x \geq \dfrac{\sqrt{33}}{3}$ or $x \leq -\dfrac{\sqrt{33}}{3}$ **d)** $x = 2$, $x \geq 2$ or $x \leq -2$



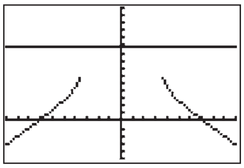


8. a) $a \approx 13.10$ **b)** $a \approx -2.25$

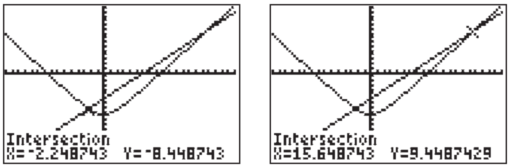




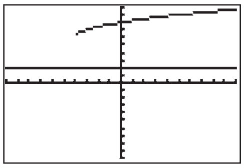
c) no solution



d) $a \approx -2.25$, $a \approx 15.65$



9. a)
$$6 + \sqrt{x + 4} = 2$$
$$\sqrt{x + 4} = -4$$
$$x + 4 = 16$$
$$x = 12$$



Left Side $= 6 + \sqrt{12 + 4}$
$= 6 + \sqrt{16}$
$= 6 + 4$
$= 10$
Right Side $= 2$
Left Side $\neq$ Right Side
Since $10 \neq 2$, there is no solution.

b) Yes, if you isolate the radical expression like $\sqrt{x + 4} = -4$, if the radical is equal to a negative value then there is no solution.

10. Greg $\rightarrow N(t) = 1.3\sqrt{t} + 4.2 = 1.3\sqrt{6} + 4.2$
≈ 7.38 million,
Yolanda $\rightarrow N(t) = 1.3\sqrt{t} + 4.2 = 1.3\sqrt{1.5} + 4.2$
≈ 5.79 million
Greg is correct, it will take more than 6 years for the entire population to be affected.

11. approximately 99 cm

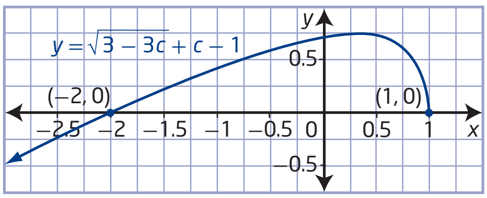
12. a) Yes **b)** 3000 kg

13. No, $\sqrt{x^2} = 9$ has two possible solutions ± 9, whereas $(\sqrt{x})^2 = 9$ has only one solution $+9$.

14. $x = \dfrac{3 + \sqrt{5}}{2}$

15. a) 5 m/s **b)** 75.2 kg

16. $c = -2$ or 1



If the function $y = \sqrt{-3(x + c)} + c$ passes through the point (0.25, 0.75), what is the value of c?

17. Lengths of sides are 55.3 cm, 60 cm, and 110.6 cm or 30.7 cm, 60 cm, and 61.4 cm.

C1 The x-intercepts of the graph of a function are the solutions to the corresponding equation. Example: A graph of the function $y = \sqrt{x - 1} - 2$ would show that the x-intercept has a value of 5. The equation that corresponds to this function is $0 = \sqrt{x - 1} - 2$ and the solution to the equation is 5.

C2 a) $s = \sqrt{9.8d}$ where s represents speed in metres per second and d represents depth in metres.

b)
$$s = \sqrt{9.8d}$$
$$s = \sqrt{(9.8 \text{ m/s}^2)(2500 \text{ m})}$$
$$s = \sqrt{24\,500 \text{ m}^2/\text{s}^2}$$
$$s \approx 156.5 \text{ m/s}$$

c) approximately 4081.6 m

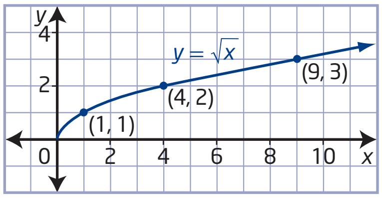
d) Example: I prefer the algebraic method because it is faster and I do not have to adjust window settings.

C3 Radical equations only have a solution in the real number system if the graph of the corresponding function has an x-intercept. For example, $y = \sqrt{x} + 4$ has no real solutions because there is no x-intercept.

C4 Extraneous roots occur when solving equations algebraically. Extraneous roots of a radical equation may occur anytime an expression is squared. For example, $x^2 = 1$ has two possible solutions, $x = \pm 1$. You can identify extraneous roots by graphing and by substituting into the original equation.

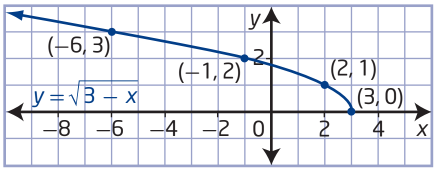
Chapter 2 Review, pages 99 to 101

1. a)



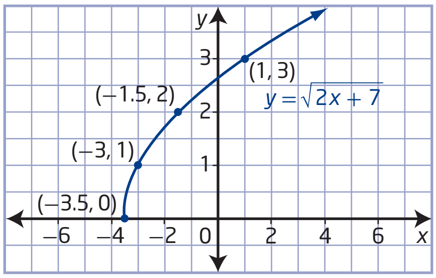
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$ All values in the table lie on the smooth curve graph of $y = \sqrt{x}$.

b)



domain $\{x \mid x \leq 3, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$ All points in the table lie on the graph of $y = \sqrt{3 - x}$.

c)



domain $\{x \mid x \geq -3.5, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$ All points in the table lie on the graph of $y = \sqrt{2x + 7}$.

2. Use $y = a\sqrt{b(x - h)} + k$ to describe transformations.

a) $a = 5 \rightarrow$ vertical stretch factor of 5
$h = -20 \rightarrow$ horizontal translation left 20 units;
domain $\{x \mid x \geq -20, x \in \mathrm{R}\}$; range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) $b = -2 \rightarrow$ horizontal stretch factor of $\frac{1}{2}$, then reflected on y-axis: $k = -8 \rightarrow$ vertical translation of 8 units down.
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$; range $\{y \mid y \geq -8, y \in \mathrm{R}\}$

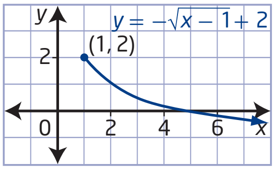
c) $a = -1 \rightarrow$ reflect in x-axis
$b = \frac{1}{6} \rightarrow$ horizontal stretch factor of 6
$h = 11 \rightarrow$ horizontal translation right 11 units; domain $\{x \mid x \geq 11, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

3. a) $y = \sqrt{\frac{1}{10}x} + 12$, domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 12, y \in \mathrm{R}\}$

b) $y = -2.5\sqrt{x + 9}$
domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

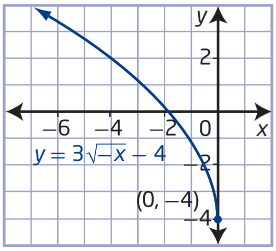
c) $y = \frac{1}{20}\sqrt{-\frac{2}{5}(x - 7)} - 3$,
domain $\{x \mid x \leq 7, x \in \mathrm{R}\}$, range $\{y \mid y \geq -3, y \in \mathrm{R}\}$

4. a)



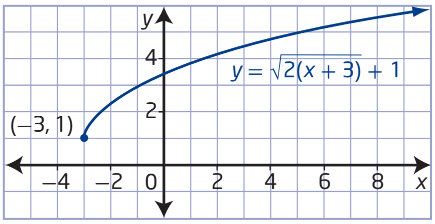
domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \leq 2, y \in \mathrm{R}\}$

b)



domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$

c)

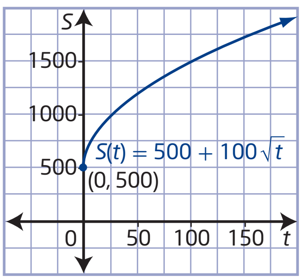


domain $\{x \mid x \geq -3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$

5. The domain is affected by a horizontal translation of 4 units right and by no reflection on the y-axis. The domain will have values of x greater than or equal to 4, due to a translation of the graph 4 units right. The range is affected by vertical translation of 9 units up and a reflection on the x-axis. The range will be less than or equal to 9, because the graph has been moved up 9 units and reflected on the x-axis, therefore the range is less than or equal to 9, instead of greater than or equal to 9.

6. a) Given the general equation $y = a\sqrt{b(x - h)} + k$ to describe transformations, $a = 100$ indicates a vertical stretch by a factor of 100, $k = 500$ indicates a vertical translation up 500 units.

b)



Since the minimum value of the graph is 500, the minimum estimated sales will be 500 units.

c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$ The domain means that time is positive in this situation.
range $\{S(t) \mid S(t) \geq 500, S(t) \in \mathrm{W}\}$. The range means that the minimum sales are 500 units.

d) about 1274 units

7. a) $y = \sqrt{\frac{1}{4}(x + 3)} + 2$ **b)** $y = -2\sqrt{x + 4} + 3$

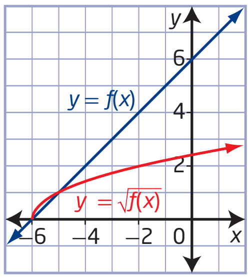
c) $y = 4\sqrt{-(x - 6)} - 4$

8. a) For $y = x - 2$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for $y = \sqrt{x - 2}$, domain $\{x \mid x \geq 2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$. The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

b) For $y = 10 - x$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for $y = \sqrt{10 - x}$, domain $\{x \mid x \leq 10, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$ The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

c) For $y = 4x + 11$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for $y = \sqrt{4x + 11}$, domain $\left\{x \mid x \geq -\frac{11}{4}, x \in \mathrm{R}\right\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$. The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

9. a)



Plot invariant points at the intersection of the graph and lines $y = 0$ and $y = 1$. Plot any points $(x, \sqrt{y})$ where the value of y is a perfect square. Sketch a smooth curve through the invariant points and points satisfying $(x, \sqrt{y})$.

b) $y = \sqrt{f(x)}$ is positive when $f(x) > 0$,
$y = \sqrt{f(x)}$ does not exist when $f(x) < 0$.
$\sqrt{f(x)} > f(x)$ when $0 < f(x) < 1$ and
$f(x) > \sqrt{f(x)}$ when $f(x) > 1$

c) For $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for $\sqrt{f(x)}$, domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$, since $\sqrt{f(x)}$ is undefined when $f(x) < 0$.

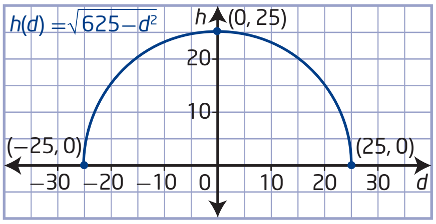
10. a) $y = 4 - x^2 \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \leq 4, y \in \mathrm{R}\}$ for $y = \sqrt{4 - x^2} \rightarrow$ domain $\{x \mid -2 \leq x \leq 2, x \in \mathrm{R}\}$, range $\{y \mid 0 \leq y \leq 2, y \in \mathrm{R}\}$, since $4 - x^2 > 0$ only between -2 and 2 then the domain of $y = \sqrt{4 - x^2}$ is $-2 \leq x \leq 2$. In the domain of $-2 \leq x \leq 2$ the maximum value of $y = 4 - x^2$ is 4, so the maximum value of $y = \sqrt{4 - x^2}$ is $\sqrt{4} = 2$ then the range of the function $y = \sqrt{4 - x^2}$ will be $0 \leq y \leq 2$.

b) $y = 2x^2 + 24 \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 24, y \in \mathrm{R}\}$ for $y = \sqrt{2x^2 + 24} \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq \sqrt{24}, y \in \mathrm{R}\}$. The domain does not change since the entire graph of $y = 2x^2 + 24$ is above the x-axis. The range changes since the entire graph moves up 24 units and the graph itself opens up, so the range becomes $y \geq \sqrt{24}$.

c) $y = x^2 - 6x \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -9, y \in \mathrm{R}\}$ for $y = \sqrt{x^2 - 6x} \rightarrow$ domain $\{x \mid x \leq 0 \text{ or } x \geq 6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$, since $x^2 - 6x < 0$ between 0 and 6, then the domain is undefined in the interval (0, 6) and exists when $x \leq 0$ or $x \geq 6$. The range changes because the function only exists above the x-axis.

11. a) $h(d) = \sqrt{625 - d^2}$

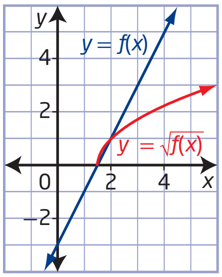
b)



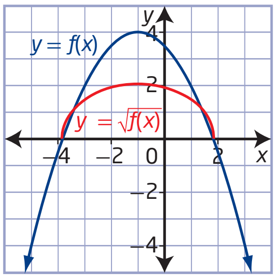
domain $\{d \mid -25 \leq d \leq 25, d \in \mathrm{R}\}$
range $\{h \mid 0 \leq h \leq 25, h \in \mathrm{R}\}$

c) In this situation, the values of h and d must be positive to express a positive distance. Therefore the domain changes to $\{d \mid 0 \leq d \leq 25, d \in \mathrm{R}\}$. Since the range of the original function $h(d) = \sqrt{625 - d^2}$ is always positive then the range does not change.

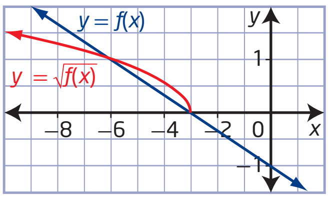
12. a)



b)

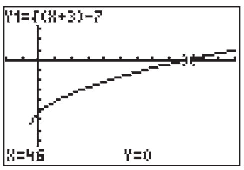


c)



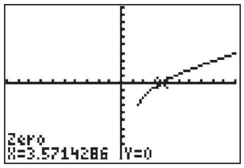
13. a) $x = 46$

b)

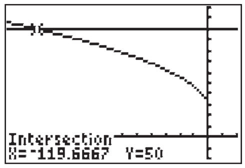


c) The root of the equation and the x value of the x-intercept are the same.

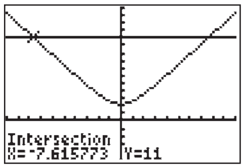
14. a) $x \approx 3.571$

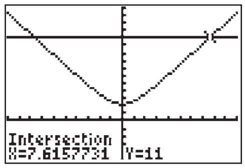


b) $x \approx -119.667$



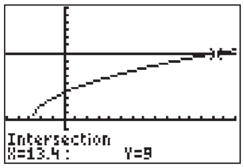
c) $x \approx -7.616$ and $x \approx 7.616$



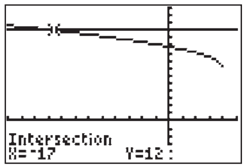


15. 4.13 m

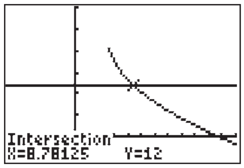
16. a) $x = 13.4$



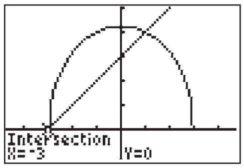
b) $x = -17$

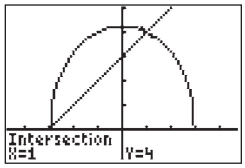


c) $x \approx 8.781$



d) $x = -3$ and 1





17. a) Jaime found two possible answers which are determined by solving a quadratic equation.

b) Carly found only one intersection at (5, 5) or x-intercept (5, 0) determined by possibly graphing.

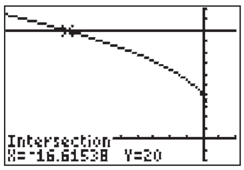
c) Atid found an extraneous root of $x = 2$.

18. a) 130 m^2 **b)** 6 m

Chapter 2 Practice Test, pages 102 to 103

1. B **2.** A **3.** A **4.** C **5.** D **6.** B

7. $x \approx -16.62$

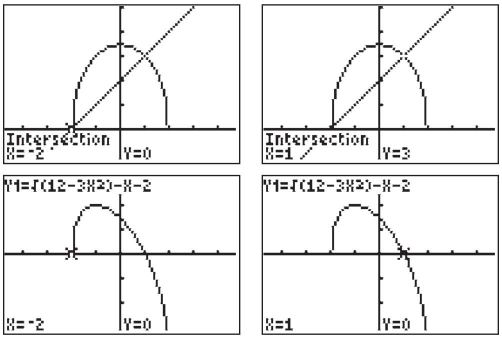


8. $y = 4\sqrt{x}$ or $y = \sqrt{16x}$

9. For $y = 7 - x \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$. Since $y = \sqrt{7 - x}$ is the square root of the y-values for the function $y = 7 - x$, then the domain and ranges of $y = \sqrt{7 - x}$ will differ. Since $7 - x < 0$ when $x > 7$, then the domain of $y = \sqrt{7 - x}$ will be $\{x \mid x \leq 7, x \in \mathrm{R}\}$ and since $\sqrt{7 - x}$ indicates positive values only, then the range of $y = \sqrt{7 - x}$ is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

10. The domain of $y = f(x)$ is $\{x \mid x \in R\}$, and the range of $y = f(x)$ is $\{y \mid y \leq 8, y \in R\}$. The domain of $y = \sqrt{f(x)}$ is $\{x \mid -2 \leq x \leq 2, x \in R\}$ and the range of $y = \sqrt{f(x)}$ is $\{y \mid 0 \leq y \leq \sqrt{8}, y \in R\}$.

11.

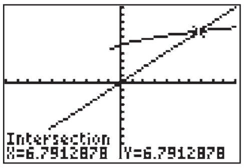


$x = -2, x = 1$

12.
$$\begin{aligned} 4 + \sqrt{x+1} &= x \\ \sqrt{x+1} &= x - 4 \\ x + 1 &= (x-4)^2 \\ x + 1 &= x^2 - 8x + 16 \\ 0 &= x^2 - 9x + 15 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(15)}}{2(1)}$$
$$\approx 2.2 \text{ or } 6.8$$

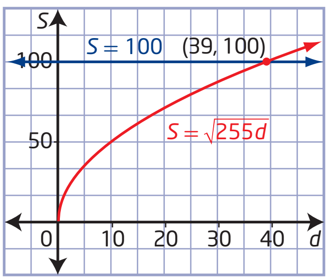
By checking, 2.2 is an extraneous root, therefore $x \approx 6.8$.



$x \approx 6.8$

13. a) Given the general equation $y = a\sqrt{b(x-h)} + k$ to describe transformations, $b = 255 \rightarrow$ indicating a horizontal stretch by a factor of $\frac{1}{255}$. To sketch the graph of $S = \sqrt{255d}$, graph the function $S = \sqrt{d}$ and apply a horizontal stretch of $\frac{1}{255}$, every point on the graph of $S = \sqrt{d}$ will become $\left(\frac{d}{255}, S\right)$.

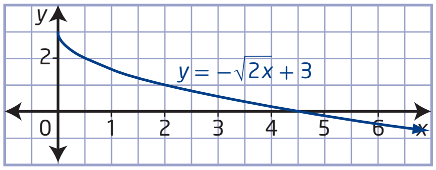
b)



$d \approx 39$ m
The skid mark of the vehicle will be approximately 39 m.

14. a) Given the general equation $y = a\sqrt{b(x-h)} + k$ to describe transformations, $a = -1 \rightarrow$ reflection of the graph in the x-axis, $b = 2 \rightarrow$ horizontal stretch by a factor of $\frac{1}{2}$, $k = 3 \rightarrow$ vertical translation up 3 units.

b)

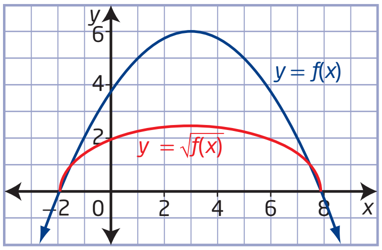


c) domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \leq 3, y \in R\}$.

d) The domain remains the same because there was no horizontal translation or reflection on the y-axis. But since the graph was reflected on the x-axis and moved up 3 units and then the range becomes $y \leq 3$.

e) The equation $5 + \sqrt{2x} = 8$ can be rewritten as $0 = -\sqrt{2x} + 3$. Therefore the x-intercept of the graph $y = -\sqrt{2x} + 3$ is the solution of the equation $5 + \sqrt{2x} = 8$.

15.



Step 1 Plot invariant points at the intersection of $y = f(x)$ and functions $y = 0$ and $y = 1$.

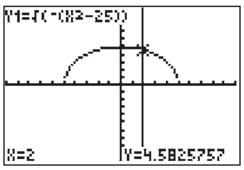
Step 2 Plot points at $\sqrt{\text{max value}}$ and $\sqrt{\text{perfect square value of } y = f(x)}$

Step 3 Join all points with a smooth curve, remember that the graph of $y = \sqrt{f(x)}$ is above the original graph for the interval $0 \leq y \leq 1$. Note that for the interval where $f(x) < 0$, the function $y = \sqrt{f(x)}$ is undefined and has no graph.

16. a) $y = (\sqrt{5})\sqrt{-(x-5)}$

b) domain $\{x \mid 0 \leq x \leq 5, x \in R\}$, range $\{y \mid 0 \leq y \leq 5, y \in R\}$
Domain: x cannot be negative nor greater than half the diameter of the base, or 5. Range: y cannot be negative nor greater than the height of the roof, or 5.

c)



The height of the roof 2 m from the centre is about 4.58 m.

Chapter 3 Polynomial Functions

3.1 Characteristics of Polynomial Functions, pages 114 to 117

1. a) No, this is a square root function.
b) Yes, this is a polynomial function of degree 1.
c) No, this is an exponential function.
d) Yes, this is a polynomial function of degree 4.
e) No, this function has a variable with a negative exponent.
f) Yes, this is a polynomial function of degree 3.

2. a) degree 1, linear, −1, 3
b) degree 2, quadratic, 9, 0
c) degree 4, quartic, 3, 1
d) degree 3, cubic, −3, 4
e) degree 5, quintic, −2, 9
f) degree 0, constant, 0, −6

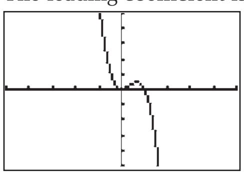
3. a) odd degree, positive leading coefficient, 3 x-intercepts, domain $\{x \mid x \in R\}$ and range $\{y \mid y \in R\}$
b) odd degree, positive leading coefficient, 5 x-intercepts, domain $\{x \mid x \in R\}$ and range $\{y \mid y \in R\}$
c) even degree, negative leading coefficient, 3 x-intercepts, domain $\{x \mid x \in R\}$ and range $\{y \mid y \leq 16.9, y \in R\}$
d) even degree, negative leading coefficient, 0 x-intercepts, domain $\{x \mid x \in R\}$ and range $\{y \mid y \leq -3, y \in R\}$

4. a) degree 2 with positive leading coefficient, parabola opens upward, maximum of 2 x-intercepts, y-intercept of −1
b) degree 3 with negative leading coefficient, extends from quadrant II to IV, maximum of 3 x-intercept, y-intercept of 5
c) degree 4 with negative leading coefficient, opens downward, maximum of 4 x-intercepts, y-intercept of 4
d) degree 5 with positive leading coefficient, extends from quadrant III to I, maximum of 5 x-intercepts, y-intercept of 0
e) degree 1 with negative leading coefficient, extends from quadrant II to IV, 1 x-intercept, y-intercept of 4
f) degree 4 with positive leading coefficient, opens upward, maximum of 4 x-intercepts, y-intercept of 0

5. Example: Jake is right as long as the leading coefficient a is a positive integer. The simplest example would be a quadratic function with $a = 2$, $b = 2$, and $n = 2$.

6. a) degree 4
b) The leading coefficient is 1 and the constant is −3000. The constant represents the initial cost.
c) degree 4 with a positive leading coefficient, opens upward, 2 x-intercepts, y-intercept of −3000
d) The domain is $\{x \mid x \geq 0, x \in R\}$, since it is impossible to have negative snowboard sales.
e) The positive x-intercept is the breakeven point.
f) Let $x = 15$, then $P(x) = 62\ 625$.

7. a) cubic function
b) The leading coefficient is −3 and the constant is 0.
c)

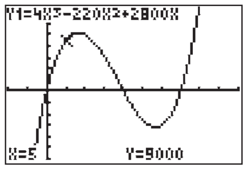


d) The domain is $\{d \mid 0 \leq d \leq 1, d \in R\}$ because you cannot give negative drug amounts and you must have positive reaction times.

8. a) For 1 ring, the total number of hexagons is given by $f(1) = 1$. For 2 rings, the total number of hexagons is given by $f(2) = 7$. For 3 rings, the total number of hexagons is given by $f(3) = 19$.
b) 397 hexagons

9. a) End behaviour: the curve extends up in quadrants I and II; domain $\{t \mid t \in R\}$; range $\{P \mid P \geq 10\ 071, P \in R\}$; the range for the period $\{t \mid 0 \leq t \leq 20, t \in R\}$ that the population model can be used is $\{P \mid 15\ 000 \leq P \leq 37\ 000, P \in R\}$. t-intercepts: none; P-intercept: 15 000
b) 15 000 people **c)** 18 000 people **d)** 18 years

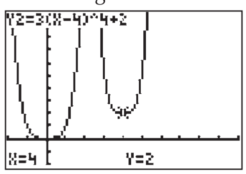
10. a)



From the graph, the height of a single box must be greater than 0 and cannot be between 20 cm and 35 cm.
b) $V(x) = 4x(x - 20)(x - 35)$. The factored form clearly shows the three possible x-intercepts.

11. a) The graphs in each pair are the same. Let n represent a whole number, then $2n$ represents an even whole number.
$y = (-x)^{2n}$
$y = (-1)^{2n}x^{2n}$
$y = 1^{n}x^{2n}$
$y = x^{2n}$
b) The graphs in each pair are reflections of each other in the y-axis. Let n represent a whole number, then $2n + 1$ represents an odd whole number.
$y = (-x)^{2n+1}$
$y = (-1)^{2n+1}x^{2n+1}$
$y = (-1)^{2n}(-1)^{1}x^{2n+1}$
$y = -(1)^{n}x^{2n+1}$
$y = -x^{2n+1}$
c) For even whole numbers, the graph of the functions are unchanged. For odd whole numbers, the graph of the functions are reflected in the y-axis.

12. a) vertical stretch by a factor of 3 and translation of 4 units right and 2 units up
b) vertical stretch by a factor of 3 and translation of 4 units right and 2 units up
c)



13. If there is only one root, $y = (x - a)^n$, then the function will only cross the x-axis once in the case of an odd-degree function and it will only touch the x-axis once if it is an even-degree function.

C1 Example: Odd degree: At least one x-intercept and up to a maximum of n x-intercepts, where n is the degree of the function. No maximum or minimum points. Domain is $\{x \mid x \in R\}$ and range is $\{y \mid y \in R\}$.
Even degree: From zero to a maximum of n x-intercepts, where n is the degree of the function. Domain is $\{x \mid x \in R\}$ and the range depends on the maximum or minimum value of the function.

C2 a) Examples:
i) $y = x^3$ **ii)** $y = x^2$
iii) $y = -x^3$ **iv)** $y = -x^2$

b) Example: Parts i) and ii) have positive leading coefficients, while parts iii) and iv) have negative leading coefficients. Parts i) and iii) are odd-degree functions, while parts ii) and iv) are even-degree functions.

C3 Example: The line $y = x$ and polynomial functions with odd degree greater than one and positive leading coefficient extend from quadrant III to quadrant I. Both have no maximums or minimums. Both have the same domain and range. Odd degree polynomial functions have at least one x-intercept.

C4 **Step 1**

Function	Degree	End Behaviour
$y = x + 2$	1	extends from quadrant III to I
$y = -3x + 1$	1	extends from quadrant II to IV
$y = x^2 - 4$	2	opens upward
$y = -2x^2 - 2x + 4$	2	opens downward
$y = x^3 - 4x$	3	extends from quadrant III to I
$y = -x^3 + 3x - 2$	3	extends from quadrant II to IV
$y = 2x^3 + 16$	3	extends from quadrant III to I
$y = -x^3 - 4x$	3	extends from quadrant II to IV
$y = x^4 - 4x^2 + 5$	4	opens upward
$y = -x^4 + x^3 + 4x^2 - 4x$	4	opens downward
$y = x^4 + 2x^2 + 1$	4	opens upward
$y = x^5 - 2x^4 - 3x^3 + 5x^2 + 4x - 1$	5	extends from quadrant III to I
$y = x^5 - 1$	5	extends from quadrant III to I
$y = -x^5 + x^4 + 8x^3 + 8x^2 - 16x - 16$	5	extends from quadrant II to IV
$y = x(x + 1)^2(x + 4)^2$	5	extends from quadrant III to I

Step 2 The leading coefficient determines if it opens upward or downward; in the case of odd functions it determines if it is increasing or decreasing.
Step 3 Always have at least one minimum or maximum. Not all functions will have the same range. Either opens upward or downward.
Step 4. Always have the same domain and range. Either extends from quadrant III to I or from quadrant II to IV. No maximum or minimum.

3.2 Remainder Theorem, pages 124 to 125

1. a) $\dfrac{x^2 + 10x - 24}{x - 2} = x + 12$
b) $x \neq 2$ **c)** $(x - 2)(x + 12)$
d) Multiplying the statement in part c) yields $x^2 + 10x - 24$.

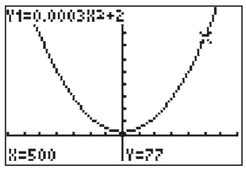
2. a) $\dfrac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{x + 1}$
$= 3x^3 - 7x^2 + x + 16 - \dfrac{24}{x + 1}$
b) $x \neq -1$
c) $(x + 1)(3x^3 - 7x^2 + x + 16) - 24$
d) Expanding the statement in part c) yields $3x^4 - 4x^3 - 6x^2 + 17x - 8$.

3. a) $Q(x) = x^2 + 4x + 1$ **b)** $Q(x) = x^2 + 4x + 1$
c) $Q(w) = 2w^2 - 3w + 4$ **d)** $Q(m) = 9m^2 + 3m + 6$
e) $Q(t) = t^3 + 5t^2 - 8t + 7$
f) $Q(y) = 2y^3 + 6y^2 + 15y + 45$

4. a) $Q(x) = x^2 - 3x + 12$ **b)** $Q(m) = m^3 + m + 14$
c) $Q(x) = -x^3 + x^2 - x + 1$ **d)** $Q(s) = 2s^2 + 7s + 5$
e) $Q(h) = h^2 - h$ **f)** $Q(x) = 2x^2 + 3x - 7$

5. a) $\dfrac{x^3 + 7x^2 - 3x + 4}{x + 2} = x^2 + 5x - 13 + \dfrac{30}{x + 2}, x \neq -2$
b) $\dfrac{11t - 4t^4 - 7}{t - 3}$
$= -4t^3 - 12t^2 - 36t - 97 - \dfrac{298}{t - 3}, t \neq 3$
c) $\dfrac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \dfrac{9}{x + 1}, x \neq -1$
d) $\dfrac{4n^3 + 7n - 5}{n + 3} = 4n - 5 + \dfrac{10}{n + 3}, n \neq -3$
e) $\dfrac{4n^3 - 15n + 2}{n - 3} = 4n^2 + 12n + 21 + \dfrac{65}{n - 3}, n \neq 3$
f) $\dfrac{x^3 + 6x^2 - 4x + 1}{x + 2} = x^2 + 4x - 12 + \dfrac{25}{x + 2}, x \neq -2$

6. a) 16 **b)** 38 **c)** −23
d) −67 **e)** −2 **f)** 8
7. a) 9 **b)** −40 **c)** 41 **d)** −4
8. a) −1 **b)** 3 **c)** 2 **d)** −1
9. 11
10. 4 and −2
11. a) $2x + 3$
b) 9, it represents the rest of the width that cannot be simplified any more.
12. a) $2n + 2 + \dfrac{9}{n - 3}$ **b)** −2 and −0.5
13. a) $9\pi x^2 + 24\pi x + 16\pi$, represents the area of the base
b) $\pi(3x + 4)^2(x + 3)$
c) 10 cm $\leq r \leq$ 28 cm and $5 \leq h \leq 11$
14. $m = -\dfrac{11}{5}, n = \dfrac{59}{5}$
15. $a = -\dfrac{14}{3}, b = -\dfrac{2}{3}$
16. Divide using the binomial $x - \dfrac{3}{2}$.
17. Examples: **a)** $x^2 - 4x - 1$
b) $x^3 + 3x^2 + 3x + 6$ **c)** $2x^4 + x^3 + x^2 + x$
C1 Example: The process is the same. Long division of polynomials results in a restriction.
C2 a) $(x - a)$ is a factor of $bx^2 + cx + d$.
b) $d + ac + a^2b$
C3 a) 77 **b)** 77
c)



The remainder is the height of the cable at the given horizontal distance.

3.3 The Factor Theorem, pages 133 to 135

1. a) $x - 1$ **b)** $x + 3$ **c)** $x - 4$ **d)** $x - a$
2. a) Yes **b)** No **c)** No **d)** Yes
e) Yes **f)** No
3. a) No **b)** No **c)** No **d)** No
e) Yes **f)** No
4. a) ±1, ±2, ±4, ±8 **b)** ±1, ±2, ±3, ±6, ±9, ±18
c) ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24
d) ±1, ±2, ±4 **e)** ±1, ±3, ±5, ±15
f) ±1, ±2, ±4
5. a) $(x - 1)(x - 2)(x - 3)$ **b)** $(x - 1)(x + 1)(x + 2)$
c) $(v - 4)(v + 4)(v + 1)$
d) $(x + 4)(x + 2)(x - 3)(x + 1)$
e) $(k - 1)(k - 2)(k + 3)(k + 2)(k + 1)$

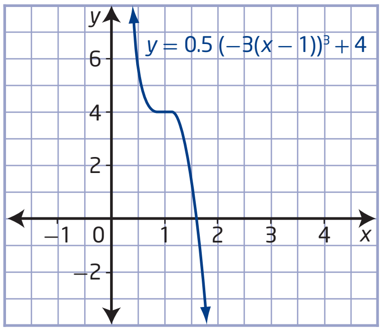
6. a) $(x + 3)(x - 2)(x - 3)$ **b)** $(t - 5)(t + 4)(t + 2)$
c) $(h - 5)(h^2 + 5h - 2)$ **d)** $x^5 + 8x^3 + 2x - 15$
e) $(q - 1)(q + 1)(q^2 + 2q + 3)$
7. a) $k = -2$ **b)** $k = 1, -7$
c) $k = -6$ **d)** $k = 6$
8. h, $h - 1$, and $h - 1$
9. $l - 5$ and $l + 3$
10. $x - 2$ cm, $x + 4$ cm, and $x + 3$ cm
11. $x + 5$ and $x + 3$
12. a) $x - 5$ is a possible factor because it is the corresponding factor for $x = 5$. Since $f(5) = 0$, $x - 5$ is a factor of the polynomial function.
b) 2-ft sections would be weak by the same principle applied in part a).
13. $x + 3$, $x + 2$, and $x + 1$
14. Synthetic division yields a remainder of $a + b + c + d + e$, which must equal 0 as given. Therefore, $x - 1$ is a possible divisor.
15. $m = -\frac{7}{10}$, $n = -\frac{51}{10}$
16. a) **i)** $(x - 1)(x^2 + x + 1)$ **ii)** $(x - 3)(x^2 + 3x + 9)$
iii) $(x + 1)(x^2 - x + 1)$ **iv)** $(x + 4)(x^2 - 4x + 16)$
b) $x + y$, $x^2 - xy + y^2$ **c)** $x - y$, $x^2 + xy + y^2$
d) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
C1 Example: Looking at the x-intercepts of the graph, you can determine at least one binomial factor, $x - 2$ or $x + 2$. The factored form of the polynomial is $(x - 2)(x + 2)(x^2 + 1)$.
C2 Example: Using the integral zero theorem, you have both ±1 and ±5 as possible integer values. The x-intercepts of the graph of the corresponding function will also give the factors.
C3 Example: Start by using the integral zero theorem to check for a first possible integer value. Apply the factor theorem using the value found from the integral zero theorem. Use synthetic division to confirm that the remainder is 0 and determine the remaining factor. Repeat the process until all factors are found.

3.4 Equations and Graphs of Polynomial Functions, pages 147 to 152

1. a) $x = -3, 0, 4$ **b)** $x = -1, 3, 5$ **c)** $x = -2, 3$
2. a) $x = -2, -1$ **b)** $x = 1$ **c)** $x = -4, -2$
3. a) $(x + 3)(x + 2)(x - 1) = 0$, roots are -3, -2 and 1
b) $-(x + 4)(x - 1)(x - 3) = 0$, roots are -4, 1 and 3
c) $-(x + 4)^2(x - 1)(x - 3) = 0$, roots are -4, 1 and 3
4. a) **i)** -4, -1, and 1
ii) positive for $-4 < x < -1$ and $x > 1$, negative for $x < -4$ and $-1 < x < 1$
iii) all three zeros are of multiplicity 1, the sign of the function changes
b) **i)** -1 and 4
ii) negative for all values of x, $x \neq -1, 4$
iii) both zeros are of multiplicity 2, the sign of the function does not change
c) **i)** -3 and 1
ii) positive for $x < -3$ and $x > 1$, negative for $-3 < x < 1$
iii) -3 (multiplicity 1) and 1 (multiplicity 3), at both the function changes sign but is flatter at $x = 1$
d) **i)** -1 and 3
ii) negative for $-1 < x < 3$ and $x > 3$, positive for $x < -1$
iii) -1 (multiplicity 3) and 3 (multiplicity 2), at $x = -1$ the function changes sign but not at $x = 3$
5. a) B **b)** D **c)** C **d)** A
6. a) $a = 0.5$ vertical stretch by a factor of 0.5, $b = -3$ horizontal stretch by a factor of $\frac{1}{3}$ and a reflection in the y-axis, $h = 1$ translation of 1 unit right, $k = 4$ translation of 4 units up
b)

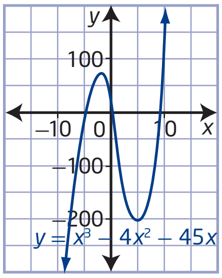
$y = x^3$	$y = (-3x)^3$	$y = 0.5(-3x)^3$	$y = 0.5(-3(x - 1))^3 + 4$
$(-2, -8)$	$\left(\frac{2}{3}, -8\right)$	$\left(\frac{2}{3}, -4\right)$	$\left(\frac{5}{3}, 0\right)$
$(-1, -1)$	$\left(\frac{1}{3}, -1\right)$	$\left(\frac{1}{3}, -\frac{1}{2}\right)$	$\left(\frac{4}{3}, \frac{7}{2}\right)$
$(0, 0)$	$(0, 0)$	$(0, 0)$	$(1, 4)$
$(1, 1)$	$\left(-\frac{1}{3}, 1\right)$	$\left(-\frac{1}{3}, \frac{1}{2}\right)$	$\left(\frac{2}{3}, \frac{9}{2}\right)$
$(2, 8)$	$\left(-\frac{2}{3}, 8\right)$	$\left(-\frac{2}{3}, 4\right)$	$\left(\frac{1}{3}, 8\right)$

c)



7. a) **i)** -5, 0, and 9
ii) degree 3 from quadrant III to I
iii) -5, 0, and 9 each of multiplicity 1
iv) 0
v) positive for $-5 < x < 0$ and $x > 9$, negative for $x < -5$ and $0 < x < 9$
b) **i)** -9, 0 and 9
ii) degree 4 opening upwards
iii) 0 (multiplicity 2), -9 and 9 each of multiplicity 1
iv) 0
v) positive for $x < -9$ and $x > 9$, negative for $-9 < x < 9$, $x \neq 0$
c) **i)** -3, -1, and 1
ii) degree 3 from quadrant III to I
iii) -3, -1, and 1 each of multiplicity 1
iv) -3
v) positive for $-3 < x < -1$ and $x > 1$, negative for $x < -3$ and $-1 < x < 1$
d) **i)** -3, -2, 1, and 2
ii) degree 4 opening downwards
iii) -3, -2, 1, and 2 each of multiplicity 1
iv) -12
v) positive for $-3 < x < -2$ and $1 < x < 2$, negative for $x < -3$ and $-2 < x < 1$ and $x > 2$

8. a)



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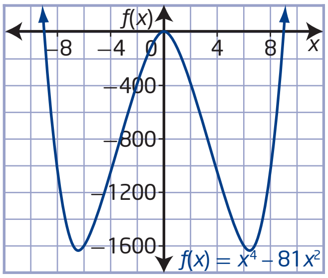

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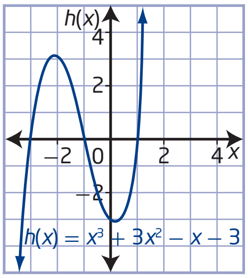
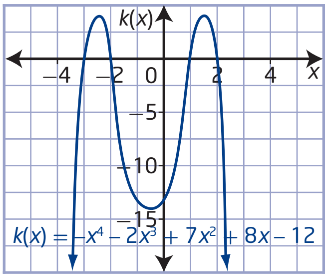
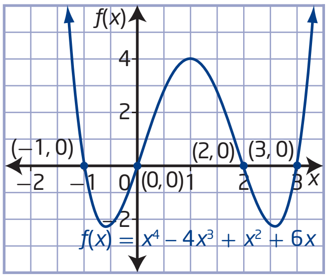


d)



9. a)



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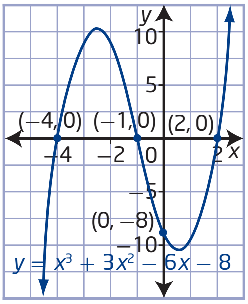

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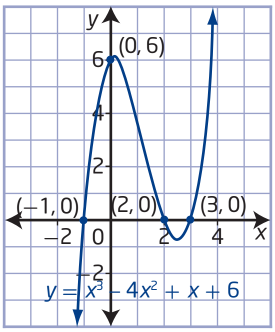

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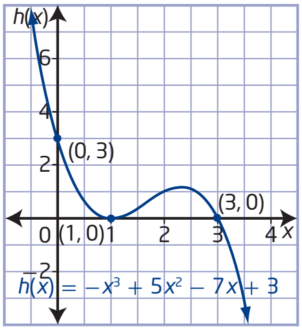
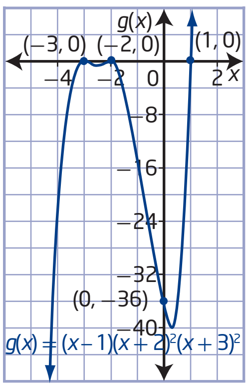
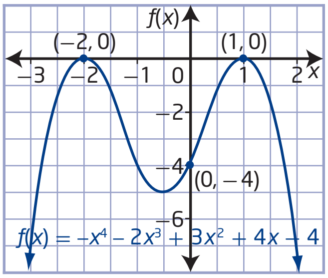


e)



f)



10. a) positive leading coefficient, x-intercepts: -2 and 3, positive for $-2 < x < 3$ and $x > 3$, negative for $x < -2$, $y = (x + 2)^3(x - 3)^2$

b) negative leading coefficient, x-intercepts: -4, -1, and 3, positive for $x < -4$ and $-1 < x < 3$, negative for $-4 < x < -1$ and $x > 3$, $y = -(x + 4)(x + 1)(x - 3)$

c) negative leading coefficient, x-intercepts: -2, -1, 2, and 3, positive for $-2 < x < -1$ and $2 < x < 3$, negative for $x < -2$ and $-1 < x < 2$ and $x > 3$, $y = -(x + 2)(x + 1)(x - 2)(x - 3)$

d) positive leading coefficient, x-intercepts: -1, 1, and 3, positive for $x < -1$ and $1 < x < 3$ and $x > 3$, negative for $-1 < x < 1$, $y = (x + 1)(x - 1)(x - 3)^2$

11. a) $a = 1$, $b = \frac{1}{2}$, $h = 2$, $k = -3$

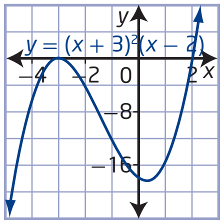
b) Horizontal stretch by a factor of 2, translation of 2 units right, and translation of 3 units down

c) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

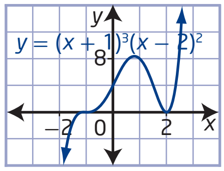
12. 2 m by 21 m by 50 m

13. 5 ft

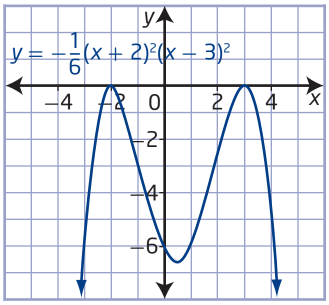
14. a) $y = (x + 3)^2(x - 2)$



b) $y = (x + 1)^3(x - 2)^2$



c) $y = -\frac{1}{6}(x + 2)^2(x - 3)^2$



15. 4 cm by 2 cm by 8 cm

16. -7, -5, and -3

17. The side lengths of the two cubes are 2 m and 3 m.

18. a) $(x^2 - 12)^2 - x^2$ **b)** 5 in. by 5 in.

c) 13 in. by 13 in.

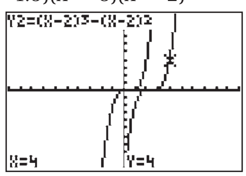
19. 4, 5, 6, and 7 or -7, -6, -5, and -4

20. $y = -\frac{1}{3}(x - \sqrt{3})(x + \sqrt{3})(x - 1)$

21. roots: -4.5, 8, and 2; $0 = (x + 4.5)(x - 8)(x - 2)$

22. a) translation of 2 units right

b)



c) $y = x^3 - x^2 = x^2(x - 1)$: 0 and 1, $y = (x - 2)^3 - (x - 2)^2 = (x - 2)^2(x - 3)$: 2 and 3

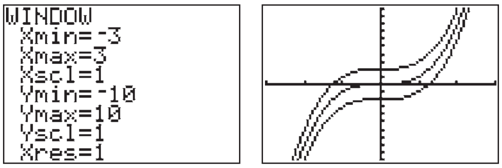
23. When $x \approx 0.65$, or when the sphere is at a depth of approximately 0.65 m.

C1 Example: It is easier to identify the roots.

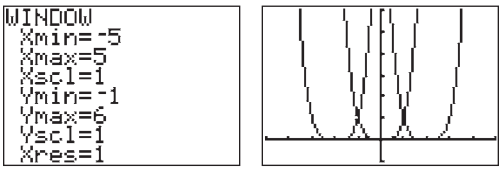
C2 Example: A root of an equation is a solution of the equation. A zero of a function is a value of x for which $f(x) = 0$. An x-intercept of a graph is the x-coordinate of the point where a line or curve crosses or touches the x-axis. They all represent the same thing.

C3 Example: If the multiplicity of a zero is 1, the function changes sign. If the multiplicity of a zero is even, the function does not change sign. The shape of a graph close to a zero of $x = a$ (order n) is similar to the shape of the graph of a function with degree equal to n of the form $y = (x - a)^n$.

C4 **Step 1** Set A



Set B

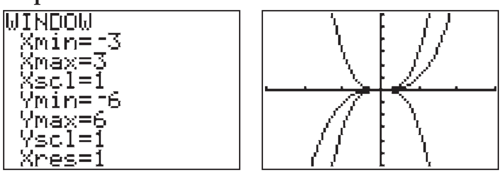


a) The graph of $y = x^3 + k$ is translated vertically k units compared to the graph of $y = x^3$.

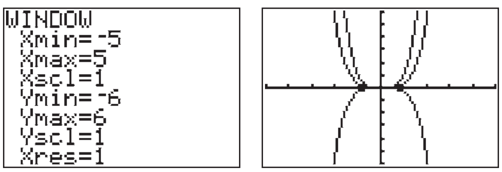
b) The graph of $y = (x - h)^4$ is translated horizontally h units compared to the graph of $y = x^4$.

Step 2 h: horizontal translation; k: vertical translation

Step 3 Set C



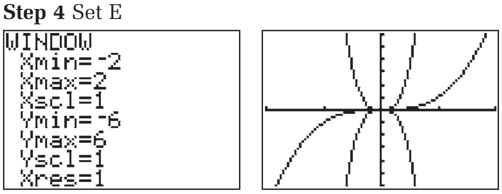
Set D



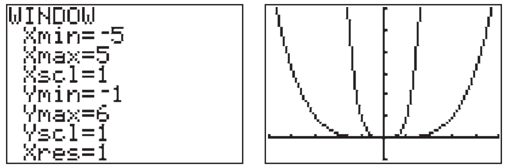
a) The graph of $y = ax^3$ is stretched vertically by a factor of $|a|$ relative to the graph of $y = x^3$. When a is negative, the graph is reflected in the x-axis.

b) When a is $-1 < a < 0$ or $0 < a < 1$, the graph of $y = ax^4$ is stretched vertically by a factor of $|a|$ relative to the graph of $y = x^4$. When a is negative, the graph is reflected in the x-axis.

Step 4 Set E



Set F



a) The graph of $y = (bx)^3$ is stretched horizontally by a factor of $\frac{1}{|b|}$ relative to the graph of $y = x^3$. When b is negative, the graph is reflected in the y-axis.

b) When b is $-1 < b < 0$ or $0 < b < 1$, the graph of $y = (bx)^4$ is stretched horizontally by a factor of $\frac{1}{|b|}$ relative to the graph of $y = x^4$. When b is negative, the graph is reflected in the y-axis.

Step 5 a: vertical stretch; reflection in the x-axis; b: horizontal stretch; reflection in the y-axis

Chapter 3 Review, pages 153 to 154

1. a) No, this is a square root function.
b) Yes, this is a polynomial function of degree 4.
c) Yes, this is a polynomial function of degree 3.
d) Yes, this is a polynomial function of degree 1.

2. a) degree 4 with positive leading coefficient, opens upward, maximum of 4 x-intercepts, y-intercept of 0
b) degree 3 with negative leading coefficient, extends from quadrant II to quadrant IV, maximum of 3 x-intercepts, y-intercept of 4
c) degree 1 with positive leading coefficient, extends from quadrant III to quadrant I, 1 x-intercept, y-intercept of -2
d) degree 2 with positive leading coefficient, opens upward, maximum of 2 x-intercepts, y-intercept of -4
e) degree 5 with positive leading coefficient, extends from quadrant III to quadrant I, maximum of 5 x-intercepts, y-intercept of 1

3. a) quadratic function **b)** 9196 ft
c) 25 s **d)** 26.81 s

4. a) 37, $\dfrac{x^3 + 9x^2 - 5x + 3}{x - 2} = x^2 + 11x + 17 + \dfrac{37}{x - 2}$, $x \neq 2$
b) 2, $\dfrac{2x^3 + x^2 - 2x + 1}{x + 1} = 2x^2 - x - 1 + \dfrac{2}{x + 1}$, $x \neq -1$
c) 9, $\dfrac{12x^3 + 13x^2 - 23x + 7}{x - 1} = 12x^2 + 25x + 2 + \dfrac{9}{x - 1}$, $x \neq 1$
d) 1, $\dfrac{-8x^4 - 4x + 10x^3 + 15}{x + 1} = -8x^3 + 18x^2 - 18x + 14 + \dfrac{1}{x + 1}$, $x \neq -1$

5. a) -3 **b)** 166

6. -34

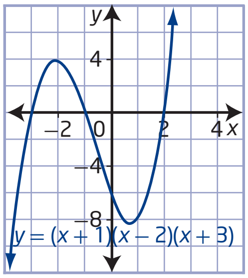
7. a) Yes, $P(1) = 0$. **b)** No, $P(-1) \neq 0$.
c) Yes, $P(-4) = 0$. **d)** Yes, $P(4) = 0$.

8. a) $(x - 2)(x - 3)(x + 1)$ **b)** $-4(x - 2)(x + 2)(x + 1)$
c) $(x - 1)(x - 2)(x - 3)(x + 2)$
d) $(x + 3)(x - 1)^2(x - 2)^2$

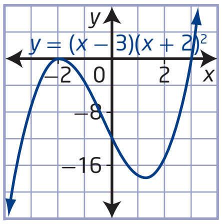
9. a) $x + 3$, $2x - 1$, and $x + 1$
b) 4 m by 1 m by 2 m

10. $k = -2$

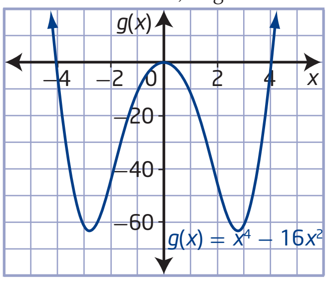
11. a) x-intercepts: -3, -1, and 2; degree 3 extending from quadrant III to quadrant I; -3, -1, and 2 each of multiplicity 1; y-intercept of -6; positive for $-3 < x < -1$ and $x > 2$, negative for $x < -3$ and $-1 < x < 2$



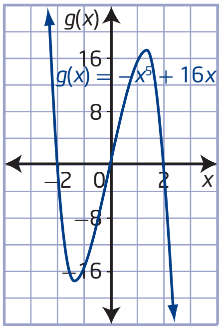
b) x-intercepts: -2 and 3; degree 3 extending from quadrant III to quadrant I; -2 (multiplicity 2) and 3 (multiplicity 1); y-intercept of -12; positive for $x > 3$, negative for $x < 3$, $x \neq -2$



c) x-intercepts: -4, 0, and 4; degree 4 opening upwards; 0 (multiplicity 2), -4 and 4 each of multiplicity 1; y-intercept of 0; positive for $x < -4$ and $x > 4$, negative for $-4 < x < 4$, $x \neq 0$



d) x-intercepts: -2, 0, and 2; degree 5 extending from quadrant II to quadrant IV; -2, 0, and 2 each of multiplicity 1; y-intercept of 0; positive for $x < -2$ and $0 < x < 2$, negative for $-2 < x < 0$ and $x > 2$

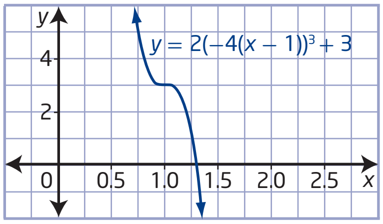


12. a) $a = 2$ vertical stretch by a factor of 2, $b = -4$ horizontal stretch by $\frac{1}{4}$ and reflection in the y-axis, $h = 1$ translation of 1 unit right, $k = 3$ translation of 3 units up

b)

Transformation	Parameter Value	Equation
horizontal stretch/ reflection in y-axis	-4	$y = (-4x)^3$
vertical stretch/ reflection in x-axis	2	$y = 2(-4x)^3$
translation right	1	$y = 2(-4(x - 1))^3$
translation up	3	$y = 2(-4(x - 1))^3 + 3$

c)



13. a) $y = (x + 1)(x + 3)^2$ **b)** $y = -(x + 1)(x - 2)^3$

14. a) Examples: $y = (x + 2)(x + 1)(x - 3)^2$ and $y = -(x + 2)(x + 1)(x - 3)^2$

b) $y = 2(x + 2)(x + 1)(x - 3)^2$

15. a) $V = 2l^2(l - 5)$ **b)** 8 cm by 3 cm by 16 cm

Chapter 3 Practice Test, pages 155 to 156

1. C **2.** B **3.** D **4.** B **5.** C

6. a) -4 and 3 **b)** -1 and 3

c) -2, 2, and 5 **d)** -3 and 3

7. a) $P(x) = (x + 2)(x + 1)^2$

b) $P(x) = (x - 1)(x^2 - 12x - 12)$

c) $P(x) = -x(x - 3)^2$

d) $P(x) = (x + 1)(x^2 - 4x + 5)$

8. a) B **b)** C **c)** A

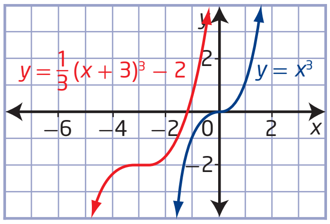
9. a) $V = x(20 - 2x)(18 - x)$

b) 2 cm by 16 cm by 16 cm

10. a) $a = \frac{1}{3}$, vertical stretch by a factor of $\frac{1}{3}$; $b = 1$, no horizontal stretch; $h = -3$, translation of 3 units left; $k = -2$, translation of 2 units down

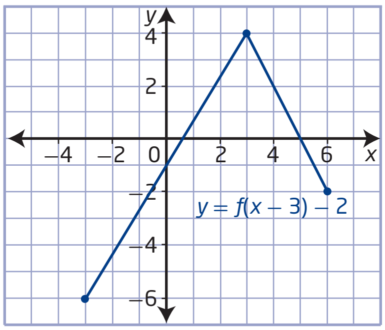
b) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

c)

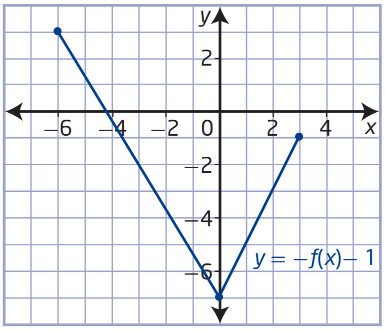


Cumulative Review, Chapters 1–3, pages 158 to 159

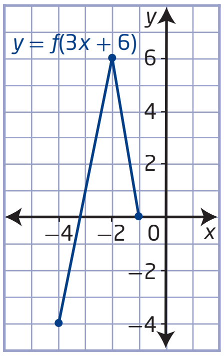
1. a)



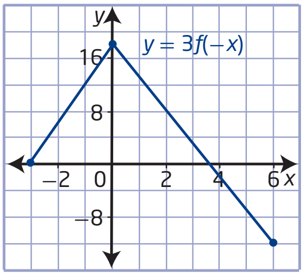
b)



c)



d)



2. $y + 4 = f(x - 3)$

3. a) translation of 1 unit left and 5 units down

b) vertical stretch by a factor of 3, reflection in the x-axis, and translation of 2 units right

c) reflection in the y-axis and translation of 1 unit right and 3 units up

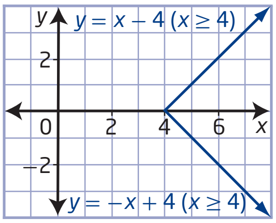
4. a) (9, 10) **b)** (6, −18) **c)** (−2, 9)

5. a) x-intercepts: $-\frac{4}{3}$ and 2, y-intercept: −3

b) x-intercepts: −4 and 6, y-intercept: 6

6. a) Yes

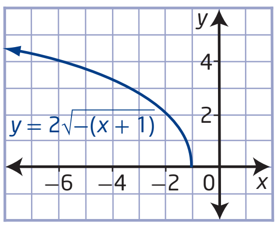
b)



c) Example: No; restrict domain of $y = |x| + 4$ to $\{x \mid x \geq 0, x \in R\}$.

7. $g(x) = \sqrt{2(x + 2)} - 3$

8. $y = 2\sqrt{-(x + 1)}$



domain $\{x \mid x \leq -1, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

9. a) $g(x) = \sqrt{9x}$ **b)** $g(x) = 3\sqrt{x}$

c) $\sqrt{9x} = \sqrt{9}(\sqrt{x}) = 3\sqrt{x}$

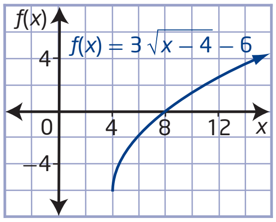
10. a) The x-intercepts are invariant points for square roots of functions, since $\sqrt{0} = 0$.

b) $f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -1, y \in R\}$; $g(x)$: domain $\{x \mid x \leq -1 \text{ or } x \geq 1, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$; The square root function has a restricted domain.

11. a) No, substituting −2.75 back into the equation does not satisfy the equation.

b) Only one solution, $x = -2$.

12. a)



The x-intercept is 8.

b) $x = 8$

c) They are the same.

13. a) $\frac{x^4 + 3x + 4}{x + 1} = x^3 - x^2 + x + 2 + \frac{2}{x + 1}$; $P(-1) = 2$

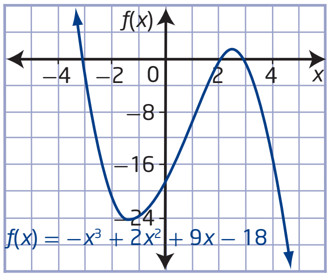
b) $\frac{x^3 + 5x^2 + x - 9}{x + 3} = x^2 + 2x - 5 + \frac{6}{x + 3}$; $P(-3) = 6$

14. ±1, ±2, ±3, ±6; P(1) = 0, P(−1) = −16, P(2) = −4, P(−2) = 0, P(3) = 0, P(−3) = 96, P(6) = 600, P(−6) = 1764

15. a) $(x + 5)(x - 1)(x - 4)$ **b)** $(x - 3)(x + 4)(x + 2)$

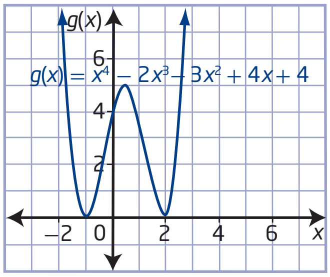
c) $-(x - 2)^2(x + 2)^2$

16. a)



x-intercepts: −3, 2, and 3; y-intercept: −18

b)



x-intercepts: −1 and 2, y-intercept: 4

17. a) $(x + 4)$ and $(x - 3)$ **b)** 4.5 m by 7.5 m by 0.5 m

18. $y = 3(-(x - 5))^3$

Unit 1 Test, pages 160 to 161

1. D **2.** C **3.** D **4.** A **5.** D **6.** C **7.** A

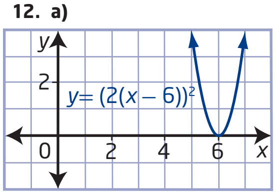
8. −13

9. $\{y \mid y \geq 8, y \in R\}$

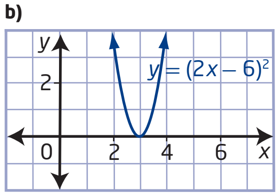
10. $g(x) = |x + 2| + 3$

11. 5

12. a)

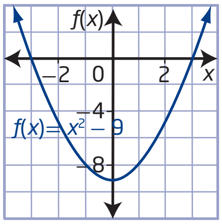


b)

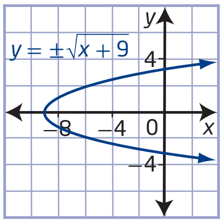


c) They both have the same shape but one of them is shifted right further.

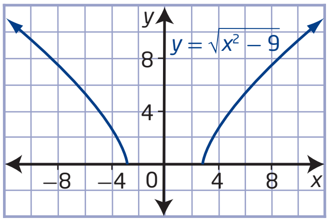
13. a)



b) $y = \pm\sqrt{x + 9}$



c) $y = \sqrt{x^2 - 9}$



d) for part a): domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -9, y \in \mathrm{R}\}$; for part b): domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for part c): domain $\{x \mid x \leq -3 \text{ or } x \geq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

14. Quadrant II: reflection in the y-axis, $y = f(-x)$; quadrant III: reflection in the y-axis and then the x-axis, $y = -f(-x)$; quadrant IV: reflection in the x-axis, $y = -f(x)$

15. a) Mary should have subtracted 4 from both sides in step 1. She also incorrectly squared the expression on the right side in step 2. The correct solution follows:

$2x = \sqrt{x + 1} + 4$

Step 1: $(2x - 4)^2 = (\sqrt{x + 1})^2$
Step 2: $4x^2 - 16x + 16 = x + 1$
Step 3: $4x^2 - 17x + 15 = 0$
Step 4: $(4x - 5)(x - 3) = 0$
Step 5: $4x - 5 = 0$ or $x - 3 = 0$
Step 6: $x = \frac{5}{4}$ $\quad x = 3$
Step 7: A check determines that $x = 3$ is the solution.

b) Yes, the point of intersection of the two graphs will yield the possible solution, $x = 3$.

16. $c = -3$; $P(x) = (x + 3)(x + 2)(x - 1)^2$

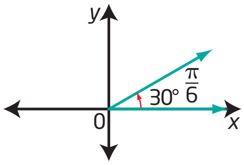
17. a) $\pm1, \pm2, \pm3, \pm6$
b) $P(x) = (x - 3)(x + 2)(x + 1)$
c) x-intercepts: -2, -1 and 3; y-intercept: -6
d) $-2 \leq x \leq -1$ and $x \geq 3$

Chapter 4 Trigonometry and the Unit Circle

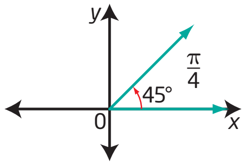
4.1 Angles and Angle Measure, pages 175 to 179

1. a) clockwise **b)** counterclockwise
c) clockwise **d)** counterclockwise

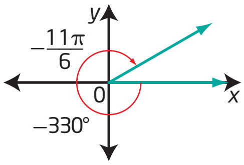
2. a) $30° = \frac{\pi}{6}$



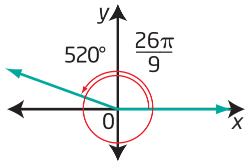
b) $45° = \frac{\pi}{4}$



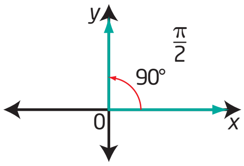
c) $-330° = -\frac{11\pi}{6}$



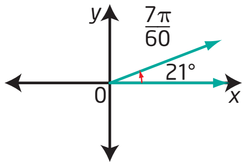
d) $520° = \frac{26\pi}{9}$



e) $90° = \frac{\pi}{2}$



f) $21° = \frac{7\pi}{60}$

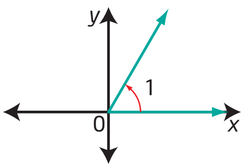


3. a) $\frac{\pi}{3}$ or 1.05 **b)** $\frac{5\pi}{6}$ or 2.62
c) $-\frac{3\pi}{2}$ or -4.71 **d)** $\frac{2\pi}{5}$ or 1.26
e) $-\frac{37\pi}{450}$ or -0.26 **f)** 3π or 9.42

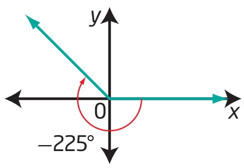
4. a) 30° **b)** 120°
c) $-67.5°$ **d)** $-450°$
e) $\frac{180°}{\pi}$ or 57.3° **f)** $\frac{495°}{\pi}$ or 157.6°

5. a) $\frac{360°}{7}$ or 51.429° **b)** $\frac{1260°}{13}$ or 96.923°
c) $\frac{120°}{\pi}$ or 38.197° **d)** $\frac{3294°}{5\pi}$ or 209.703°
e) $\frac{-1105.2°}{\pi}$ or $-351.796°$ **f)** $\frac{-3600°}{\pi}$ or $-1145.916°$

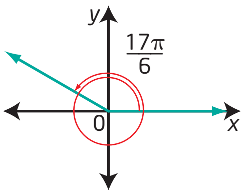
6. a) quadrant I



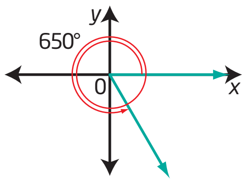
b) quadrant II



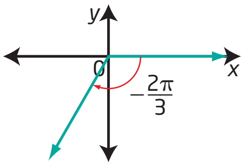
c) quadrant II



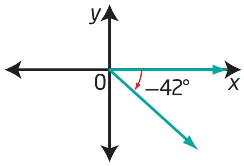
d) quadrant IV



e) quadrant III



f) quadrant IV



7. Examples:

a) 432°, −288° **b)** $\frac{11\pi}{4}$, $-\frac{5\pi}{4}$

c) 240°, −480° **d)** $\frac{7\pi}{2}$, $-\frac{\pi}{2}$

e) 155°, −565° **f)** 1.5, −4.8

8. a) coterminal, $\frac{17\pi}{6} = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{5\pi}{6} + 2\pi$

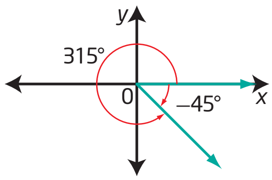
b) not coterminal **c)** not coterminal

d) coterminal, $-493° = 227° - 2(360°)$

9. a) $135° \pm (360°)n,\ n \in \mathrm{N}$ **b)** $-\frac{\pi}{2} \pm 2\pi n,\ n \in \mathrm{N}$

c) $-200° \pm (360°)n,\ n \in \mathrm{N}$ **d)** $10 \pm 2\pi n,\ n \in \mathrm{N}$

10. Example:



$-45° + 360° = 315°$, $-45° \pm (360°)n,\ n \in \mathrm{N}$

11. a) 425° **b)** 320°

c) −400°, 320°, 680° **d)** $-\frac{5\pi}{4}$

e) $-\frac{23\pi}{6}, \frac{\pi}{6}, \frac{13\pi}{6}$ **f)** $-\frac{5\pi}{3}, \frac{\pi}{3}$

g) −3.9 **h)** −0.9, 5.4

12. a) 13.30 cm **b)** 4.80 m

c) 15.88 cm **d)** 30.76 in.

13. a) 2.25 radians **b)** 10.98 ft

c) 3.82 cm **d)** 17.10 m

14. a) $\frac{25\pi}{3}$ or 26.18 m

b) $\frac{A_{\text{sector}}}{A_{\text{circle}}} = \frac{\text{sector angle}}{2\pi}$

$A_{\text{sector}} = \frac{\pi r^2\left(\frac{5\pi}{3}\right)}{2\pi}$

$A_{\text{sector}} = \frac{5\pi(5)^2}{6}$

$A_{\text{sector}} = \frac{125\pi}{6}$

The area watered is approximately 65.45 m².

c) 16π radians or 2880°

15. a) Examples: $\frac{\pi}{12}$ radians/h, 1 revolution per day, 15°/h

b) $\frac{100\pi}{3}$ or 104.72 radians/s

c) 54 000°/min

16. a) 2.36 **b)** 135.3°

17.

	Revolutions	Degrees	Radians
a)	1 rev	360°	2π
b)	0.75 rev	270°	$\frac{3\pi}{2}$ or 4.7
c)	0.4 rev	150°	$\frac{5\pi}{6}$
d)	−0.3 rev	−97.4°	−1.7
e)	−0.1 rev	−40°	$-\frac{2\pi}{9}$ or −0.7
f)	0.7 rev	252°	$\frac{7\pi}{5}$ or 4.4
g)	−3.25 rev	−1170°	$-\frac{13\pi}{2}$ or −20.4
h)	$\frac{23}{18}$ or 1.3 rev	460°	$\frac{23\pi}{9}$ or 8.0
i)	$-\frac{3}{16}$ or −0.2 rev	−67.5°	$-\frac{3\pi}{8}$

18. Jasmine is correct. Joran's answer includes the solution when $k = 0$, which is the reference angle 78°.

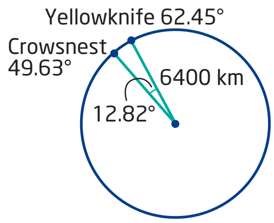
19. a) 55.6 grad

b) Use a proportion: $\frac{\text{gradians}}{\text{degrees}} = \frac{400 \text{ grad}}{360°}$.

So, measure in gradians $= \frac{10(\text{number of degrees})}{9}$.

c) The gradian was developed to express a right angle as a metric measure. A right angle is equivalent to 100 grads.

20. a)



b) 1432.01 km

c) Example: Bowden (51.93° N, 114.03° W) and Airdrie (51.29° N, 114.01° W) are 71.49 km apart.

21. a) 2221.4 m/min **b)** 7404.7 radians/min

22. 8.5 km/h

23. 66 705.05 mph

24. a) $69.375° = 69° + 0.375(60')$
$= 69°\ 22.5'$
$= 69°\ 22'\ 30''$

b) **i)** 40° 52′ 30″ **ii)** 100° 7′ 33.6″

iii) 14° 33′ 54″ **iv)** 80° 23′ 6″

25. a) $69°\ 22'\ 30'' = 69°22.5'$
$= 69° + \left(\frac{22.5}{60}\right)^{\circ}$
$= 69.375°$

b) **i)** 45.508° **ii)** 72.263°

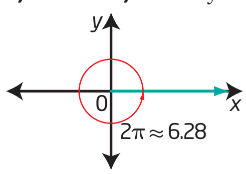
iii) 105.671° **iv)** 28.167°

26. $A_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin\theta)$

27. a) 120° **b)** 65° **c)** Examples: 3:00 and 9:00

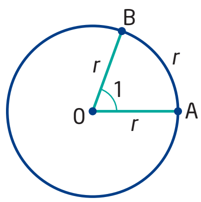
d) 2 **e)** shortly after 4:05

C1



π is 180° and 2π is 360°. 2(3.14) = 6.282 which is more than 6. Therefore, 6 radians must be less than 360°.

C2

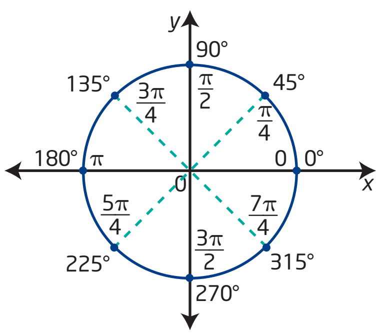


1° is a very small angle, it is $\frac{1}{360}$ of one rotation. One radian is much larger than 1°; 1 radian is the angle whose arc is the same as the radius, it is nearly $\frac{1}{6}$ of one rotation.

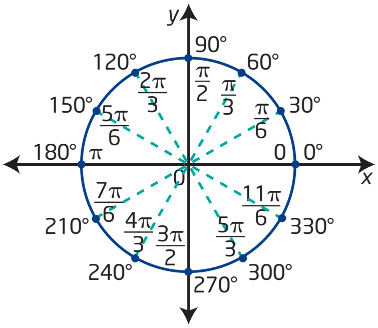
C3 a) 40°; $140° \pm (360°)n,\ n \in \mathrm{N}$

b) 0.72; $0.72 \pm 2\pi n,\ n \in \mathrm{N}$

C4 a)



b)



C5 a) $x = 3$ **b)** $y = x - 3$

4.2 The Unit Circle, pages 186 to 190

1. a) $x^2 + y^2 = 16$ **b)** $x^2 + y^2 = 9$
c) $x^2 + y^2 = 144$ **d)** $x^2 + y^2 = 6.76$

2. a) No; $\left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{5}{8} \neq 1$

b) No; $\left(\frac{\sqrt{5}}{8}\right)^2 + \left(\frac{7}{8}\right)^2 = \frac{27}{32} \neq 1$

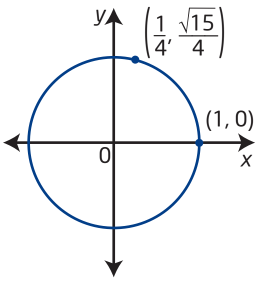
c) Yes; $\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = 1$

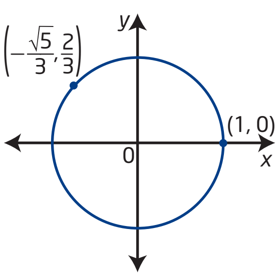
d) Yes; $\left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2 = 1$

e) Yes; $\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 = 1$

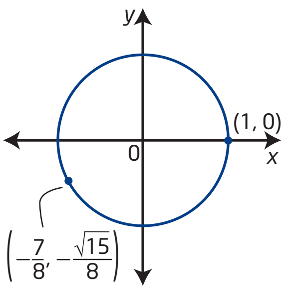
f) Yes; $\left(\frac{\sqrt{7}}{4}\right)^2 + \left(\frac{3}{4}\right)^2 = 1$

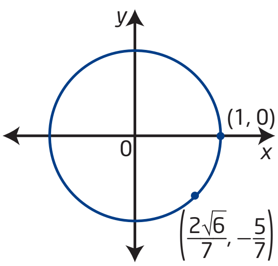
3. a) $y = \frac{\sqrt{15}}{4}$ **b)** $x = -\frac{\sqrt{5}}{3}$



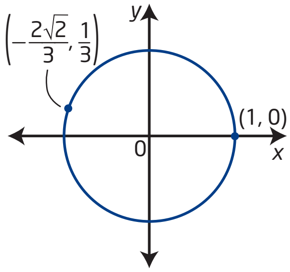


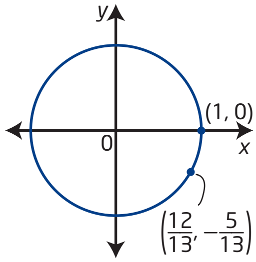
c) $y = -\frac{\sqrt{15}}{8}$ **d)** $x = \frac{2\sqrt{6}}{7}$





e) $x = -\frac{2\sqrt{2}}{3}$ **f)** $y = -\frac{5}{13}$



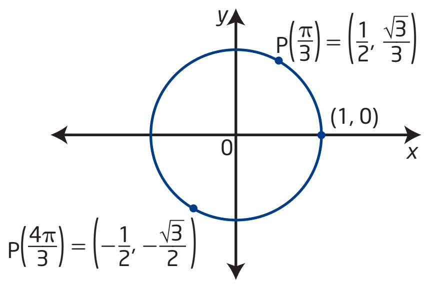


4. a) $(-1, 0)$ **b)** $(0, -1)$
c) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **d)** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
e) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **f)** $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
g) $(1, 0)$ **h)** $(0, 1)$
i) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **j)** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

5. a) $\frac{3\pi}{2}$ **b)** 0 **c)** $\frac{\pi}{4}$ **d)** $\frac{3\pi}{4}$
e) $\frac{\pi}{3}$ **f)** $\frac{5\pi}{3}$ **g)** $\frac{5\pi}{6}$ **h)** $\frac{7\pi}{6}$
i) $\frac{5\pi}{4}$ **j)** π

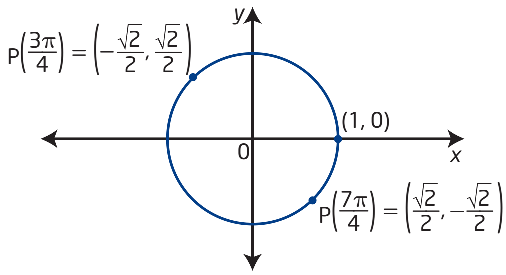
6. $\frac{5\pi}{6}$ and $-\frac{7\pi}{6}$

7. a)



If $\theta = \frac{\pi}{3}$ then $\theta + \pi = \frac{\pi}{3} + \pi$ or $\frac{4\pi}{3}$ since
$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $P\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

b)



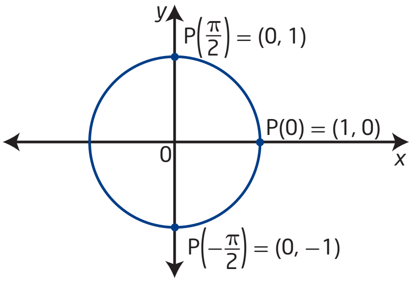
If $\theta = \frac{3\pi}{4}$ then $\theta + \pi = \frac{3\pi}{4} + \pi$ or $\frac{7\pi}{4}$ since
$P\left(\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $P\left(\frac{7\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

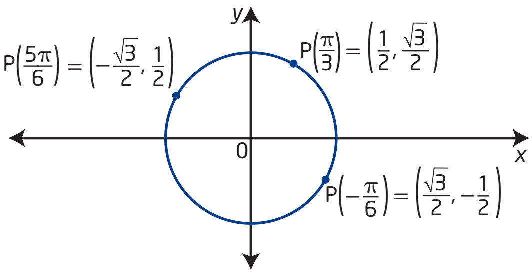
8.

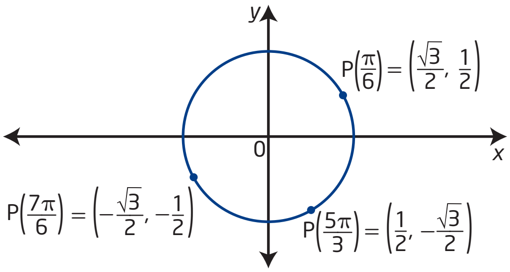
Point	$+\frac{1}{4}$ rotation	$-\frac{1}{4}$ rotation	Step 4: Description
$P(0) = (1, 0)$	$P\left(\frac{\pi}{2}\right) = (0, 1)$	$P\left(-\frac{\pi}{2}\right) = (0, -1)$	x- and y-values change places and take signs of new quadrant
$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$P\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$P\left(\frac{\pi}{3} - \frac{\pi}{2}\right) = P\left(-\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	x- and y-values change places and take signs of new quadrant
$P\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	$P\left(\frac{5\pi}{3} + \frac{\pi}{2}\right) = P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$P\left(\frac{5\pi}{3} - \frac{\pi}{2}\right) = P\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	x- and y-values change places and take signs of new quadrant

Diagrams:

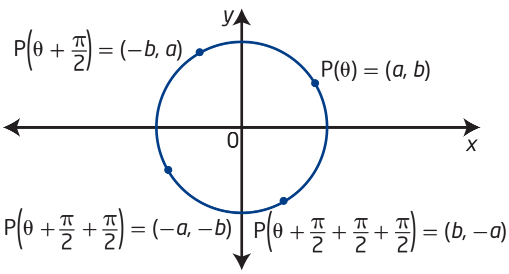
Steps 1–3







Step 4



9. a) $x^2 + y^2 = 1$ **b)** $\left(\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$

c) $\theta + \frac{\pi}{2}$ **d)** quadrant IV

e) maximum value is $+1$, minimum value is -1

10. a) Yes. In quadrant I the values of $\cos\theta$ decrease from 1 at $\theta = 0°$ to 0 at $\theta = 90°$, since the x-coordinate on the unit circle represents $\cos\theta$, in the first quadrant the values of x will range from 1 to 0.

b) Substitute the values of x and y into the equation $x^2 + y^2 = 1$, Mya was not correct, the correct answer is $y = \sqrt{1 - (0.807)^2}$
$= \sqrt{0.348\ 751}$
$\approx 0.590\ 551$

c) $x = 0.9664$

11. b) All denominators are 2.

c) The numerators of the x-coordinates decrease from $\sqrt{3}$, $\sqrt{2}$, $\sqrt{1} = 1$, the numerators of the y-coordinates increase from $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$. The x-coordinates are moving closer to the y-axis and therefore decrease in value, whereas the y-coordinates are moving further away from the x-axis and therefore increase in value.

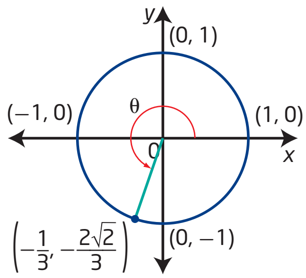
d) Since $x^2 + y^2 = 1$ then $x = \sqrt{1 - y^2}$ and $y = \sqrt{1 - x^2}$, all solutions involve taking square roots.

12. a) $-2\pi \le \theta < 4\pi$ represents three rotations around the unit circle and includes three coterminal angles for each point on the unit circle.

b) If $P(\theta) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, then $\theta = -\frac{4\pi}{3}$ when $-2\pi \le \theta \le 0$, $\theta = \frac{2\pi}{3}$ when $0 \le \theta \le 2\pi$, and $\theta = \frac{8\pi}{3}$ when $2\pi \le \theta < 4\pi$.

c) All these angles are coterminal since they are all 2π radians apart.

13. a)

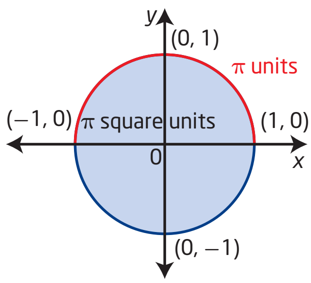


This point represents the terminal point of an angular rotation on the unit circle.

b) quadrant III **c)** $P\left(\theta + \frac{\pi}{2}\right) = \left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$

d) $P\left(\theta - \frac{\pi}{2}\right) = \left(-\frac{2\sqrt{2}}{3}, \frac{1}{3}\right)$

14.



π units is the perimeter of half of a unit circle since $a = r\theta = (1)\pi = \pi$ units. π square units is the area of a unit circle since $A = \pi r^2 = \pi(1)^2$
$= \pi$ square units.

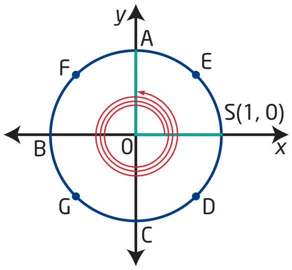
15. a) $B(-a, b)$, $C(-a, -b)$, $D(a, -b)$

b) **i)** $\theta + \pi = C(-a, -b)$ **ii)** $\theta - \pi = C(-a, -b)$
iii) $-\theta + \pi = B(-a, b)$ **iv)** $-\theta - \pi = B(-a, b)$

c) They do not differ.

16. a) $\theta = \frac{5\pi}{4}$; $a = r\theta = (1)\left(\frac{5\pi}{4}\right) = \frac{5\pi}{4}$

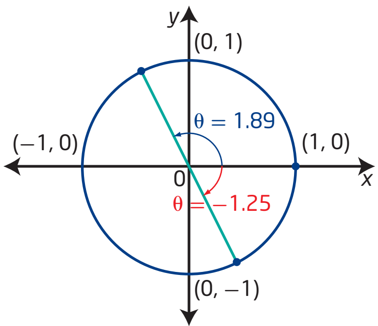
b) $P\left(\frac{13\pi}{2}\right)$ represents the ordered pair of the point where the terminal arm of the angle $\frac{13\pi}{2}$ intersects the unit circle. Since one rotation of the unit circle is 2π, then $\frac{13\pi}{2}$ represents three complete rotations with an extra $\frac{\pi}{2}$ or quarter rotation, therefore ending at point A.



c) Point $C = P\left(\frac{3\pi}{2}\right) \approx P(4.71)$ and point $D = P\left(\frac{7\pi}{4}\right) \approx P(5.50)$. Therefore P(5), lies between points C and D.

17. a) $\left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$ and $\left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$

b)

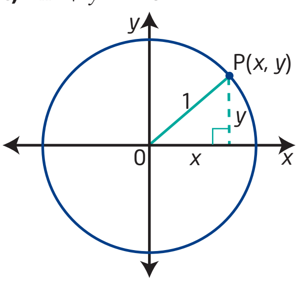


θ represents the angle in standard position.

18. a) $\left(\frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right)$ **b)** $\sqrt{29}$

c) $x^2 + y^2 = 29$

19.

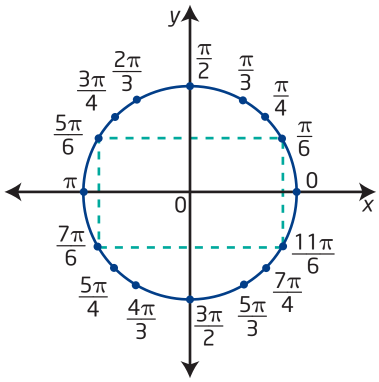


From the diagram: opposite side = y, adjacent side = x and hypotenuse = 1.
Since $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ then $\sin \theta = \frac{y}{1} = y$ or $y = \sin \theta$. Similarly, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, so $\cos \theta = \frac{x}{1} = x$ or $x = \cos \theta$. Therefore any point on the unit circle can be represented by the coordinates $(\cos \theta, \sin \theta)$.

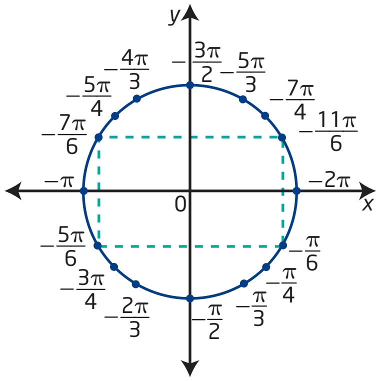
20. a) $\left(1, \frac{\pi}{4}\right)$ **b)** $\left(\frac{\sqrt{31}}{6}, 3.509\right)$

c) $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ **d)** $(5, 5.640)$

C1 a)



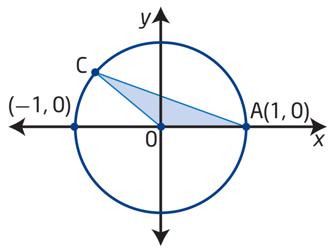
b)



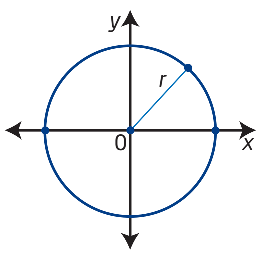
c) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right), \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

d) Example: The circumference is divided into eighths by successive quarter rotations, each eighth of the circumference measures $\frac{\pi}{4}$. The exact coordinates of the points can be determined using the special right triangles $(1 : 1 : \sqrt{2}$ and $1 : \sqrt{3} : 2)$ with signs adjusted according to the quadrant.

C2 a) $\frac{\pi}{5}$ **b)** $\frac{3\pi}{20}$



C3 a) $x^2 + y^2 = r^2$



b) Compare with a quadratic function. When $y = x^2$ is translated so its vertex moves from (0, 0) to (h, k), its equation becomes $y = (x - h)^2 + k$. So, a reasonable conjecture for the circle centre (0, 0) moving its centre to (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. Test some key points on the circle centre (0, 0) such as $(r, 0)$. When the centre moves to (h, k) the test point moves to $(r + h, k)$. Substitute into the left side of the equation.
$(r + h - h)^2 + (k - k)^2 = r^2 + 0 =$ right side.

C4 a) 21.5% **b)** $\pi : 4$

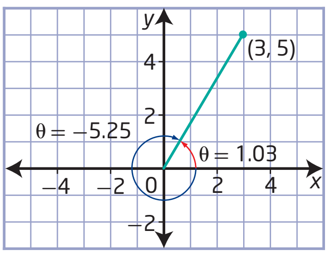
4.3 Trigonometric Ratios, pages 201 to 205

1. a) $\frac{\sqrt{2}}{2}$ **b)** $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ **c)** $-\frac{\sqrt{2}}{2}$
d) $\sqrt{3}$ **e)** -2 **f)** -2
g) undefined **h)** -1 **i)** $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
j) $\frac{\sqrt{3}}{2}$ **k)** $-\frac{\sqrt{3}}{2}$ **l)** $\sqrt{2}$

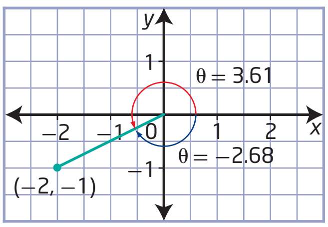
2. a) 0.68 **b)** -2.75 **c)** 1.04
d) -1.00 **e)** -0.96 **f)** 1.37
g) 0.78 **h)** 0.71 **i)** 0.53
j) -0.97 **k)** -3.44 **l)** undefined

3. a) I or IV **b)** II or IV **c)** III or IV
d) II **e)** II **f)** I

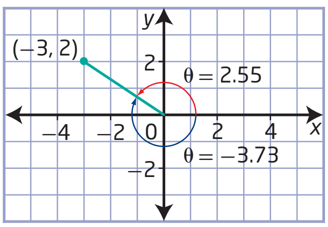
4. a) $\sin 250° = -\sin 70°$ **b)** $\tan 290° = -\tan 70°$
c) $\sec 135° = -\sec 45°$ **d)** $\cos 4 = -\cos (4 - \pi)$
e) $\csc 3 = \csc (\pi - 3)$ **f)** $\cot 4.95 = \cot (4.95 - \pi)$

5. a)


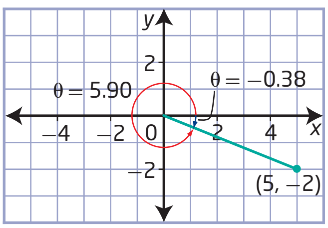
1.03, −5.25

b)


3.61, −2.68

c)


2.55, −3.73

d)


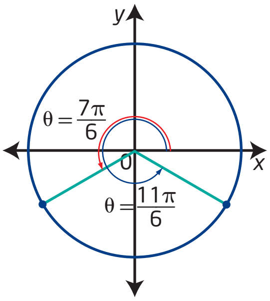
5.90, −0.38

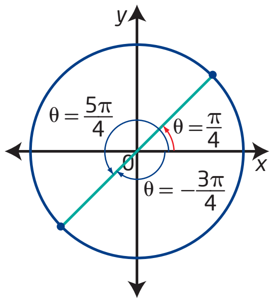
6. a) positive **b)** negative **c)** negative
d) positive **e)** positive **f)** positive

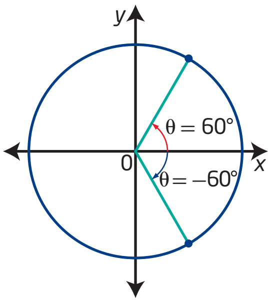
7. a) $\sin^{-1} 0.2014 = 0.2$; an angle of 0.2 radians has a sine ratio of 0.2014
b) $\tan^{-1} 1.429 = 7$; an angle of 7 radians has a tangent ratio of 1.429
c) sec 450° is undefined; an angle of 450° has a secant ratio that is undefined
d) cot (−180°) is undefined; an angle of −180° has a cotangent ratio that is undefined

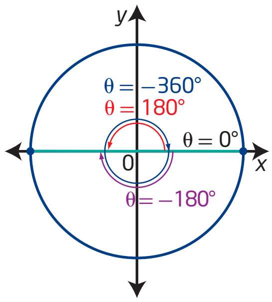
8. a) $-\frac{4}{5}$ **b)** $-\frac{4}{3}$ **c)** -1.25

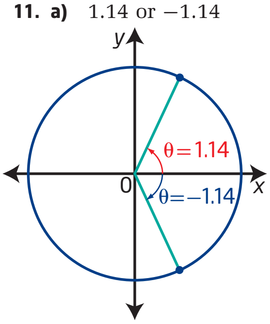
9. a) 1 **b)** 2 **c)** 1
d) -1 **e)** 1 **f)** 3

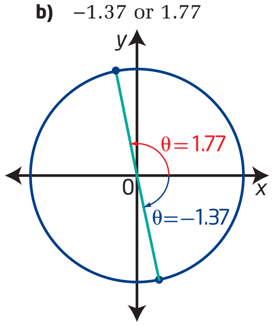
10. a) $\frac{7\pi}{6}, \frac{11\pi}{6}$


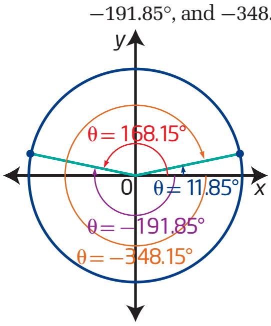
b) $-\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$


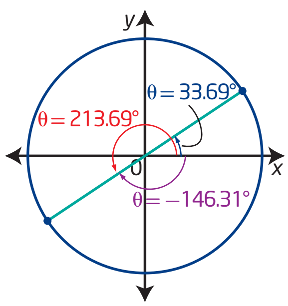
c) −60°, 60°


d) −360°, −180°, 0°, 180°


11. a) 1.14 or −1.14


b) −1.37 or 1.77


c) 11.85°, 168.15°, −191.85°, and −348.15°


d) 33.69°, 213.69° and −146.31°


12. a) $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$, $\csc\theta = \frac{5}{3}$,
$\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

b) $\sin\theta = \pm\frac{1}{3}$, $\tan\theta = \pm\frac{\sqrt{2}}{4}$, $\csc\theta = \pm 3$,
$\sec\theta = -\frac{3\sqrt{2}}{4}$, $\cot\theta = \pm 2\sqrt{2}$

c) $\sin\theta = \pm\frac{2}{\sqrt{13}}$, $\cos\theta = \pm\frac{3}{\sqrt{13}}$,
$\csc\theta = \pm\frac{\sqrt{13}}{2}$, $\sec\theta = \pm\frac{\sqrt{13}}{3}$, $\cot\theta = \frac{3}{2}$

d) $\sin\theta = \pm\frac{\sqrt{39}}{4\sqrt{3}}$ or $\pm\frac{\sqrt{13}}{4}$, $\cos\theta = \frac{3}{4\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$,
$\csc\theta = \pm\frac{4\sqrt{3}}{\sqrt{39}}$ or $\pm\frac{4\sqrt{13}}{13}$, $\tan\theta = \pm\frac{\sqrt{39}}{3}$,
$\cot\theta = \pm\frac{3}{\sqrt{39}}$ or $\pm\frac{\sqrt{39}}{13}$

13. Sketch the point and angle in standard position. Draw the reference triangle. Find the missing value of the hypotenuse by using the equation $x^2 + y^2 = r^2$. Use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ to find the exact value.
Therefore, $\cos\theta = -\frac{2}{\sqrt{13}}$ or $-\frac{2\sqrt{13}}{13}$.

14. a) $\frac{4900°}{360°} = 13\frac{11}{18}$ revolutions counterclockwise

b) quadrant III **c)** 40°

d) $\sin 4900° = -0.643$, $\cos 4900° = -0.766$, $\tan 4900° = 0.839$, $\csc 4900° = -1.556$, $\sec 4900° = -1.305$, $\cot 4900° = 1.192$

15. a) 0.8; For an angle whose cosine is 0.6, think of a 3-4-5 right triangle, or in this case a 0.6-0.8-1 right triangle. The x-coordinate is the same as the cosine or 0.6, the sine is the y-coordinate which will be 0.8.

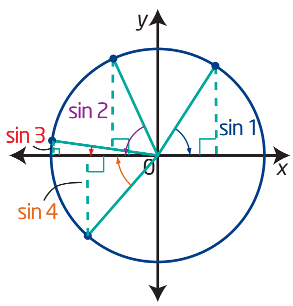
b) 0.8; Since $\cos^{-1} 0.6 = 90° - \sin^{-1} 0.6$ and $\sin^{-1} 0.6 = 90° - \cos^{-1} 0.6$, then $\cos(\sin^{-1} 0.6) = \sin(\cos^{-1} 0.6)$. Alternatively use similar reasoning as in part a) except the x- and y-coordinates are switched.

16. a) He is not correct. His calculator was in degree measure but the angle is expressed in radians.

b) Set calculator to radian mode and find the value of $\cos\left(\frac{40\pi}{7}\right)$. Since $\sec\theta = \frac{1}{\cos\theta}$, take the reciprocal of $\cos\left(\frac{40\pi}{7}\right)$ to get $\sec\left(\frac{40\pi}{7}\right) \approx 1.603\ 875\ 472$.

17. a) sin 4, sin 3, sin 1, sin 2

b)

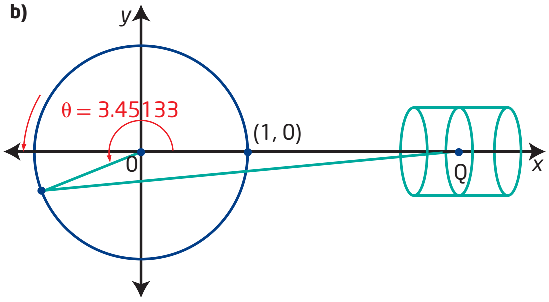


Sin 4 is in quadrant III and has a negative value, therefore it has the least value. Sin 3 is in quadrant II but has the smallest reference angle and is therefore the second smallest. Sin 1 has a smaller reference angle than sin 2.

c) cos 3, cos 4, cos 2, cos 1

18. a) 2 units

b)

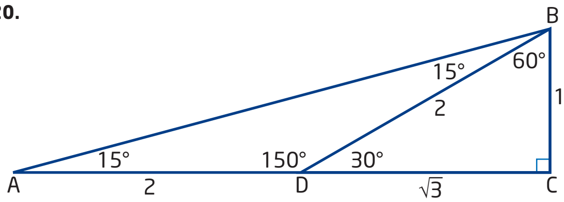


c) 0.46 units

19. a) 2.21, 8.50 **b)** −11.31°, 348.69°

c) −2.16, 4.12, 10.41

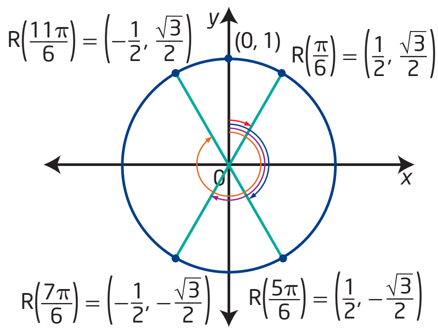
20.



$\triangle$BCD is a 30°-60°-90° triangle, so DC = $\sqrt{3}$ units and BD = 2 units. $\triangle$ABD has two equal angles of 15°, so AD = BD = 2. Then
$\tan 15° = \frac{BC}{AC} = \frac{BC}{CD + DA} = \frac{1}{\sqrt{3} + 2}$.

21. Since $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2.5}{5.0} = \frac{1}{2}$ then $\theta = 60°$.
Since 60° is $\frac{2}{3}$ of 90° then the point is $\frac{1}{3}$ the distance on the arc from (0, 5) to (5, 0).

22. a)



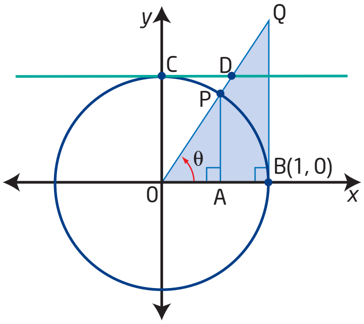
b) $R\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $R\left(\frac{5\pi}{6}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

c) $R\left(\frac{\pi}{6}\right) = P\left(\frac{\pi}{3}\right)$, $R\left(\frac{5\pi}{6}\right) = P\left(\frac{5\pi}{3}\right)$, $R\left(\frac{7\pi}{6}\right) = P\left(\frac{4\pi}{3}\right)$, $R\left(\frac{11\pi}{6}\right) = P\left(\frac{2\pi}{3}\right)$, where R(θ) represents the new angle and P(θ) represents the conventional angle in standard position.

d) The new system is the same as bearings in navigation, except bearings are measured in degrees, not radians.

23. a) In $\triangle$OBQ, $\cos\theta = \frac{OB}{OQ} = \frac{1}{OQ}$.
So, $\sec\theta = \frac{1}{\cos\theta} = OQ$.

b)



In $\triangle OCD$, $\angle ODC = \theta$ (alternate angles). Then, $\sin \theta = \frac{OC}{OD} = \frac{1}{OD}$. So, $\csc \theta = \frac{1}{\sin \theta} = OD$.
Similarly, $\cot \theta = CD$.

C1 a) Paula is correct. Examples: $\sin 0° = 0$, $\sin 10° \approx 0.1736$, $\sin 25° \approx 0.4226$, $\sin 30° = 0.5$, $\sin 45° \approx 0.7071$, $\sin 60° \approx 0.8660$, $\sin 90° = 1$.

b) In quadrant II, sine decreases from $\sin 90° = 1$ to $\sin 180° = 0$. This happens because the y-value of points on the unit circle are decreasing toward the horizontal axis as the value of the angle moves from 90° to 180°.

c) Yes, the sine ratio increases in quadrant IV, from its minimum value of -1 at 270° up to 0 at 0°.

C2 When you draw its diagonals, the hexagon is composed of six equilateral triangles. On the diagram shown, each vertex will be 60° from the previous one. So, the coordinates, going in a positive direction from (1, 0) are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $(-1, 0)$, $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, and $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

C3 a) $\text{slope}_{OP} = \frac{\sin \theta}{\cos \theta}$ or $\tan \theta$

b) Yes, this formula applies in each quadrant. In quadrant II, $\sin \theta$ is negative, which makes the slope negative, as expected. Similar reasoning applies in the other quadrants.

c) $y = \left(\frac{\sin \theta}{\cos \theta}\right)x$ or $y = (\tan \theta)x$

d) Any line whose slope is defined can be translated vertically by adding the value of the y-intercept b. The equation will be $y = \left(\frac{\sin \theta}{\cos \theta}\right)x + b$ or $y = (\tan \theta)x + b$.

C4 a) $\frac{4}{5}$ **b)** $\frac{3}{5}$ **c)** $\frac{5}{4}$ **d)** $-\frac{4}{5}$

4.4 Introduction to Trigonometric Equations, pages 211 to 214

1. a) two solutions; $\sin \theta$ is positive in quadrants I and II

b) four solutions; $\cos \theta$ is positive in quadrants I and IV, giving two solutions for each of the two complete rotations

c) three solutions; $\tan \theta$ is negative in quadrants II and IV, and the angle rotates through these quadrants three times from $-360°$ to 180°

d) two solutions; $\sec \theta$ is positive in quadrants I and IV and the angle is in each quadrant once from $-180°$ to 180°

2. a) $\theta = \frac{\pi}{3} + 2\pi n, n \in I$ **b)** $\theta = \frac{5\pi}{3} + 2\pi n, n \in I$

3. a) $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$ **b)** $\theta = 0°, 180°$

c) $\theta = -135°, -45°, 45°, 135°, 225°, 315°$

d) $\theta = -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

4. a) $\theta = 1.35, 4.49$ **b)** $\theta = 1.76, 4.52$

c) $\theta = 1.14, 2.00$ **d)** $\theta = 0.08, 3.22$

e) 1.20 and 5.08 **f)** 3.83 and 5.59

5. a) $\theta = \pi$ **b)** $\theta = -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

c) $x = -315°, -225°, 45°, 135°$

d) $x = -150°, -30°$

e) $x = -45°, 135°, 315°$

f) $\theta = -\frac{5\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}$

6. a) $\theta \in [-2\pi, 2\pi]$ **b)** $\theta \in \left[-\frac{\pi}{3}, \frac{7\pi}{3}\right]$

c) $\theta \in [0°, 270°]$ **d)** $0 \le \theta < \pi$

e) $0° < \theta < 450°$ **f)** $-2\pi < \theta \le 4\pi$

7. a) $\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}$

b) $\theta = 63.435°, 243.435°, 135°, 315°$

c) $\theta = 0, \frac{\pi}{2}, \pi$

d) $\theta = -180°, -70.529°, 70.529°$

8. Check for $\theta = 180°$.
Left Side $= 5(\cos 180°)^2 = 5(-1)^2 = 5$
Right Side $= -4 \cos 180° = -4(-1) = 4$
Since Left Side $\neq$ Right Side, $\theta = 180°$ is not a solution.
Check for $\theta = 270°$.
Left Side $= 5(\cos 270°)^2 = 5(0)^2 = 0$
Right Side $= -4 \cos 270° = -4(0) = 0$
Since Left Side = Right Side, $\theta = 270°$ is a solution.

9. a) They should not have divided both sides of the equation by $\sin \theta$. This will eliminate one of the possible solutions.

b)
$$2 \sin^2 \theta = \sin \theta$$
$$2 \sin^2 \theta - \sin \theta = 0$$
$$\sin \theta(2 \sin \theta - 1) = 0$$
$$\sin \theta = 0 \quad \text{and} \quad 2 \sin \theta - 1 = 0$$
$$\sin \theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \pi$$

10. $\sin \theta = 0$ when $\theta = 0$, π, and 2π but none of these values are in the interval $(\pi, 2\pi)$.

11. $\sin \theta$ is only defined for the values $-1 \le \sin \theta \le 1$, and 2 is outside this range, so $\sin \theta = 2$ has no solution.

12. Yes, the general solutions are $\theta = \frac{\pi}{3} + 2\pi n, n \in I$ and $\theta = \frac{5\pi}{3} + 2\pi n, n \in I$. Since there are an infinite number of integers, there will be an infinite number of solutions coterminal with $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

13. a) Helene can check her work by substituting π for θ in the original equation.
$$\begin{aligned}\text{Left Side} &= 3(\sin \pi)^2 - 2 \sin \pi \\ &= 3(0)^2 - 2(0) \\ &= 0 \\ &= \text{Right Side}\end{aligned}$$

b) $\theta = 0, 0.7297, 2.4119, \pi$

14. 25.56°

15. a) June **b)** December

c) Yes. Greatest sales of air conditioners be expected to happen before the hottest months (June) and the least sales before the coldest months (December).

16. The solution is correct as far as the statement "Sine is negative in quadrants II and III." Sine is actually negative in quadrants III and IV. Quadrant III solution is $180° + 41.8° = 221.8°$ and quadrant IV solution is $360° - 41.8° = 318.2°$.

17. Examples: Tan 90° has no solution since division by 0 is undefined. $\sin \theta = 2$ does not have a solution. The range of $y = \sin \theta$ is $-1 \le y \le 1$ and 2 is beyond this range.

18. $\sec \theta = -\frac{5}{3}$

19. **a)** 0 s, 3 s, 6 s, 9 s **b)** 1.5 s, $1.5 + 6n$, $n \in W$
c) 1.4 m below sea level

20. **a)** Substitute I = 0, then $0 = 4.3 \sin 120\pi t$
$0 = \sin 120\pi t$
$\sin \theta = 0$ at $\theta = 0, \pi, 2\pi, \ldots$
$0 = 120\pi t \rightarrow t = 0$
$\pi = 120\pi t \rightarrow t = \frac{1}{120}$
$2\pi = 120\pi t \rightarrow t = \frac{1}{60}$
Since the current must alternate from 0 to positive back to 0 and then negative back to 0, it will take $\frac{1}{60}$ s for one complete cycle or 60 cycles in one second.
b) $t = 0.004\ 167 + \frac{1}{60}n$, $n \in W$ seconds
c) $t = 0.0125 + \frac{1}{60}n$, $n \in W$ seconds
d) 4.3 amps

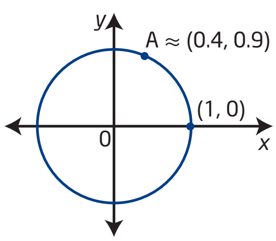
21. $x = \frac{\pi}{3}, \frac{2\pi}{3}$

22. **a)** No. **b)** $\sin \theta = \frac{-1 + \sqrt{5}}{2}$ and $\frac{-1 - \sqrt{5}}{2}$
c) 0.67, 2.48

23. **a)** The height of the trapezoid is $4 \sin \theta$ and its base is $4 + 2(4 \cos \theta)$. Use the formula for the area of a trapezoid:
$A = \frac{\text{sum of parallel sides}}{2} \times \text{height}$
$A = \left(\frac{4 + 4 + 8 \cos \theta}{2}\right)(4 \sin \theta)$
$A = 8(1 + \cos \theta)(2 \sin \theta)$
$A = 16 \sin \theta(1 + \cos \theta)$
b) $\frac{\pi}{3}$
c) Example: Graph $y = 16 \sin \theta(1 + \cos \theta)$ and find the maximum for domain in the first quadrant.

C1 The principles involved are the same up to the point where you need to solve for a trigonometric ratio.

C2 **a)** Check if $x^2 + y^2 = 1$. Yes, A is on the unit circle.
b) $\cos \theta = 0.385$, $\tan \theta = 2.400$, $\csc \theta = 1.083$
c) 67.4°; this angle measure seems reasonable as shown on the diagram.



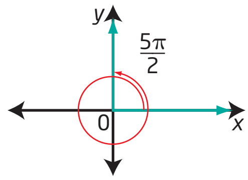
C3 **a)** Non-permissible values are values that the variable can never be because the expression is not defined in that case. For a rational expression, this occurs when the denominator is zero.
Example: $\frac{3}{x}$, $x \ne 0$
b) Example: $\tan \frac{\pi}{2}$ **c)** $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$
d) $\frac{\pi}{2} + \pi n$, $n \in I$

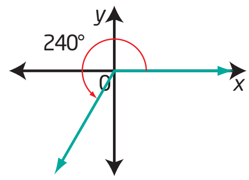
C4 **a)** 30°, 150°, 270°
b) Exact, because $\sin^{-1}(0.5)$ and $\sin^{-1}(-1)$ correspond to exact angle measures.
c) Example: Substitute $\theta = 30°$ in each side. Left side $= 2 \sin^2 30° = 2(0.5)^2 = 0.5$. Right side $= 1 - \sin 30° = 1 - 0.5 = 0.5$. The value checks.

Chapter 4 Review, pages 215 to 217

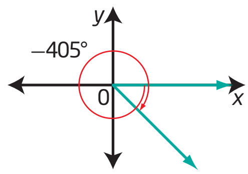
1. **a)** quadrant II **b)** quadrant II
c) quadrant III **d)** quadrant II

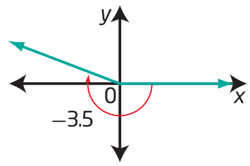
2. **a)** 450° **b)** $\frac{4\pi}{3}$





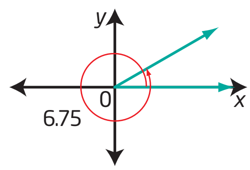
c) $-\frac{9\pi}{4}$ **d)** $-\frac{630°}{\pi}$

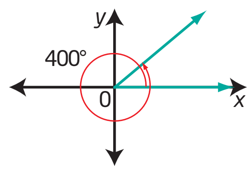




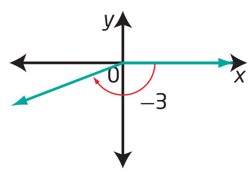
3. **a)** 0.35 **b)** −3.23 **c)** −100.27° **d)** 75°

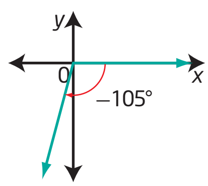
4. **a)** 0.467 **b)** 40°





c) 3.28 **d)** 255°





5. **a)** $250° \pm (360°)n$, $n \in N$ **b)** $\frac{5\pi}{2} \pm 2\pi n$, $n \in N$
c) $-300° \pm (360°)n$, $n \in N$ **d)** $6 \pm 2\pi n$, $n \in N$

6. **a)** $160\ 000\pi$ radians/minute **b)** 480 000°/s

7. **a)** $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$ **b)** $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ **c)** (0, 1)
d) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **e)** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **f)** $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

8. **a)** Reflect $P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ in the y-axis to give $P\left(\frac{2\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; then reflect this point in the x-axis to give $P\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. Reflect about the original point in the x-axis to give $P\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.
b) $\left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$

c) quadrant IV; $P\left(\frac{5\pi}{6}\right)$ lies in quadrant II and $P\left(\frac{5\pi}{6} + \pi\right)$ is a half circle away, so it lies in quadrant IV. $\theta = \frac{11\pi}{6}$ $P\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

9. a) $P\left(\frac{\pi}{2}\right)$ and $P\left(-\frac{3\pi}{2}\right)$ **b)** $P\left(\frac{11\pi}{6}\right)$ and $P\left(-\frac{\pi}{6}\right)$

c) $P\left(\frac{3\pi}{4}\right)$ and $P\left(-\frac{5\pi}{4}\right)$ **d)** $P\left(\frac{2\pi}{3}\right)$ and $P\left(-\frac{4\pi}{3}\right)$

10. a) P(–150°) and P(210°) **b)** P(180°)

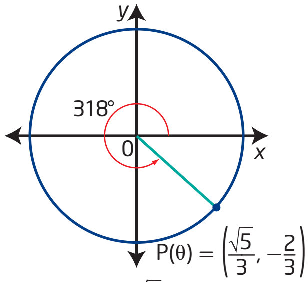
c) P(135°) **d)** P(–60°) and P(300°)

11. a) $\theta = 318°$ or 5.55 **b)** IV

c) $\left(-\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$

d) $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

e) $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

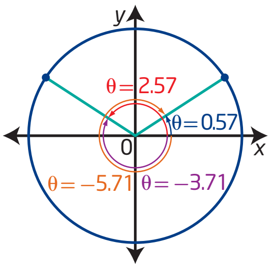


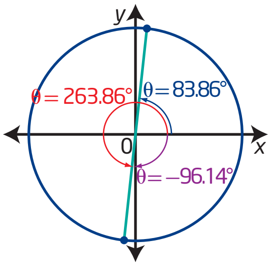
12. $\sin\theta = \frac{2\sqrt{2}}{3}$, $\tan\theta = 2\sqrt{2}$, $\sec\theta = 3$, $\csc\theta = \frac{3}{2\sqrt{2}}$ or $\frac{3\sqrt{2}}{4}$, $\cot\theta = \frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$

13. a) 1 **b)** $-\frac{\sqrt{2}}{2}$ **c)** $\sqrt{3}$

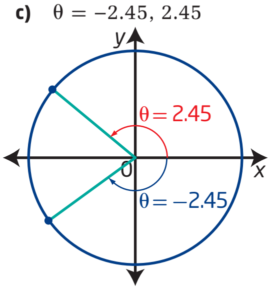
d) $-\frac{2\sqrt{3}}{3}$ **e)** 0 **f)** $-\frac{2\sqrt{3}}{3}$

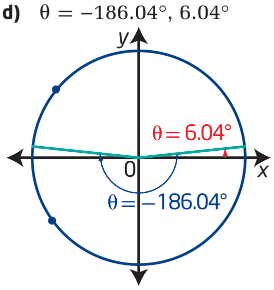
14. a) $\theta = -5.71, -3.71, 0.57$, and 2.57 **b)** $\theta = -96.14°, 83.86°, 263.86°$





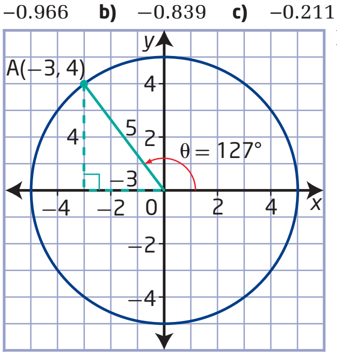
c) $\theta = -2.45, 2.45$ **d)** $\theta = -186.04°, 6.04°$





15. a) –0.966 **b)** –0.839 **c)** –0.211 **d)** 2.191

16. a) Example: 127°



b) $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = -\frac{3}{5}$ **c)** $-\frac{1}{12}$

d) 126.9° or 2.2

17. a) $\cos\theta(\cos\theta + 1)$ **b)** $(\sin\theta - 4)(\sin\theta + 1)$

c) $(\cot\theta + 3)(\cot\theta - 3)$ **d)** $(2\tan\theta - 5)(\tan\theta - 2)$

18. a) 2 is not a possible value for $\sin\theta$, $|\sin\theta| \le 1$

b) $\tan 90° = \frac{\sin 90°}{\cos 90°} = \frac{1}{0}$, but division by 0 is undefined, so tan 90° has no solutions

19. a) 2 solutions **b)** 2 solutions

c) 1 solution **d)** 6 solutions

20. a) $\theta = 45°, 135°$ **b)** $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

c) $\theta = -150°, 30°, 210°$ **d)** $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}$

21. a) $\theta = \frac{\pi}{2}$

b) $\theta = 108.435°, 180°, 288.435°, 360°$

c) $\theta = 70.529°, 120°, 240°$, and 289.471°

d) $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

22. Examples:

a) $0 \le \theta < 2\pi$ **b)** $-2\pi \le \theta < \frac{\pi}{2}$

c) $-720° \le \theta < 0°$ **d)** $-270° \le \theta < 450°$

23. a) $x = \frac{7\pi}{6} + 2\pi n, n \in I$ and $x = \frac{11\pi}{6} + 2\pi n, n \in I$

b) $x = 90° + (360°)n, n \in I$ and $x = (180°)n, n \in I$

c) $x = 120° + (360°)n, n \in I$ and $x = 240° + (360°)n, n \in I$

d) $x = \frac{\pi}{4} + \pi n, n \in I$ and $x = \frac{\pi}{3} + \pi n, n \in I$

Chapter 4 Practice Test, pages 218 to 219

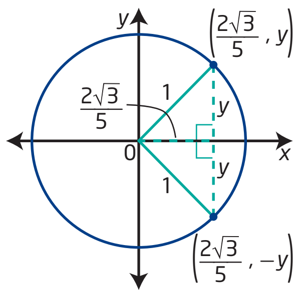
1. D **2.** C **3.** A **4.** B **5.** B

6. a) 4668.5° or 81.5

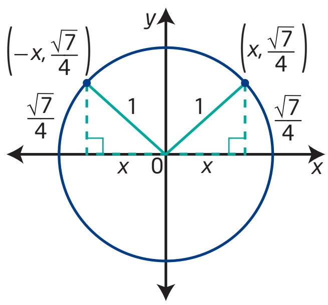
b) 92.6 Yes; a smaller tire requires more rotations to travel the same distance so it will experience greater tire wear.

7. a) $x^2 + y^2 = 1$

b) i) $y = \pm\frac{\sqrt{13}}{5}$



ii) $x = -\frac{3}{4}$



c) In the expression $\sin \theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, substitute the y-value for the opposite side and 1 for the hypotenuse. Since $x^2 + y^2 = 1$ then $\cos^2 \theta + \sin^2 \theta = 1$. Substitute the value you determined for $\sin \theta$ into $\cos^2 \theta + \sin^2 \theta = 1$ and solve for $\cos \theta$.

8. a) Cosine is negative in quadrants II and III. Find the reference angle by subtracting π from the given angle in quadrant III. To find the solution in quadrant II, subtract the reference angle from π.

b) Given each solution θ, add $2\pi n$, $n \in I$ to obtain each general solution $\theta + 2\pi n$, $n \in I$.

9. $\theta = \dfrac{3\pi}{4} + 2\pi n, n \in I$ or $\theta = \dfrac{5\pi}{4} + 2\pi n, n \in I$

10. Since $1° = \dfrac{\pi}{180}$, then $3° = \dfrac{3\pi}{180}$ or $\dfrac{\pi}{60}$.

$3 = \dfrac{3(180°)}{\pi} \approx 172°$.

11. a) quadrant III **b)** 40°

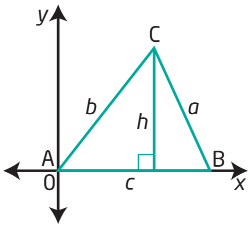
c) $\sin(-500°) = -0.6$, $\cos(-500°) = -0.8$, $\tan(-500°) = 0.8$, $\csc(-500°) = -1.6$, $\sec(-500°) = -1.3$, $\cot(-500°) = 1.2$

12. a) $\dfrac{5\pi}{4}, -\dfrac{3\pi}{4}; \dfrac{5\pi}{4} \pm 2\pi n, n \in N$

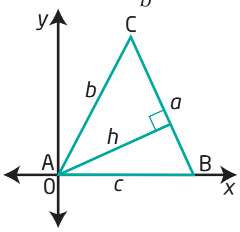
b) 145°, −215°, 145° ± (360°)n, $n \in N$

13. 7.7 km

14.



Given $A = \dfrac{1}{2}bh$, b = side c, since $\sin \theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ then $\sin A = \dfrac{h}{b}$ or $h = b \sin A$ and $A = \dfrac{1}{2}bc \sin A$ or



Given $A = \dfrac{1}{2}bh$, b = side a, since $\sin \theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ then $\sin B = \dfrac{h}{c}$ or $h = c \sin B$, therefore $A = \dfrac{1}{2}ac \sin B$.

15. a) $\theta = -\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{7\pi}{4}, -2.21, 0.93, 4.07$

b) 0.67, 2.48 **c)** 0, π, 2π, 4.47, 1.33

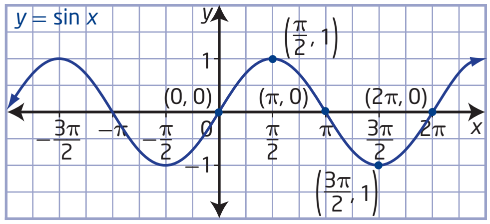
16. $\dfrac{28\pi}{3}$ m or 29.32 m

Chapter 5 Trigonometric Functions and Graphs

5.1 Graphing Sine and Cosine Functions, pages 233 to 237

1. a) $(0, 0), \left(\dfrac{\pi}{2}, 1\right), (\pi, 0), \left(\dfrac{3\pi}{2}, -1\right), (2\pi, 0)$

b)



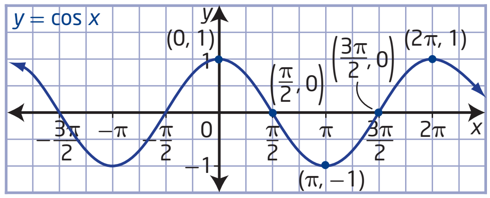
c) x-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$

d) y-intercept: 0

e) The maximum value is 1, and the minimum value is −1.

2. a) $(0, 1), \left(\dfrac{\pi}{2}, 0\right), (\pi, -1), \left(\dfrac{3\pi}{2}, 0\right), (2\pi, 1)$

b)



c) x-intercepts: $-\dfrac{3\pi}{2}, -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

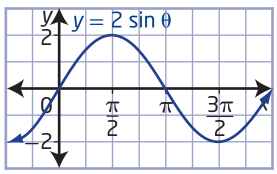
d) y-intercept: 1

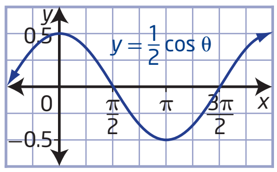
e) The maximum value is 1, and the minimum value is −1.

3.

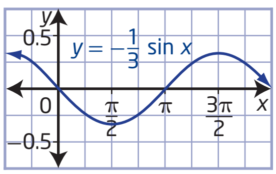
Property	$y = \sin x$	$y = \cos x$
maximum	1	1
minimum	−1	−1
amplitude	1	1
period	2π	2π
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid -1 \le y \le 1, y \in R\}$	$\{y \mid -1 \le y \le 1, y \in R\}$
y-intercept	0	1
x-intercepts	$\pi n, n \in I$	$\dfrac{\pi}{2} + \pi n, n \in I$

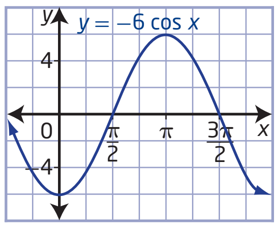
4. a) 2 **b)** $\dfrac{1}{2}$



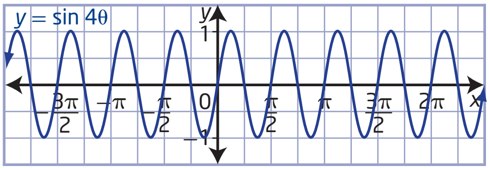


c) $\dfrac{1}{3}$ **d)** 6

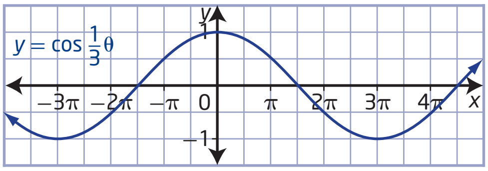




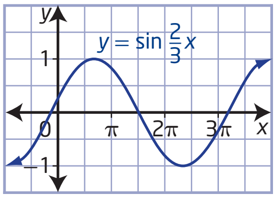
5. a) $\frac{\pi}{2}$ or 90°



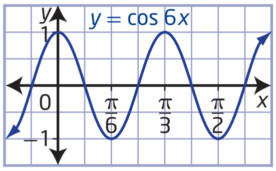
b) 6π or 1080°



c) 3π or 540°



d) $\frac{\pi}{3}$ or 60°



6. a) A **b)** D **c)** C **d)** B

7. a) Amplitude is 3; stretched vertically by a factor of 3 about the x-axis.

b) Amplitude is 5; stretched vertically by a factor of 5 about the x-axis and reflected in the x-axis.

c) Amplitude is 0.15; stretched vertically by a factor of 0.15 about the x-axis.

d) Amplitude is $\frac{2}{3}$; stretched vertically by a factor of $\frac{2}{3}$ about the x-axis and reflected in the x-axis.

8. a) Period is 180°; stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis.

b) Period is 120°; stretched horizontally by a factor of $\frac{1}{3}$ about the y-axis and reflected in the y-axis.

c) Period is 1440°; stretched horizontally by a factor of 4 about the y-axis.

d) Period is 540°; stretched horizontally by a factor of $\frac{3}{2}$ about the y-axis.

9. a) Amplitude is 2; period is 360° or 2π.

b) Amplitude is 4; period is 180° or π.

c) Amplitude is $\frac{5}{3}$; period is 540° or 3π.

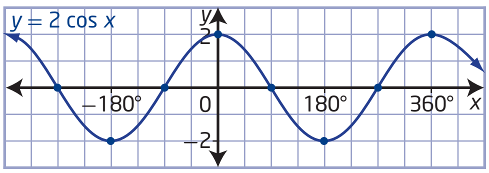
d) Amplitude is 3; period is 720° or 4π.

10. a) Graph A: Amplitude is 2 and period is 4π. Graph B: Amplitude is 0.5 and period is π.

b) Graph A: $y = 2 \sin \frac{1}{2}x$; Graph B: $y = 0.5 \cos 2x$

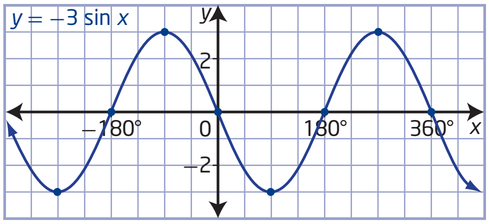
c) Graph A starts at 0, so the sine function is the obvious choice. Graph B starts at 1, so the cosine function is the obvious choice.

11. a)



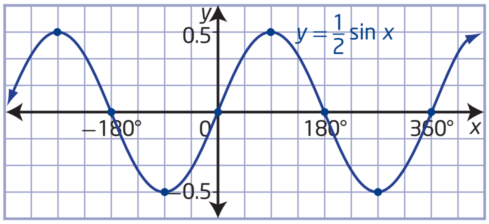
Property	Points on the Graph of $y = 2 \cos x$
maximum	(−360°, 2), (0°, 2), (360°, 2)
minimum	(−180°, −2), (180°, −2)
x-intercepts	(−270°, 0), (−90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, 2)
period	360°
range	$\{y \mid -2 \le y \le 2, y \in R\}$

b)



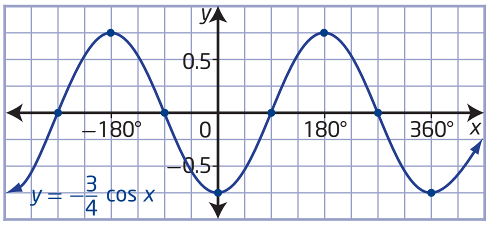
Property	Points on the Graph of $y = -3 \sin x$
maximum	(−90°, 3), (270°, 3)
minimum	(−270°, −3), (90°, −3)
x-intercepts	(−360°, 0), (−180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -3 \le y \le 3, y \in R\}$

c)



Property	Points on the Graph of $y = \frac{1}{2} \sin x$
maximum	(−270°, 0.5), (90°, 0.5)
minimum	(−90°, −0.5), (270°, −0.5)
x-intercepts	(−360°, 0), (−180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -0.5 \le y \le 0.5, y \in R\}$

d)



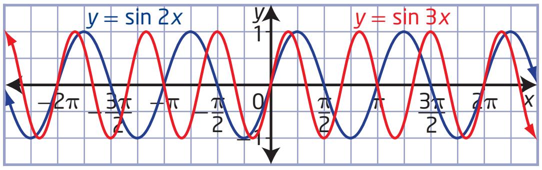
Property	Points on the Graph of $y = -\frac{3}{4} \cos x$
maximum	(−180°, 0.75), (180°, 0.75)
minimum	(−360°, −0.75), (0°, −0.75), (360°, −0.75)
x-intercepts	(−270°, 0), (−90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, −0.75)
period	360°
range	$\{y \mid -0.75 \le y \le 0.75, y \in R\}$

12. a) $B\left(\frac{\pi}{4}, 3\right)$, $C\left(\frac{\pi}{2}, 0\right)$, $D\left(\frac{3\pi}{4}, -3\right)$, $E(\pi, 0)$

b) $C\left(\frac{\pi}{2}, 0\right)$, $D(\pi, -2)$, $E\left(\frac{3\pi}{2}, 0\right)$, $F(2\pi, 2)$

c) $B(-3\pi, 1)$, $C(-2\pi, 0)$, $D(-\pi, -1)$, $E(0, 0)$

13.



The amplitude, maximum, minimum, y-intercepts, domain, and range are the same for both graphs. The period and x-intercepts are different.

14. a) Amplitude is 5; period is $\frac{4\pi}{3}$.

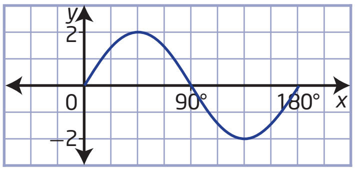
b) Amplitude is 4; Period is $\frac{2\pi}{3}$.

15. a) Amplitude is 20 mm Hg; Period is 0.8 s.

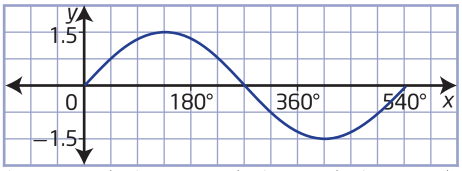
b) 75 bpm

16. Answers may vary.

17. a)



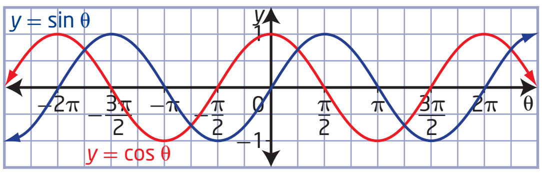
b)



18. a) $\left(-\frac{7\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(-\frac{5\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{9\pi}{4}, \frac{\sqrt{2}}{2}\right)$;
Find the points of intersection of $y = \sin\theta$ and $y = \frac{\sqrt{2}}{2}$.

b) $\left(-\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{13\pi}{6}, \frac{\sqrt{3}}{2}\right)$;
Find the points of intersection of $y = \cos\theta$ and $y = \frac{\sqrt{3}}{2}$.

19.



a) The graphs have the same maximum and minimum values, the same period, and the same domain and range.

b) The graphs have different x- and y-intercepts.

c) A horizontal translation could make them the same graph.

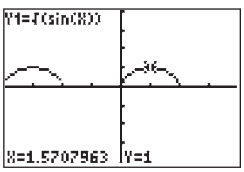
20. 12

21. a) $\frac{2\pi}{3}$ **b)** 12

22. 0.9

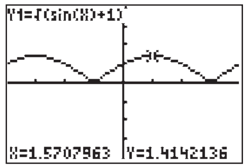
23. a) Example: The graph of $y = \sqrt{\sin x}$ will contain the portions of the graph of $y = \sin x$ that lie on or above the x-axis.

b)



c) Example: The function $y = \sqrt{\sin x + 1}$ is defined for all values of x, while the function $y = \sqrt{\sin x}$ is not.

d)



24. It is sinusoidal and the period is 2π.

C1 Step 5

a) The x-coordinate of each point on the unit circle represents $\cos\theta$. The y-coordinate of each point on the unit circle represents the $\sin\theta$.

b) The y-coordinates of the points on the sine graph are the same as the y-coordinates of the points on the unit circle. The y-coordinates of the points on the cosine graph are the same as the x-coordinates of the points on the unit circle.

C2 The constant is 1. The sum of the squares of the legs of each right triangle is equal to the radius of the unit circle, which is always 1.

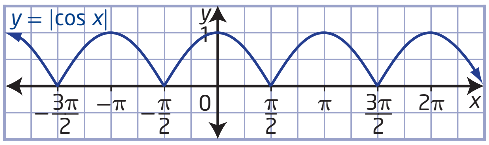
C3 a) Cannot determine because the amplitude is not given.

b) $f(4) = 0$; given in the question.

c) $f(84) = 0$; the period is 40° so it returns to 0 every 40°.

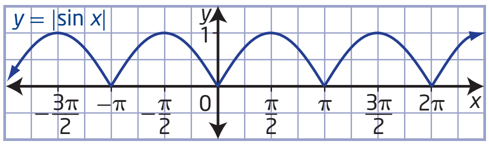
C4 a) Sine and Cosine **b)** Sine and Cosine
c) Sine and Cosine **d)** Sine and Cosine
e) Sine **f)** Cosine **g)** Cosine **h)** Sine
i) Cosine **j)** Sine **k)** Cosine **l)** Sine
m) Sine **n)** Cosine

C5 a)



The parts of the graph below the x-axis have been reflected across the x-axis.

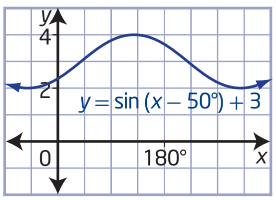
b)



The parts of the graph below the x-axis have been reflected across the x-axis.

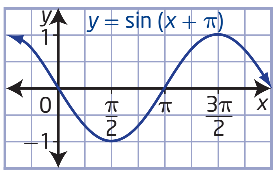
5.2 Transformations of Sinusoidal Functions, pages 250 to 255

1. a)

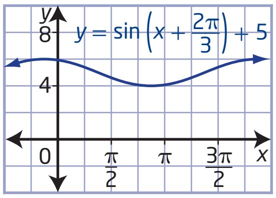


The phase shift is 50° right. The vertical displacement is 3 units up.

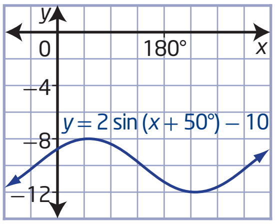
b)



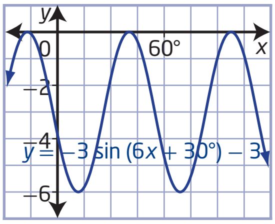
The phase shift is π units left. There is no vertical displacement.

c) 

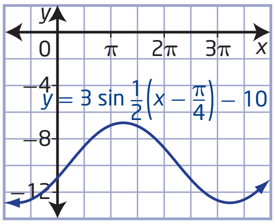
The phase shift is $\frac{2\pi}{3}$ units left. The vertical displacement is 5 units up.

d) 

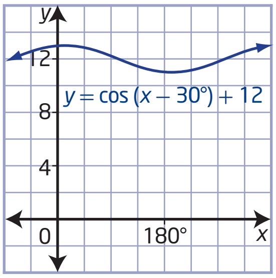
The phase shift is 50° left. The vertical displacement is 10 units down.

e) 

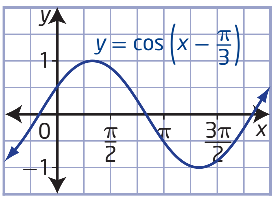
The phase shift is 5° left. The vertical displacement is 3 units down.

f) 

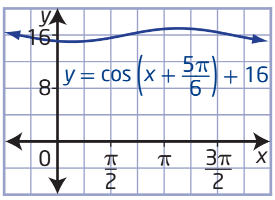
The phase shift is $\frac{\pi}{4}$ units right. The vertical displacement is 10 units down.

2. a) 

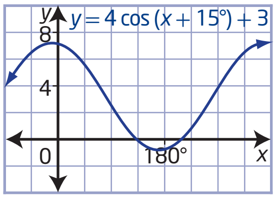
The phase shift is 30° right. The vertical displacement is 12 units up.

b) 

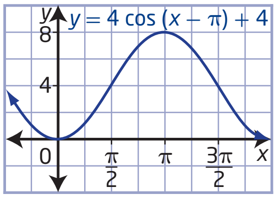
The phase shift is $\frac{\pi}{3}$ units right. There is no vertical displacement.

c) 

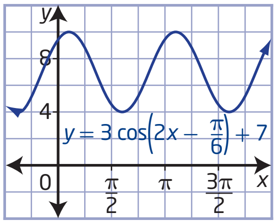
The phase shift is $\frac{5\pi}{6}$ units left. The vertical displacement is 16 units up.

d) 

The phase shift is 15° left. The vertical displacement is 3 units up.

e) 

The phase shift π units right. The vertical displacement is 4 units up.

f) 

The phase shift is $\frac{\pi}{12}$ units right. The vertical displacement is 7 units up.

3. a) **i)** $\{y \mid 2 \le y \le 8, y \in R\}$
ii) $\{y \mid -5 \le y \le -1, y \in R\}$
iii) $\{y \mid 2.5 \le y \le 5.5, y \in R\}$
iv) $\left\{y \mid \frac{1}{12} \le y \le \frac{17}{12}, y \in R\right\}$

b) Take the vertical displacement and add and subtract the amplitude to it. The region in between these points is the range.

4. a) D **b)** C **c)** B **d)** A **e)** E

5. a) D **b)** B **c)** C **d)** A

6. a) $y = 4\sin 2\left(x - \frac{\pi}{2}\right) - 6$

b) $y = 0.5\sin\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$

c) $y = \frac{3}{4}\sin\frac{1}{2}x - 5$

7. a) $a = 3, b = \frac{1}{2}, c = -2, d = 3; y = 3\cos\frac{1}{2}(x + 2) + 3$

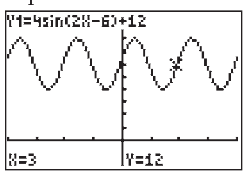
b) $a = \frac{1}{2}, b = 4, c = 3, d = -5;$
$y = \frac{1}{2}\cos 4(x - 3) - 5$

c) $a = -\frac{3}{2}, b = \frac{1}{3}, c = \frac{\pi}{4}, d = -1;$
$y = -\frac{3}{2}\cos\frac{1}{3}\left(x - \frac{\pi}{4}\right) - 1$

8. red, orange, yellow, green, blue, indigo, violet

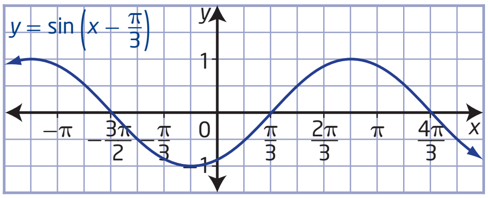
9. *b*

10. a) Stewart is correct. He remembered to factor the expression in brackets first.

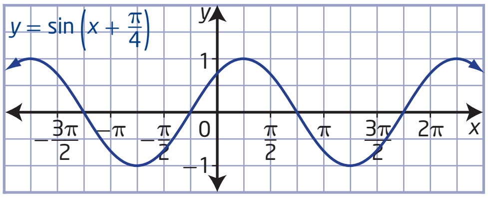
b) 

11. a) $\{y \mid -1 \le y \le 5, y \in R\}$ **b)** $\{y \mid -6 \le y \le 0, y \in R\}$
c) $\{y \mid -13 \le y \le -7, y \in R\}$
d) $\{y \mid 5 \le y \le 11, y \in R\}$

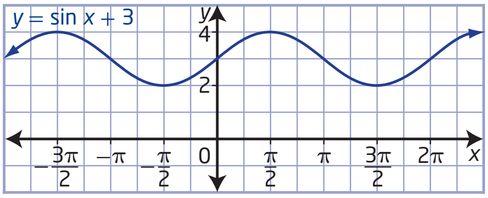
12. a)



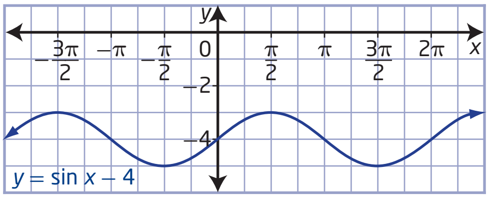
b)



c)



d)



13. $a = 9$, $d = -4$

14. a) **i)** 3 **ii)** 2π
iii) $\frac{\pi}{4}$ units right **iv)** none
v) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid -3 \le y \le 3, y \in \mathrm{R}\}$
vi) The maximum value of 3 occurs at $x = \frac{3\pi}{4}$.
vii) The minimum value of -3 occurs at $x = \frac{7\pi}{4}$.

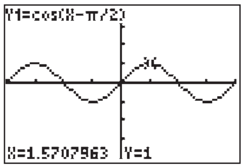
b) **i)** 2 **ii)** 2π
iii) $\frac{\pi}{2}$ units right **iv)** 2 units down
v) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid -4 \le y \le 0, y \in \mathrm{R}\}$
vi) The maximum value of 0 occurs at $x = \frac{\pi}{2}$.
vii) The minimum value of -4 occurs at $x = \frac{3\pi}{2}$.

c) **i)** 2 **ii)** π
iii) $\frac{\pi}{4}$ units right **iv)** 1 unit up
v) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid -1 \le y \le 3, y \in \mathrm{R}\}$
vi) The maximum value of 3 occurs at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.
vii) The minimum value of -1 occurs at $x = 0$, $x = \pi$, and $x = 2\pi$.

15. a) $y = 2 \sin x - 1$ **b)** $y = 3 \sin 2x + 1$
c) $y = 2 \sin 4\left(x - \frac{\pi}{4}\right) + 2$

16. a) $y = 2 \cos 2\left(x - \frac{\pi}{4}\right) + 1$
b) $y = 2 \cos \left(x + \frac{\pi}{2}\right) - 1$ **c)** $y = \cos (x - \pi) + 1$

17. a)



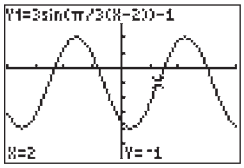
b) $y = \sin x$
c) The graph of the cosine function shifted $\frac{\pi}{2}$ units right is equivalent to the graph of the sine function.

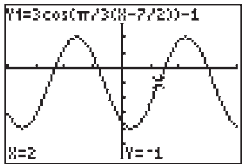
18. phase shift of $\frac{\pi}{2}$ units left

19. a) **i)** Phase shift is 30° right; period is 360°; x-intercepts are at 120° and 300°.
ii) Maximums occur at (30°, 3) and (390°, 3); minimum occurs at (210°, −3).
b) **i)** Phase shift is $\frac{\pi}{4}$ units right; period is π; x-intercepts are at $\frac{\pi}{2}$ and π
ii) Maximums occur at $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{5\pi}{4}, 3\right)$; minimum occurs at $\left(\frac{3\pi}{4}, -3\right)$.

20. $y = 50 \cos \frac{\pi}{2640}(x - 9240) + 5050$

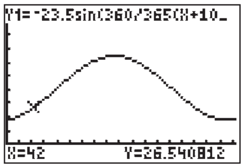
21. The graphs are equivalent.





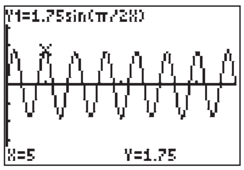
22. $y = 4 \sin 4(x + \pi)$

23. a)



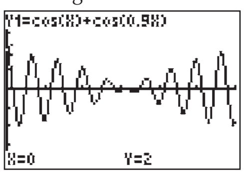
b) approximately 26.5°
c) day 171 or June 21

24. a) 4 s **b)** 15 cycles per minute
c)



d) The air flow velocity is 0 L/s. This corresponds to when the lungs are either completely full or completely empty.
e) The air flow velocity is −1.237 L/s. This corresponds to part of a cycle when the lungs are blowing out air.

25. a)



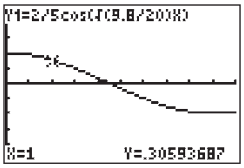
b) The amplitude is 2. The period is 20π.

26. a) **i)** 120° **ii)** $\frac{3\pi}{4}$ **iii)** π **iv)** $\frac{\pi}{4}$
b) Example: When graphed, a cosine function is ahead of the graph of a sine function by 90°. So, adding 90° to the phase shift in part a) works.

27. a) $y = 3\sin(x + \pi) + 2$ **b)** $y = 3\sin 2\left(x - \frac{\pi}{2}\right) + 1$

c) $y = 2\sin\left(x + \frac{\pi}{2}\right) + 5$ **d)** $y = 5\sin 3(x - 120°) - 1$

28. a) $P = \frac{2}{5}\cos\sqrt{\frac{9.8}{20}}t$



b) approximately −0.20 radians or 3.9 cm along the arc to the left of the vertical

C1 a changes the amplitude, b changes the period, c changes the phase shift, d changes the vertical translation; Answers may vary.

C2 a) They are exactly same.

b) This is because the sine of a negative number is the same as the negative sine of the number.

c) They are mirror images reflected in the x-axis.

d) It is correct.

C3 $\frac{5\pi}{4}$ square units

C4 a) $0 < b < 1$ **b)** $a > 1$

c) Example: $c = 0$, $d = 0$ **d)** $d > a$

e) Example: $c = -\frac{\pi}{2}$, $b = 1$, $d = 0$ **f)** $b = 3$

5.3 The Tangent Function, pages 262 to 265

1. a) 1, 45° **b)** −1.7, 120.5°

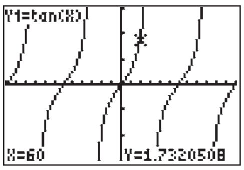
c) −1.7, 300.5° **d)** 1, 225°

2. a) undefined **b)** −1 **c)** 1

d) 0 **e)** 0 **f)** 1

3. No. The tangent function has no maximum or minimum, so there is no amplitude.

4.



−300°, −120°, 240°

5. $\frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta}$; $\tan\theta = \frac{\sin\theta}{\cos\theta}$

6. a) slope $= \frac{y}{x}$

b) Since y is equal to $\sin\theta$ and x is equal to $\cos\theta$, then $\tan\theta = \frac{y}{x}$.

c) slope $= \frac{\sin\theta}{\cos\theta}$ **d)** $\tan\theta = \frac{y}{x}$

7. a) $\tan\theta = \frac{y}{x}$ **b)** $\tan\theta = \frac{\sin\theta}{\cos\theta}$

c) $\sin\theta$ and $\cos\theta$ are equal to y and x, respectively.

8. a)

θ	tan θ
89.5°	114.59
89.9°	572.96
89.999°	57 295.78
89.999 999°	57 295 779.51

b) The value of tan θ increases to infinity.

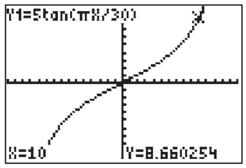
c)

θ	tan θ
90.5°	−114.59
90.01°	−5729.58
90.001°	−57 295.78
90.000 001°	−57 295 779.51

The value of tan θ approaches negative infinity.

9. a) $d = 5\tan\frac{\pi}{30}t$

b)

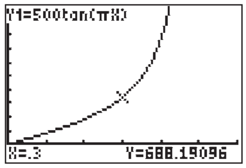


c) 8.7 m

d) At $t = 15$ s, the camera is pointing along a line parallel to the wall and is turning away from the wall.

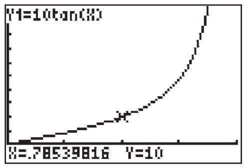
10. a) $d = 500\tan\pi t$

b)



c) The asymptote represents the moment when the ray of light shines along a line that is parallel to the shore.

11. $d = 10\tan x$



12. a) a tangent function

b) The slope would be undefined. It represents the place on the graph where the asymptote is.

13. Example:

a) (4, 3) **b)** 0.75

c) tan θ is the slope of the graph.

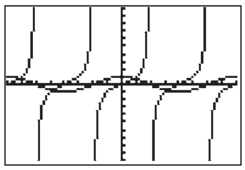
14. a) $\tan 0.5 \approx 0.5463$, power series ≈ 0.5463

b) $\sin 0.5 \approx 0.4794$, power series ≈ 0.4794

c) $\cos 0.5 \approx 0.8776$, power series ≈ 0.8776

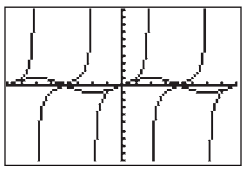
C1 The domain of $y = \sin x$ and $y = \cos x$ is all real numbers. The tangent function is not defined at $x = \frac{\pi}{2} + n\pi$, $n \in I$. Thus, these numbers must be excluded from the domain of $y = \tan x$.

C2 a)



Example: The tangent function has asymptotes at the same x-values where zeros occur on the cosine function.

b)



Example: The tangent function has zeros at the same x-values where zeros occur on the sine function.

C3 Example: A circular or periodic function repeats its values over a specific period. In the case of $y = \tan x$, the period is π. So, the equation $\tan(x + \pi) = \tan x$ is true for all x in the domain of $\tan x$.

5.4 Equations and Graphs of Trigonometric Functions, pages 275 to 281

1. a) $x = 0, \pi, 2\pi$ **b)** $x = \pi n$ where n is an integer

c) $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$

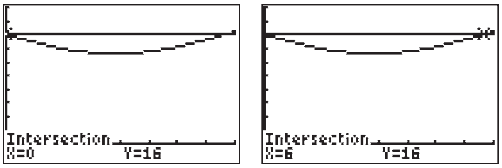
2. Examples:

a) 1.25, 4.5

b) −3, −1.9, 0.1, 1.2, 3.2, 4.1, 6.3, 7.2

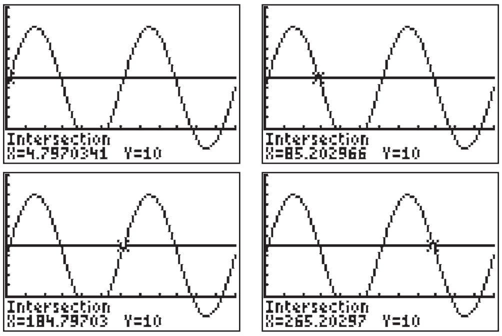
3. Examples: −50°, −10°, 130°, 170°, 310°, 350°

4. a)



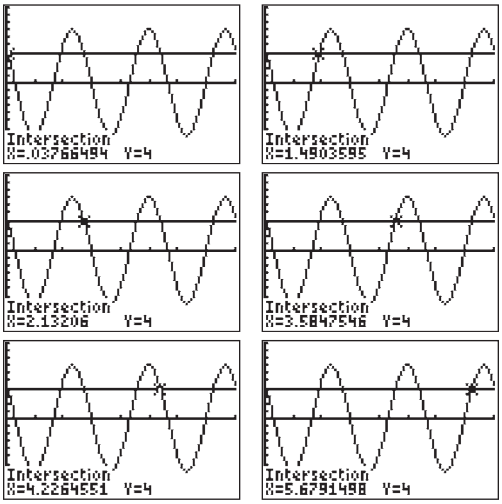
$x = 0$ and $x = 6$

b)



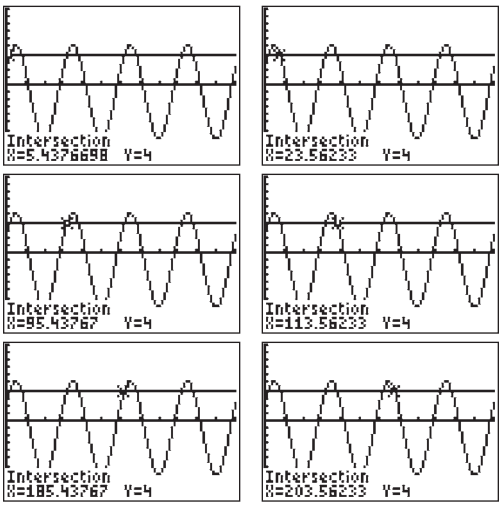
$x \approx 4.80°$, $x \approx 85.20°$, $x \approx 184.80°$, $x \approx 265.20°$

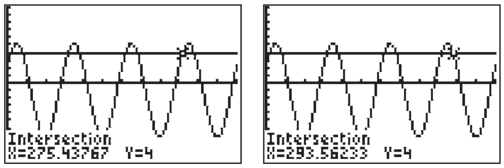
c)



$x \approx 0.04$, $x \approx 1.49$, $x \approx 2.13$, $x \approx 3.58$, $x \approx 4.23$, and $x \approx 5.68$

d)





$x \approx 5.44°$, $x \approx 23.56°$, $x \approx 95.44°$, $x \approx 113.56°$, $x \approx 185.44°$, $x \approx 203.56°$, $x \approx 275.44°$, and $x \approx 293.56°$

5. a) $x \approx 1.33$

b) $x \approx 3.59°$ and $x \approx 86.41°$

c) $x \approx 1.91 + \pi n$ and $x \approx 3.09 + \pi n$, where n is an integer

d) $x \approx 4.50° + (8°)n$ and $x \approx 7.50° + (8°)n$, where n is an integer

6. a) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{\mathrm{P} \mid 2000 \leq \mathrm{P} \leq 14\,000, \mathrm{P} \in \mathrm{N}\}$

b) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{h \mid 1 \leq h \leq 13, h \in \mathrm{R}\}$

c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{h \mid 6 \leq h \leq 18, h \in \mathrm{R}\}$

d) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{h \mid 5 \leq h \leq 23, h \in \mathrm{R}\}$

7. $\frac{1}{200}$ s or 5 ms

8. a) Period is 100°; sinusoidal axis is at $y = 15$; amplitude is 9.

b) Period is $\frac{4\pi}{3}$; sinusoidal axis is at $y = -6$; amplitude is 10.

c) Period is $\frac{1}{50}$ s or 20 ms; sinusoidal axis is at $y = 0$; amplitude is 10.

9. a) 28 m **b)** 0 min, 0.7 min, 1.4 min, …

c) 2 m **d)** 0.35 min, 1.05 min, 1.75 min, …

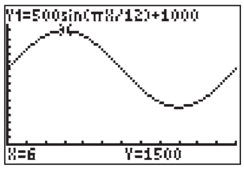
e) 0.18 min **f)** approximately 23.1 m

10. 78.5 cm

11. $V = 155 \sin 120\pi t$

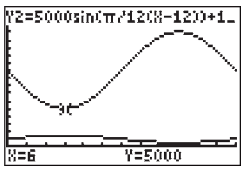
12. a) $\frac{1}{14}$ days **b)** 102.9 min **c)** 14 revolutions

13. a)



It takes approximately 15 months for the fox population to drop to 650.

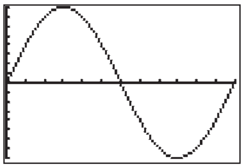
b)



c)

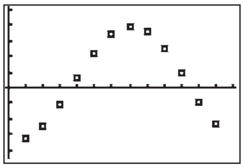
	Arctic Fox	Lemming
Maximum Population	1500	15 000
Month	6	18
Minimum Population	500	5000
Month	18	6

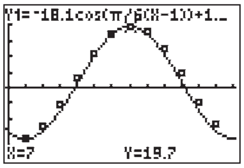
d) Example: The maximum for the predator occurs at a minimum for the prey and vice versa. The predators population depends on the prey, so every time the lemming's population changes the arctic fox population changes in accordance.

14. a) 

b) 35.1 cm
c) 1 s

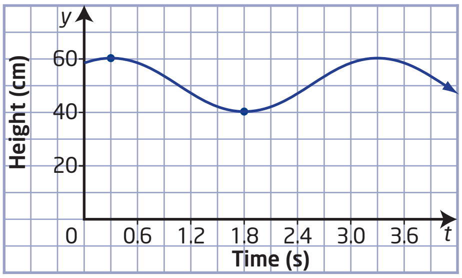
15. a) Maximum is 7.5 Sun widths; minimum is 1 Sun width.
b) 24 h
c) $y = -3.25 \sin \frac{\pi}{12}x + 4.25$, where x represents the time, in hours, and y represents the number of Sun widths

16. a) 

b) 1.6 °C
c) $y = -18.1 \cos \frac{\pi}{6}(x - 1) + 1.6$, where x represents the time, in months, and y represents the average monthly temperature, in degrees Celsius, for Winnipeg, Manitoba
d) 

e) about 2.5 months

17. a) $T = -4.5 \cos \frac{\pi}{30}t + 38.5$
b) 36.25 °C

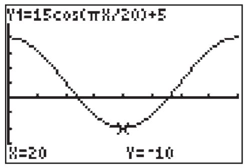
18. a) 

b) $y = 10 \sin \frac{2\pi}{3}(t + 0.45) + 50$, where t represents the time, in seconds, and y represents the height of the mass, in centimetres, above the floor
c) 43.3 cm
d) 0.0847 s

19. a) $h = -10 \cos \frac{\pi}{30}t + 12$, where t represents the time, in seconds, and h represents the height of a passenger, in metres, above the ground
b) 15.1 m
c) approximately 21.1 s, 38.9 s

20. a) $h = 7 \sin \frac{2\pi}{5}(t + 1.75) + 15$ or $h = 7 \cos \frac{2\pi}{5}(t + 0.5) + 15$, where t represents the time, in seconds, and h represents the height of the tip of the blade, in metres, above the ground
b) 20.66 m
c) 4.078 s

21. a) $y = -9.7 \cos \frac{\pi}{183}(t - 26) + 13.9$, where t represents the time, in days, and y represents the average daily maximum temperature, in degrees Celsius
b) 18.6 °C
c) 88 days

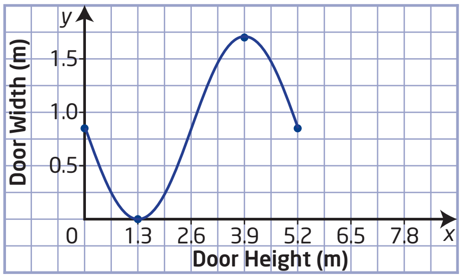
22. a) $y = 15 \cos \frac{\pi}{20}t + 5$
b) 

c) approximately +9.6% of the total assets
d) Example: No, because it fluctuates too much.

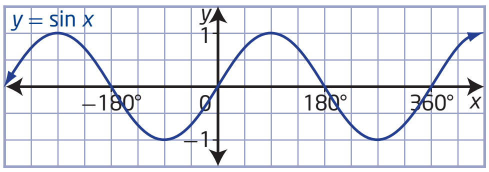
23. a) $y = 1.2 \sin \frac{\pi}{2}t$, where t represents the time, in seconds, and y represents the distance for a turn, in metres, from the midline
b) $y = 1.2 \sin \frac{2\pi}{5}t$; The period increases.

C1 Examples:
a) Use a sine function as a model when the curve or data begins at or near the intersection of the vertical axis and the sinusoidal axis.
b) Use a cosine function as a model when the curve or data has a maximum or minimum near or at the vertical axis.

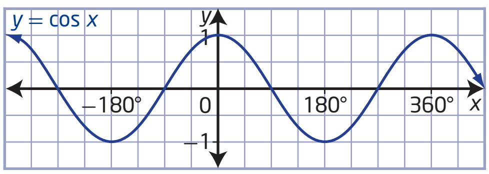
C2 Example:
a)–b) The parameter b has the greatest influence on the graph of the function. It changes the period of the function. Parameters c and d change the location of the curve, but not the shape. Parameter a changes the maximum and minimum values.

C3 Examples:
a) $y = -0.85 \sin \frac{2\pi}{5.2}x + 0.85$, where x represents the height of the door, in metres, and y represents the width of the door, in metres
b) 

Chapter 5 Review, pages 282 to 285

1. a) 

x-intercepts: −360°, −180°, 0°, 180°, 360°
b) y-intercept: 0
c) domain $\{x \mid x \in R\}$, range $\{y \mid -1 \leq y \leq 1, y \in R\}$, period is 2π
d) $y = 1$

2. a) 

x-intercepts: −270°, −90°, 90°, 270°
b) y-intercept: 1

c) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$, period is 2π

d) $y = 1$

3. a) A **b)** D **c)** B **d)** C

4. a) Amplitude is 3; period is π or $180°$.

b) Amplitude is 4; period is 4π or $720°$.

c) Amplitude is $\frac{1}{3}$; period is $\frac{12\pi}{5}$ or $432°$.

d) Amplitude is 5; period is $\frac{4\pi}{3}$ or $240°$.

5. a) Compared to the graph of $y = \sin x$, the graph of $y = \sin 2x$ completes two cycles in $0° \le x \le 360°$ and the graph of $y = 2 \sin x$ has an amplitude of 2.

b) Compared to the graph of $y = \sin x$, the graph of $y = -\sin x$ is reflected in the x-axis and the graph of $y = \sin(-x)$ is reflected in the y-axis. The graphs of $y = -\sin x$ and $y = \sin(-x)$ are the same.

c) Compared to the graph of $y = \cos x$, the graph of $y = -\cos x$ is reflected in the x-axis and the graph of $y = \cos(-x)$ is reflected in the y-axis. The graph of $y = \cos(-x)$ is the same as $y = \cos x$.

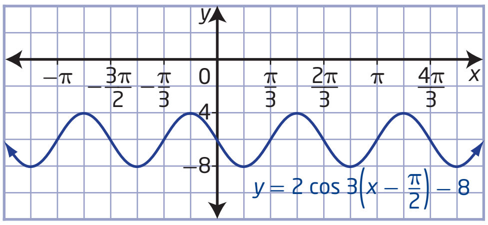
6. a) $y = 3 \cos 2x$ **b)** $y = 4 \cos \frac{12}{5}x$

c) $y = \frac{1}{2} \cos \frac{1}{2}x$ **d)** $y = \frac{3}{4} \cos 12x$

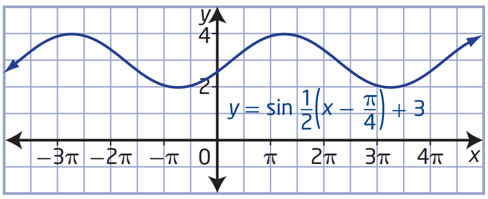
7. a) $y = 8 \sin 2x$ **b)** $y = 0.4 \sin 6x$

c) $y = \frac{3}{2} \sin \frac{1}{2}x$ **d)** $y = 2 \sin 3x$

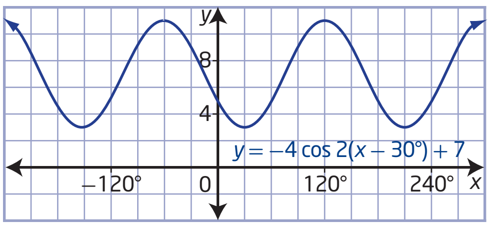
8. a) Amplitude is 2; period is $\frac{2\pi}{3}$; phase shift is $\frac{\pi}{2}$ units right; vertical displacement is 8 units down



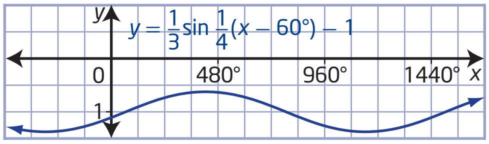
b) Amplitude is 1; period is 4π; phase shift is $\frac{\pi}{4}$ units right; vertical displacement is 3 units up



c) Amplitude is 4; period is $180°$; phase shift is $30°$ right; vertical displacement is 7 units up



d) Amplitude is $\frac{1}{3}$; period is $1440°$; phase shift is $60°$ right; vertical displacement is 1 unit down



9. a) They both have periods of π.

b) $f(x)$ has a phase shift of $\frac{\pi}{2}$ units right; $g(x)$ has a phase shift of $\frac{\pi}{4}$ units right

c) π units right **d)** $\frac{\pi}{b}$ units right

10. a) $y = 3 \sin 2(x - 45°) + 1$, $y = -3 \cos 2x + 1$

b) $y = 2 \sin 2x - 1$, $y = 2 \cos 2(x - 45°) - 1$

c) $y = 2 \sin 2\left(x - \frac{\pi}{4}\right) - 1$, $y = -2 \cos 2x - 1$

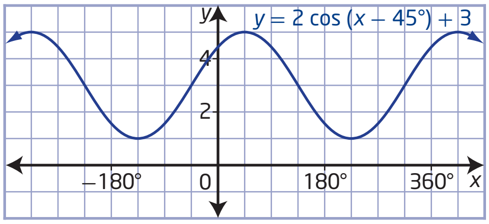
d) $y = 3 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right) + 1$, $y = 3 \cos \frac{1}{2}\left(x - \frac{3\pi}{2}\right) + 1$

11. a) $y = 4 \sin 2\left(x - \frac{\pi}{3}\right) - 5$

b) $y = \frac{1}{2} \cos \frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$

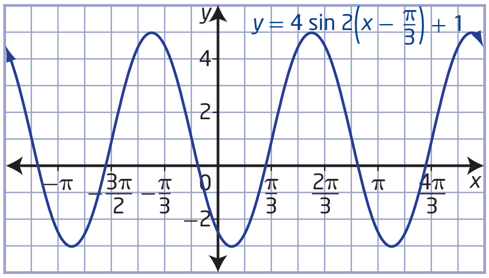
c) $y = \frac{2}{3} \sin \frac{2}{3}x - 5$

12. a)



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid 1 \le y \le 5, y \in \mathrm{R}\}$, maximum value is 5, minimum value is 1, no x-intercepts, y-intercept of approximately 4.41

b)



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid -3 \le y \le 5, y \in \mathrm{R}\}$, maximum value is 5, minimum value is -3, x-intercepts: approximately $0.92 + n\pi$, $2.74 + n\pi$, $n \in \mathrm{I}$, y-intercept: approximately -2.5

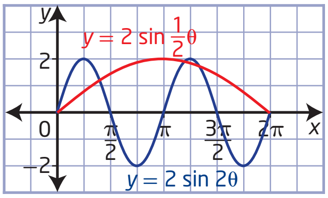
13. a) vertically stretched by a factor of 3 about the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated $\frac{\pi}{3}$ units right and 6 units up

b) vertically stretched by a factor of 2 about the x-axis, reflected in the x-axis, horizontally stretched by a factor of 2 about the y-axis, translated $\frac{\pi}{4}$ units left and 3 units down

c) vertically stretched by a factor of $\frac{3}{4}$ about the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated $30°$ right and 10 units up

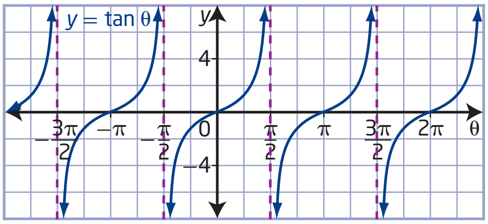
d) reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated 45° left and 8 units down

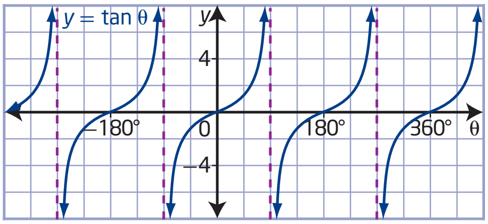
14. a)



b) Compared to the graph of $y = \sin \theta$, the graph of $y = 2 \sin 2\theta$ is vertically stretched by a factor of 2 about the x-axis and half the period. Compared to the graph of $y = \sin \theta$, the graph of $y = 2 \sin \frac{1}{2}\theta$ is vertically stretched by a factor of 2 about the x-axis and double the period.

15. a)





b) **i)** domain $\{x \mid -2\pi \le x \le 2\pi, x \ne -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, x \in \mathrm{R}\}$ or $\{x \mid -360° \le x \le 360°, x \ne -270°, -90°, 90°, 270°, x \in \mathrm{R}\}$
ii) range $\{y \mid y \in \mathrm{R}\}$ **iii)** y-intercept: 0
iv) x-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$ or $-360°, -180°, 0°, 180°, 360°$
v) asymptotes: $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = -270°, -90°, 90°, 270°$

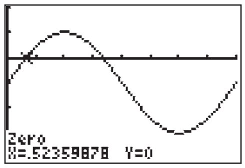
16. a) $\left(1, \frac{1}{\sqrt{3}}\right)$ **b)** $\tan \theta = \frac{\sin \theta}{\cos \theta}$
c) As θ approaches 90°, tan θ approaches infinity.
d) tan 90° is not defined.

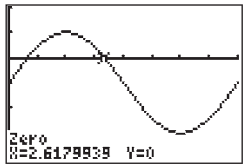
17. a) Since cos θ is the denominator, when it is zero tan θ becomes undefined.
b) Since sin θ is the numerator, when it is zero tan θ becomes zero.

18. The shadow has no length which makes the slope infinite. This relates to the asymptotes on the graph of $y = \tan \theta$.

19. A vertical asymptote is an imaginary line that the graph comes very close to touching but in fact never does. If a trigonometric function is represented by a quotient, such as the tangent function, asymptotes generally occur at values for which the function is not defined; that is, when the function in the denominator is equal to zero.

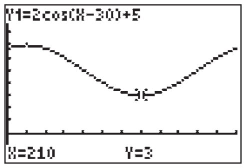
20. a)





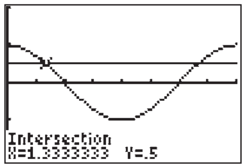
$x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$ or $x \approx 0.52$ and $x \approx 2.62$

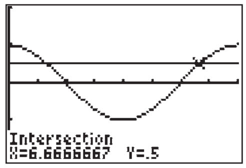
b)



no solution

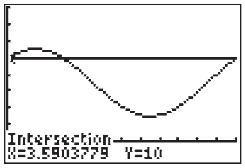
c)

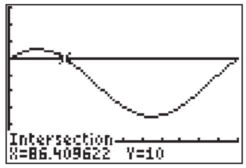




$x \approx 1.33 + 8n$ radians and
$x \approx 6.67 + 8n$ radians, where n is an integer

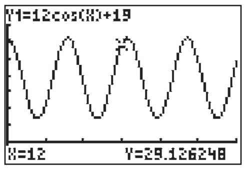
d)





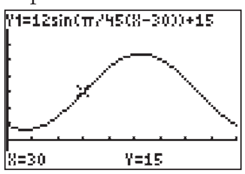
$x \approx 3.59° + (360°)n$ and
$x \approx 86.41° + (360°)n$, where n is an integer

21. a)



b) 9.4 h
c) Example: A model for temperature variance is important for maintaining constant temperatures to preserve artifacts.

22. a)



b) maximum height: 27 m, minimum height: 3 m
c) 90 s **d)** approximately 25.4 m

23. a) $L = -3.7 \cos \frac{2\pi}{365}(t + 10) + 12$
b) approximately 12.8 h of daylight

24. a) approximately 53 sunspots
b) around the year 2007
c) around the year 2003

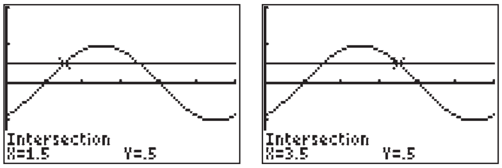
Chapter 5 Practice Test, pages 286 to 287

1. A **2.** D **3.** C **4.** D **5.** B **6.** A **7.** C **8.** $\frac{\pi}{2}$

9. asymptotes: $x = \frac{\pi}{2} + n\pi, n \in \mathrm{I}$,
domain $\left\{x \mid x \ne \frac{\pi}{2} + n\pi, x \in \mathrm{R}, n \in \mathrm{I}\right\}$,
range $\{y \mid y \in \mathrm{R}\}$, period is π

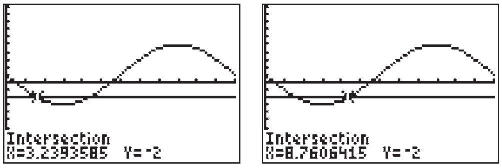
10. Example: They have the same maximum and minimum values. Neither function has a horizontal or vertical translation.

11. Amplitude is 120; period is 0.0025 s or 2.5 ms.
12. The minimum depth of 2 m occurs at 0 h, 12 h, and 24 hour. The maximum depth of 8 m occurs at 6 h and 18 h.
13. a)



$x = 1.5 + 6n$ radians and $x = 3.5 + 6n$ radians, where n is an integer

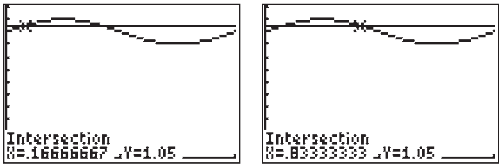
b)



$x \approx 3.24° + (24°)n$ and
$x \approx 8.76° + (24°)n$, where n is an integer

14. Example: Graph II has half the period of graph I. Graph I represents a cosine curve with no phase shift. Graph II represents a sine curve with no phase shift. Graph I and II have the same amplitude and both graphs have no vertical translations.
15. a) $h = 0.1 \sin \pi t + 1$, where t represents the time, in seconds, and h represents the height of the mass, in metres, above the floor

b)



approximately 0.17 s and 0.83 s

c) $t = \frac{1}{6}$ or 0.1666... and $t = \frac{5}{6}$ or 0.8333

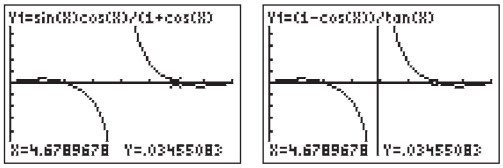
16. a) $y = 3 \sin 2\left(x - \frac{\pi}{4}\right) - 1$ b) $y = -3 \cos 2x - 1$
17. a) A, B b) A, B or C, D, E c) B

Chapter 6 Trigonometric Identities

6.1 Reciprocal, Quotient, and Pythagorean Identities, pages 296 to 298

1. a) $x \neq \pi n$; $n \in I$ b) $x \neq \left(\frac{\pi}{2}\right)n$, $n \in I$
c) $x \neq \frac{\pi}{2} + 2\pi n$ and $x \neq \pi n$, $n \in I$
d) $x \neq \frac{\pi}{2} + \pi n$ and $x \neq \pi + 2\pi n$, $n \in I$
2. Some identities will have non-permissible values because they involve trigonometric functions that have non-permissible values themselves or a function occurs in a denominator. For example, an identity involving $\sec \theta$ has non-permissible values $\theta \neq 90° + 180°n$, where $n \in I$, because these are the non-permissible values for the function.
3. a) $\tan x$ b) $\sin x$ c) $\sin x$
4. a) $\cot x$ b) $\csc x$ c) $\sec x$
5. a) When substituted, both values satisfied the equation.
b) $x \neq 0°, 90°, 180°, 270°$
6. a) $x \neq \pi + 2\pi n$, $n \in I$; $x \neq \frac{\pi}{2} + \pi n$, $n \in I$

b)

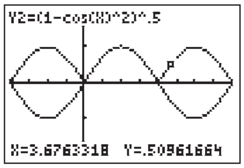


Yes, it appears to be an identity.

c) The equation is verified for $x = \frac{\pi}{4}$.

7. a) $\cos^2 \theta$ b) 0.75 c) 25%
8. a) All three values check when substituted.

b)



c) The equation is not an identity since taking the square then the square root removes the negative sign and $\sin x$ is negative from π to 2π.

9. a) $E = \frac{I \cos \theta}{R^2}$ b) $E = \frac{I \cot \theta}{R^2 \csc \theta}$

$$E = \frac{I\left(\frac{\cos \theta}{\sin \theta}\right)}{R^2\left(\frac{1}{\sin \theta}\right)}$$

$$E = \left(\frac{I \cos \theta}{\sin \theta}\right)\left(\frac{\sin \theta}{R^2}\right)$$

$$E = \frac{I \cos \theta}{R^2}$$

10. $\cos x$, $x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
11. a) It appears to be equivalent to $\sec x$.
b) $x \neq \frac{\pi}{2} + \pi n$, $n \in I$
c)

$$\frac{\csc^2 x - \cot^2 x}{\cos x} = \frac{\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}}{\cos x}$$
$$= \frac{\frac{1 - \cos^2 x}{\sin^2 x}}{\cos x}$$
$$= \frac{\frac{\sin^2 x}{\sin^2 x}}{\cos x}$$
$$= \frac{1}{\cos x}$$
$$= \sec x$$

12. a) Yes, it could be an identity.
b)

$$\frac{\cot x}{\sec x} + \sin x = \frac{\cos x}{\sin x} \div \frac{1}{\cos x} + \sin x$$
$$= \frac{\cos^2 x}{\sin x} + \sin x$$
$$= \frac{\cos^2 x + \sin^2 x}{\sin x}$$
$$= \csc x$$

13. a) $1 = 1$
b) The left side = 1, but the right side is undefined.
c) The chosen value is not permissible for the tan x function.
d) The left side $= \frac{2}{\sqrt{2}}$, but the right side = 2.
e) Giselle has found a permissible value for which the equation is not true, so they can conclude that it is not an identity.
14. 2
15. 7.89

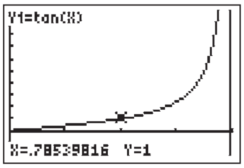
16. $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=\frac{1-\sin\theta+1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$
$=\frac{2}{(1-\sin^2\theta)}$
$=2\sec^2\theta$

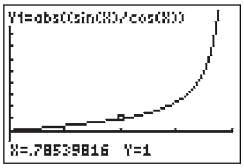
17. $m=\csc x$

C1 $\cot^2 x+1$
$=\frac{\cos^2 x}{\sin^2 x}+\frac{\sin^2 x}{\sin^2 x}$
$=\frac{\cos^2 x+\sin^2 x}{\sin^2 x}$
$=\frac{1}{\sin^2 x}$
$=\csc^2 x$

C2 $\left(\frac{\sin\theta}{1+\cos\theta}\right)\left(\frac{1-\cos\theta}{1-\cos\theta}\right)$
$=\frac{\sin\theta-\sin\theta\cos\theta}{1-\cos^2\theta}$
$=\frac{\sin\theta-\sin\theta\cos\theta}{\sin^2\theta}$
$=\frac{1-\cos\theta}{\sin\theta}$
It helps to simplify by creating an opportunity to use the Pythagorean identity.

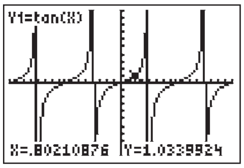
C3 **Step 1**

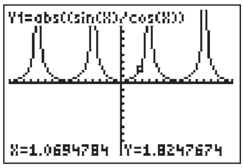




Yes, over this domain it is an identity.

Step 2





The equation is not an identity since the graphs of the two sides are not the same.

Step 3 Example: $y=\cot\theta$ and $y=\left|\frac{\cos\theta}{\sin\theta}\right|$ are identities over the domain $0\le\theta\le\frac{\pi}{2}$ but not over the domain $-2\pi\le\theta\le 2\pi$

Step 4 The weakness with this approach is that for some more complicated identities you may think it is an identity when really it is only an identity over that domain.

6.2 Sum, Difference, and Double-angle Identities, pages 306 to 308

1. a) $\cos 70°$ b) $\sin 35°$ c) $\cos 38°$
d) $\sin\frac{\pi}{4}$ e) $4\sin\frac{2\pi}{3}$

2. a) $\cos 60°=0.5$ b) $\sin 45°=\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$
c) $\cos\frac{\pi}{3}=0.5$ d) $\cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}$

3. $\cos 2x=1-2\sin^2 x$;
$1-\cos 2x=1-1+2\sin^2 x=2\sin^2 x$

4. a) $\sin\frac{\pi}{2}$ b) $6\sin 48°$ c) $\tan 152°$ d) $\cos\frac{\pi}{3}$
e) $-\cos\frac{\pi}{6}$

5. a) $\sin\theta$ b) $\cos x$ c) $\cos\theta$ d) $\cos x$

6. Example: When $x=60°$ and $y=30°$, then left side $=0.5$, but right side ≈ 0.366.

7. $\cos(90°-x)=\cos 90°\cos x+\sin 90°\sin x$
$=\sin x$

8. a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$ b) $\frac{-\sqrt{3}+1}{\sqrt{3}+1}$ or $\sqrt{3}-2$
c) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$ d) $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{-\sqrt{6}-\sqrt{2}}{4}$
e) $\sqrt{2}(1+\sqrt{3})$ f) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}-\sqrt{6}}{4}$

9. a) $P=1000\sin(x+113.5°)$
b) i) 101.056 W/m^2 ii) 310.676 W/m^2
iii) -50.593 W/m^2
c) The answer in part iii) is negative which means that there is no sunlight reaching Igloolik. At latitude 66.5°, the power received is 0 W/m^2.

10. $-2\cos x$

11. a) $\frac{119}{169}$ b) $-\frac{120}{169}$ c) $-\frac{12}{13}$

12. a) Both sides are equal for this value.
b) Both sides are equal for this value.
c) $\tan 2x=\frac{2\tan x}{1-\tan^2 x}$
$=\frac{2\tan x}{1-\tan^2 x}\left(\frac{\cos^2 x}{\cos^2 x}\right)$
$=\frac{2\left(\frac{\sin x}{\cos x}\right)(\cos^2 x)}{\left(1-\frac{\sin^2 x}{\cos^2 x}\right)\cos^2 x}$
$=\frac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$

13. a) $d=\frac{v_o^2\sin 2\theta}{g}$ b) 45°
c) It is easier after applying the double-angle identity since there is only one trigonometric function whose value has to be found.

14. $k-1$

15. a) $\cos^4 x-\sin^4 x=(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)$
$=\cos^2 x-\sin^2 x$
$=\cos 2x$
b) $\frac{\csc^2 x-2}{\csc^2 x}=1-\frac{2}{\csc^2 x}$
$=1-2\sin^2 x$
$=\cos 2x$

16. a) $\frac{1-\cos 2x}{2}=\frac{1-1+2\sin^2 x}{2}=\sin^2 x$
b) $\frac{4-8\sin^2 x}{2\sin x\cos x}=\frac{4\cos 2x}{\sin 2x}=\frac{4}{\tan 2x}$

17. $-\frac{2}{\sqrt{29}}$

18. $k=3$

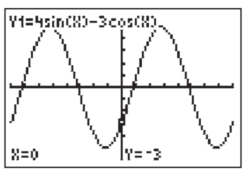
19. a) $0.9928, -0.392\ 82$ or $\frac{\pm 4\sqrt{3}+3}{10}$
b) 0.9500 or $\frac{\sqrt{5}+2\sqrt{3}}{6}$

20. a) $\frac{56}{65}$ b) $\frac{63}{65}$ c) $\frac{-7}{25}$ d) $\frac{24}{25}$

21. a) $\sin x$ b) $\tan x$

22. $\cos x=2\cos^2\left(\frac{x}{2}\right)-1$
$\frac{\cos x+1}{2}=\cos^2\left(\frac{x}{2}\right)$
$\pm\sqrt{\frac{\cos x+1}{2}}=\cos\frac{x}{2}$

23. a)



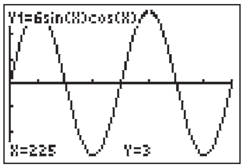
b) $a=5$, $c=37°$
c) $y=5\sin(x-36.87°)$

24. $y = 3 \sin 2x - 3$

C1 a) **i)** $\frac{120}{169}$ or 0.7101 **ii)** $\frac{120}{169}$ or 0.7101

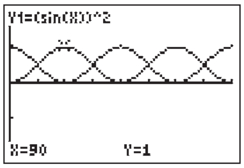
b) Using identities is more straightforward.

C2 a)

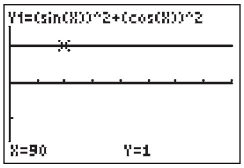


b) To find the sine function from the graph, compare the amplitude and the period to that of a base sine curve. The alternative equation is $y = 3 \sin 2x$.

C3 a)

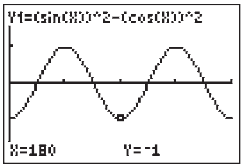


b)



The graph will be the horizontal line $y = 1$.

c)



The resultant graph is a cosine function reflected over the x-axis and the period becomes π.

d) $f(x) = -\cos 2x$. Using trigonometric identities,

$$\begin{aligned}\sin^2 x - \cos^2 x &= 1 - \cos^2 x - \cos^2 x.\\ &= 1 - 2\cos^2 x\\ &= -\cos 2x\end{aligned}$$

6.3 Proving Identities, pages 314 to 315

1. a) $\sin x$ **b)** $\frac{\cos x + 1}{6}$

c) $\frac{\sin x}{\cos x + 1}$ **d)** $\sec x - 4 \csc x$

2. a)
$$\begin{aligned}\cos x + \cos x \tan^2 x &= \cos x + \frac{\sin^2 x}{\cos x}\\ &= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}\\ &= \frac{1}{\cos x}\\ &= \sec x\end{aligned}$$

b)
$$\begin{aligned}\frac{\sin^2 x - \cos^2 x}{\sin x + \cos x} &= \frac{(\sin x - \cos x)(\sin x + \cos x)}{\sin x + \cos x}\\ &= \sin x - \cos x\end{aligned}$$

c)
$$\begin{aligned}\frac{\sin x \cos x - \sin x}{\cos^2 x - 1} &= \frac{\sin x \cos x - \sin x}{-\sin^2 x}\\ &= \frac{-\sin x(1 - \cos x)}{-\sin^2 x}\\ &= \frac{1 - \cos x}{\sin x}\end{aligned}$$

d)
$$\begin{aligned}\frac{1 - \sin^2 x}{1 + 2\sin x - 3\sin^2 x} &= \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)(1 + 3\sin x)}\\ &= \frac{1 + \sin x}{1 + 3\sin x}\end{aligned}$$

3. a) $\frac{\sin x + 1}{\cos x}$ **b)** $\frac{-2\tan x}{\cos x}$

c) $\csc x$ **d)** $2\cot^2 x$

4. a) $\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$ **b)** $\sin x$

5. $\frac{\sin 2x}{2 \sin x} = \frac{2 \sin x \cos}{2 \sin x} = \cos x$, $x \neq \pi n$; $n \in \mathrm{I}$

6. $\cos x$

7. a)
$$\begin{aligned}\frac{\csc x}{2\cos x} &= \frac{1}{2\sin x \cos x}\\ &= \frac{1}{\sin 2x}\\ &= \csc 2x\end{aligned}$$

b)
$$\begin{aligned}\sin x + \cos x \cot x &= \sin x + \frac{\cos^2 x}{\sin x}\\ &= \frac{1}{\sin x}\\ &= \csc x\end{aligned}$$

8. Hannah's choice takes fewer steps.

9. a) 42.3 m

b)
$$\begin{aligned}\frac{v_o^2 \sin 2\theta}{g} &= \frac{v_o^2 2 \sin\theta \cos\theta}{g}\\ &= \frac{2v_o^2 \sin^2\theta \cos\theta}{g \sin\theta}\\ &= \frac{2v_o^2 \sin^2\theta}{g \tan\theta}\\ &= \frac{2v_o^2(1 - \cos^2\theta)}{g \tan\theta}\end{aligned}$$

10. a) Left Side
$$\begin{aligned}&= \frac{\csc x}{2\cos x}\\ &= \frac{1}{2\sin x \cos x}\\ &= \frac{1}{\sin 2x}\\ &= \csc 2x\\ &= \text{Right Side}\end{aligned}$$

b) Left Side
$$\begin{aligned}&= \frac{\sin x \cos x}{1 + \cos x}\\ &= \frac{(\sin x \cos x)(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}\\ &= \frac{\sin x \cos x - \sin x \cos^2 x}{\sin^2 x}\\ &= \frac{\cos x - \cos^2 x}{\sin x}\\ &= \frac{1 - \cos x}{\tan x}\\ &= \text{Right Side}\end{aligned}$$

c)
$$\begin{aligned}\text{Left Side} &= \frac{\sin x + \tan x}{1 + \cos x}\\ &= \left(\frac{\sin x}{1} + \frac{\sin x}{\cos x}\right) \div (1 + \cos x)\\ &= \left(\frac{\sin x \cos x + \sin x}{\cos x}\right) \times \frac{1}{1 + \cos x}\\ &= \left(\frac{\sin x(1 + \cos x)}{\cos x}\right) \times \frac{1}{1 + \cos x}\\ &= \frac{\sin x}{\cos x}\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= \frac{\sin 2x}{2\cos^2 x}\\ &= \frac{2\sin x \cos x}{2\cos^2 x}\\ &= \frac{\sin x}{\cos x}\end{aligned}$$

Left Side = Right Side

11. a)
$$\begin{aligned}\text{Left Side} &= \frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x}\\ &= \frac{2\sin x \cos x}{\cos x} + \frac{1 - 2\sin^2 x}{\sin x}\\ &= 2\sin x + \csc x - 2\sin x\\ &= \csc x\\ &= \text{Right Side}\end{aligned}$$

b) Left Side

$= \csc^2 x + \sec^2 x$

$= \dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$

$= \dfrac{1}{\sin^2 x \cos^2 x}$

$= \csc^2 x \sec^2 x$

= Right Side

c) Left Side

$= \dfrac{\cot x - 1}{1 - \tan x}$

$= \dfrac{\dfrac{1 - \tan x}{\tan x}}{1 - \tan x}$

$= \dfrac{1 - \tan x}{\tan x(1 - \tan x)}$

$= \dfrac{1}{\tan x}$

$= \dfrac{\csc x}{\sec x}$

= Right Side

12. a) Left Side $= \sin (90° + \theta)$

$= \sin 90° \cos \theta + \cos 90° \sin \theta$

$= \cos \theta$

Right Side $= \sin (90° - \theta)$

$= \sin 90° \cos \theta - \cos 90° \sin \theta$

$= \cos \theta$

b) Left Side $= \sin (2\pi - \theta)$

$= \sin (2\pi) \cos (\theta) - \cos (2\pi) \sin (\theta)$

$= -\sin \theta$

= Right Side

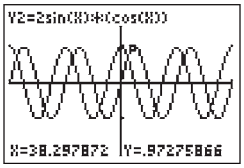
13. Left Side $= 2 \cos x \cos y$

Right Side $= \cos(x + y) + \cos(x - y)$

$= \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y$

$= 2 \cos x \cos y$

14. a)



No, this is not an identity.

b) Replacing the variable with 0 is a counter example.

15. a) $x \neq \pi n$; $n \in \mathrm{I}$

b) Left Side $= \dfrac{\sin 2x}{1 - \cos 2x}$

$= \dfrac{2 \sin x \cos x}{1 - 1 + 2 \sin^2 x}$

$= \dfrac{\cos x}{\sin x}$

$= \cot x$

= Right Side

16. Right Side

$= \dfrac{\sin 4x - \sin 2x}{\cos 4x + \cos 2x}$

$= \dfrac{2 \sin 2x \cos 2x - 2 \sin x \cos x}{\cos 4x + 2 \cos^2 x - 1}$

$= \dfrac{2(2 \sin x \cos x)(2 \cos^2 x - 1) - 2 \sin x \cos x}{2 \cos^2 2x - 1 + 2 \cos^2 x - 1}$

$= \dfrac{(2 \sin x \cos x)(2(2 \cos^2 x - 1) - 1)}{2(2 \cos^2 x - 1)^2 + 2 \cos^2 x - 2}$

$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{2(4 \cos^4 x - 4 \cos^2 x + 1) + 2 \cos^2 x - 2}$

$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{8 \cos^4 x - 6 \cos^2 x}$

$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{2 \cos^2 x(4 \cos^2 x - 3)}$

$= \dfrac{2 \sin x \cos x}{2 \cos^2 x}$

$= \tan x$

= Left Side

17. Left Side $= \dfrac{\sin 2x}{1 - \cos 2x}$

$= \dfrac{\sin 2x}{1 - \cos 2x}\left(\dfrac{1 + \cos 2x}{1 + \cos 2x}\right)$

$= \dfrac{\sin 2x + \sin 2x \cos 2x}{1 - \cos^2 2x}$

$= \dfrac{\sin 2x + \sin 2x \cos 2x}{\sin^2 2x}$

$= \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$

$= \dfrac{1}{\sin 2x} + \dfrac{1 - 2 \sin^2 x}{\sin 2x}$

$= \dfrac{2}{\sin 2x} - \dfrac{2 \sin^2 x}{\sin 2x}$

$= 2 \csc 2x - \dfrac{2 \sin^2 x}{2 \sin x \cos x}$

$= 2 \csc 2x - \tan x$

= Right Side

18. Left Side $= \dfrac{1 - \sin^2 x - 2 \cos x}{\cos^2 x - \cos x - 2}$

$= \dfrac{\cos^2 x - 2 \cos x}{\cos^2 x - \cos x - 2}$

$= \dfrac{\cos x(\cos x - 2)}{(\cos x - 2)(\cos x + 1)}$

$= \dfrac{\cos x}{\cos x + 1}$

$= \dfrac{\dfrac{\cos x}{\cos x}}{\dfrac{\cos x + 1}{\cos x}}$

$= \dfrac{1}{1 + \sec x}$

= Right Side

19. a) $\sin \theta_t = \dfrac{n_1 \sin \theta_i}{n_2}$

b) Using $\sin^2 x + \cos^2 x = 1$, $\cos x = \sqrt{1 - \sin^2 x}$. Then, replace this in the equation.

c) Substitute $\sin \theta_t = \dfrac{n_1 \sin \theta_i}{n_2}$.

C1 Graphing gives a visual approximation, so some functions may look the same but actually are not. Verifying numerically is not enough since it may not hold for other values.

C2 Left Side $= \cos\left(\dfrac{\pi}{2} - x\right)$

$= \cos\left(\dfrac{\pi}{2}\right) \cos x + \sin\left(\dfrac{\pi}{2}\right) \sin x$

$= \sin x$

= Right Side

C3 a) $\cos x \geq 0$, $\dfrac{\pi}{2} + 2\pi n < x < \dfrac{3\pi}{2} + 2\pi n$, $n \in \mathrm{I}$

b) $x = 1$

c) $x = \pi$, $\cos x$ will give a negative answer and radical functions always give a positive answer, so the equation is not an identity.

d) An identity is always true whereas an equation is true for certain values or a restricted domain.

6.4 Solving Trigonometric Equations Using Identities, pages 320 to 321

1. a) $0, \dfrac{\pi}{4}, \pi, \dfrac{5\pi}{4}$

b) $0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

c) $\dfrac{3\pi}{2}$

d) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

2. a) 0°, 120°, 240°

b) 270°

c) no solution

d) 0°, 120°, 180°, 300°

3. a) $2\sin^2 x + 3\sin x + 1 = 0; \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

b) $2\sin^2 x + 3\sin x + 1 = 0; \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

c) $\sin^2 x + 2\sin x - 3 = 0, \frac{\pi}{2}$

d) $2 - \sin^2 x = 0$; no solution

4. $-150°, -30°, 30°, 150°$

5. 0.464, 2.034, 3.605, 5.176

6. There are two more solutions that Sanesh did not find since she divided by cos (x). The extra solutions are $x = 90° + 360°n$ and $x = 270° + 360°n$.

7. a) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ **b)** $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

8. $x = \frac{\pi}{2} + \pi n, n \in \mathrm{I}$

9. $x = \frac{\pi}{2} + 2\pi n, n \in \mathrm{I}$

10. 7. Inspection of each factor shows that there are 2 + 1 + 4 solutions, which gives a total of 7 solutions over the interval $0° < x \leq 360°$.

11. $\frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

12. $B = -3, C = -2$

13. Example: $\sin 2x - \sin 2x\cos^2 x = 0; x = \left(\frac{\pi}{2}\right)n, n \in \mathrm{I}$

14. $x = \left(\frac{\pi}{2}\right)(2n + 1), n \in \mathrm{I}, x = \frac{\pi}{6} + 2\pi n, n \in \mathrm{I}$,
$x = \frac{5\pi}{6} + 2\pi n, n \in \mathrm{I}$

15. 12 solutions

16. $x = \pi + 2\pi n, n \in \mathrm{I}, x = \pm 0.955\ 32 + n\pi, n \in \mathrm{I}$

17. $x = \frac{\pi}{4} + \pi n, n \in \mathrm{I}, x = -\frac{\pi}{4} + \pi n, n \in \mathrm{I}$

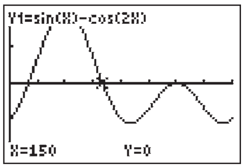
18. $-1.8235, 1.8235$

19. $x = 2\pi n, n \in \mathrm{I}, x = \pm\frac{\pi}{3} + 2\pi n, n \in \mathrm{I}$

20. 1 and -2

C1 a) $\cos 2x = 1 - 2\sin^2 x$ **b)** $(2\sin x - 1)(\sin x + 1)$

c) 30°, 150°, 270° **d)**



C2 a) You cannot factor the left side of the equation because there are no two integers whose product is -3 and whose sum is 1.

b) $-0.7676, 0.4343$

c) 64.26°, 140.14°, 219.86°, 295.74°, 424.26°, 500.14°, 579.86°, 655.74°

C3 Example: $\sin 2x\cos x + \cos x = 0$; The reason this is not an identity is that it is not true for all replacement values of the variable. For example, if $x = 30°$, the two sides are not equal. The solutions are $90° + 180°n, n \in \mathrm{I}$ and $135° + 180°n, n \in \mathrm{I}$.

Chapter 6 Review, pages 322 to 323

1. a) $x \neq \frac{\pi}{2} + n\pi, n \in \mathrm{I}$ **b)** $x \neq \left(\frac{\pi}{2}\right)n, n \in \mathrm{I}$

c) $x = \pm\frac{\pi}{3} + 2\pi n, n \in \mathrm{I}$ **d)** $x \neq \frac{\pi}{2} + n\pi, n \in \mathrm{I}$

2. a) $\cos x$ **b)** $\tan x$ **c)** $\tan x$ **d)** $\cos x$

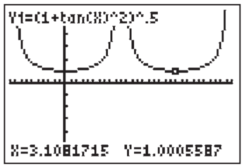
3. a) 1 **b)** 1 **c)** 1

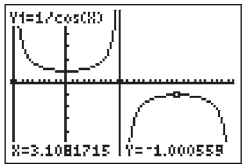
4. a) Both sides have the same value so the equation is true for those values.

b) $x \neq 90°, 270°$

5. a) Example: $x = 0, 1$

b)





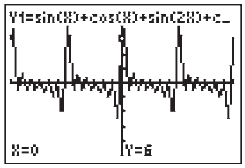
c) The graphs are the same for part of the domain. Outside of this interval they are not the same.

6. a) $f(0) = 2, f\left(\frac{\pi}{6}\right) = 1 + \sqrt{3}$

b) $\sin x + \cos x + \sin 2x + \cos 2x$
$= \sin x + \cos x + 2\sin x\cos x + 1 - \sin^2 x$

c) No, because you cannot write the first two terms as anything but the way they are.

d) You cannot get a perfect saw tooth graph but the approximation gets closer as you increase the amount of iterations. Six terms give a reasonable approximation.



7. a) $\sin 90° = 1$ **b)** $\sin 30° = 0.5$

c) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ **d)** $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

8. a) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} - \sqrt{2}}{4}$ **b)** $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} + \sqrt{2}}{4}$

c) $\sqrt{3} - 2$ **d)** $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} + \sqrt{2}}{4}$

9. a) $\frac{7}{13\sqrt{2}}$ or $\frac{7\sqrt{2}}{26}$ **b)** $\frac{12 - 5\sqrt{3}}{26}$

c) $-\frac{120}{169}$

10. $1 + \frac{1}{\sqrt{2}}$

11. $\tan x$

12. a) $\frac{\cos x}{\sin x - 1}$ or $\frac{-1 - \sin x}{\cos x}$

b) $\tan^2 x\sin^2 x$

13. a) Left Side
$= 1 + \cot^2 x$
$= 1 + \frac{\cos^2 x}{\sin^2 x}$
$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$
$= \frac{1}{\sin^2 x}$
$= \csc^2 x$
= Right Side

b) Right Side
$= \csc 2x - \cot 2x$
$= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$
$= \frac{1 - (2\cos^2 x - 1)}{2\sin x\cos x}$
$= \frac{2\sin^2 x}{2\sin x\cos x}$
$= \tan x$
= Left Side

c) Left Side
$= \sec x + \tan x$
$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$
$= \frac{1 + \sin x}{\cos x}$
$= \frac{1 - \sin^2 x}{(1 - \sin x)\cos x}$
$= \frac{\cos x}{1 - \sin x}$
= Right Side

d) Left Side
$= \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$
$= \frac{1 - \cos x}{1 - \cos^2 x} + \frac{1 + \cos x}{1 - \cos^2 x}$
$= \frac{2}{\sin^2 x}$
$= 2\csc^2 x$
= Right Side

14. a) It is true when $x = \frac{\pi}{4}$. The equation is not necessarily an identity. Sometimes equations can be true for a small domain of x.

b) $x = \frac{\pi}{2} + n\pi, n \in \mathrm{I}$

c) Left Side $= \sin 2x$
$$= 2\sin x\cos x$$
$$= \frac{2\sin x\cos^2 x}{\cos x}$$
$$= \frac{2\tan x}{\sec^2 x}$$
$$= \frac{2\tan x}{1+\tan^2 x}$$
$=$ Right Side

15. a) Left Side
$$= \frac{\cos x + \cot x}{\sec x + \tan x}$$
$$= \frac{\cos x + \frac{\cos x}{\sin x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}$$
$$= \frac{\frac{\sin x\cos^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}}{1+\sin x}$$
$$= \frac{\frac{(\sin x + 1)\cos^2 x}{\sin x}}{1+\sin x}$$
$$= \frac{\cos x\cos x}{\sin x}$$
$= \cos x \cot x$
$=$ Right Side

b) Left Side
$= \sec x + \tan x$
$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$
$$= \frac{1+\sin x}{\cos x}$$
$$= \frac{1-\sin^2 x}{(1-\sin x)\cos x}$$
$$= \frac{\cos x}{1-\sin x}$$
$=$ Right Side

16. a) You can disprove it by trying a value of x or by graphing.
b) Substituting $x = 0$ makes the equation fail.

17. a) $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ **b)** $x = \frac{5\pi}{6}, \frac{11\pi}{6}$
c) $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ **d)** $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$

18. a) $x = 15°, 75°, 195°, 255°$ **b)** $x = 90°, 270°$
c) $x = 30°, 150°, 270°$ **d)** $x = 0°, 180°$

19. $x = \pm\frac{\pi}{3} + n\pi, n \in I$

20. $\cos x = \pm\frac{4}{5}$

21. $x = -2\pi, -\pi, 0, \pi, 2\pi$

Chapter 6 Practice Test, page 324

1. A **2.** A **3.** D **4.** D **5.** A **6.** D

7. a) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}-\sqrt{6}}{4}$

b) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ or $\frac{\sqrt{6}+\sqrt{2}}{4}$

8. Left Side $= \cot\theta - \tan\theta$
$$= \frac{1}{\tan\theta} - \tan\theta$$
$$= \frac{1-\tan^2\theta}{\tan\theta}$$
$$= 2\left(\frac{1-\tan^2\theta}{2\tan\theta}\right)$$
$= 2\cot 2\theta$
$=$ Right Side
$\theta = \left(\frac{\pi}{2}\right)n, n \in I$

9. Theo's Formula $= I_0\cos^2\theta$
$= I_0 - I_0\sin^2\theta$
$$= I_0 - \frac{I_0}{\csc^2\theta}$$
$=$ Sany's Formula

10. a) $A = \frac{2\pi}{3} + 2\pi n, n \in I, A = \frac{4\pi}{3} + 2\pi n, n \in I$
b) $B = \pi n, n \in I, B = \frac{\pi}{6} + 2\pi n, n \in I,$
$B = \frac{5\pi}{6} + 2\pi n, n \in I$
c) $\theta = \pi n, n \in I, \theta = \pm\frac{\pi}{3} + 2\pi n, n \in I$

11. $x = \frac{\pi}{2} + n\pi, n \in I$

12. $\frac{-4-3\sqrt{3}}{10}$

13. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

14. $x = 0°, 90°, 270°$

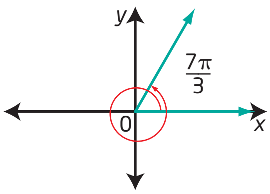
15. a) Left Side $= \frac{\cot x}{\csc x - 1}$
$$= \frac{\cot x(\csc x + 1)}{\csc^2 x - 1}$$
$$= \frac{\cot x(\csc x + 1)}{1 + \cot^2 x - 1}$$
$$= \frac{(\csc x + 1)}{\cot x}$$
$=$ Right Side

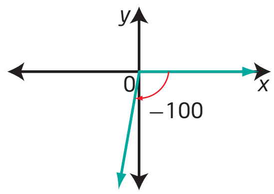
b) Left Side $= \sin(x+y)\sin(x-y)$
$= (\sin x\cos y + \sin y\cos x) \times$
$(\sin x\cos y - \sin y\cos x)$
$= \sin^2 x\cos^2 y - \sin^2 y\cos^2 x$
$= \sin^2 x(1-\sin^2 y) - \sin^2 y(1-\sin^2 x)$
$= \sin^2 x - \sin^2 y$
$=$ Right Side

16. $x = \frac{\pi}{2} + 2\pi n, n \in I, x = \frac{\pi}{6} + 2\pi n, n \in I,$
$x = \frac{5\pi}{6} + 2\pi n, n \in I$

Cumulative Review, Chapters 4–6, pages 326 to 327

1. a) $\frac{7\pi}{3} \pm 2\pi n, n \in N$ **b)** $-100° \pm (360°)n, n \in N$





2. a) 229° **b)** −300°

3. a) $\frac{7\pi}{6}$ **b)** $-\frac{25\pi}{9}$

4. a) 13.1 ft **b)** 106.9 ft

5. a) $x^2 + y^2 = 25$ **b)** $x^2 + y^2 = 16$

6. a) quadrant III **b)** $-\frac{2\pi}{3}, \frac{4\pi}{3}$

c) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$; when the given quadrant III angle is rotated through $\frac{\pi}{2}$, its terminal arm is in quadrant IV and its coordinates are switched and the signs adjusted.

d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; when the given quadrant III angle is rotated through $-\pi$, its terminal arm is in quadrant I and its coordinates are the same but the signs adjusted.

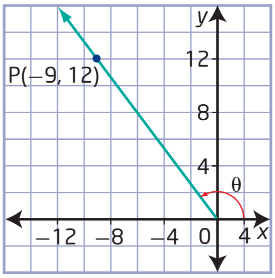
7. a) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; the points have the same x-coordinates but opposite y-coordinates.

b) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; the points have the same x-coordinates but opposite y-coordinates.

8. a) $-\frac{\sqrt{3}}{2}$ **b)** $\frac{1}{2}$ **c)** $-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

d) $\sqrt{2}$ **e)** undefined **f)** $-\sqrt{3}$

9. a)



b) $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$,
$\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

c) $\theta = 126.87° + (360°)n, n \in I$

10. a) $-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $-30°, 30°$

c) $\frac{3\pi}{4}, \frac{7\pi}{4}$

11. a) $\theta = \frac{3\pi}{4} + 2\pi n, n \in I$; $\frac{5\pi}{4} + 2\pi n, n \in I$

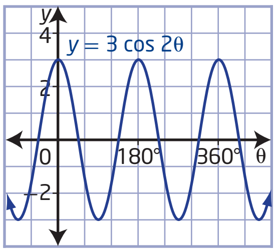
b) $\theta = \frac{\pi}{2} + 2\pi n, n \in I$ **c)** $\theta = \frac{\pi}{2} + \pi n, n \in I$

12. a) $\theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$ **b)** $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

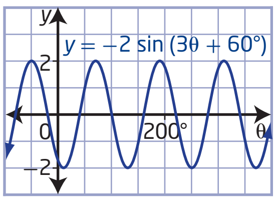
13. a) $\theta = 27°, 153°, 207°, 333°$
b) $\theta = 90°, 199°, 341°$

14. $y = 3\sin\frac{1}{2}\left(x + \frac{\pi}{4}\right)$

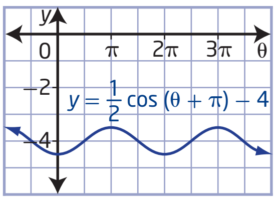
15. a) amplitude 3,
period 180°,
phase shift 0,
vertical displacement 0



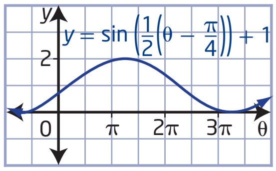
b) amplitude 2,
period 120°,
phase shift 20° left,
vertical displacement 0



c) amplitude $\frac{1}{2}$,
period 2π,
phase shift
π units left,
vertical displacement
4 units down



d) amplitude 1,
period 4π,
phase shift
$\frac{\pi}{4}$ units right,
vertical displacement
1 unit up

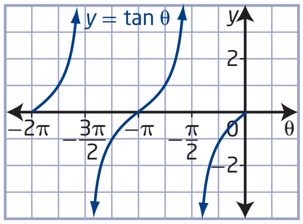


16. a) $y = 2\sin(x - 30°) + 3$, $y = 2\cos(x - 120°) + 3$

b) $y = \sin 2\left(x + \frac{\pi}{3}\right) - 1$, $y = \cos 2\left(x + \frac{\pi}{12}\right) - 1$

17. $y = 4\cos 1.2(x + 30°) - 3$

18. a)



b) $x = -\frac{\pi}{2}$,
$x = -\frac{3\pi}{2}$

19. a) $h(x) = -25\cos\frac{2\pi}{11}x + 26$ **b)** $x = 3.0$ min

20. a) $\theta \neq \frac{\pi}{2} + \pi n, n \in I, \tan^2\theta$

b) $x \neq \left(\frac{\pi}{2}\right)n, n \in I, \sec^2 x$

21. a) $-\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $-\frac{\sqrt{6} - \sqrt{2}}{4}$

b) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} - \sqrt{2}}{4}$

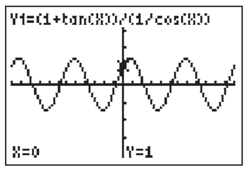
22. a) $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ **b)** $\sin 90° = 1$

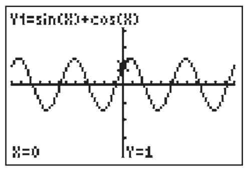
c) $\tan\frac{7\pi}{3} = \sqrt{3}$

23. a) Both sides have the same value for A = 30°.

b)
$$\begin{aligned}\text{Left Side} &= \sin^2 A + \cos^2 A + \tan^2 A \\ &= 1 + \tan^2 A \\ &= \sec^2 A \\ &= \text{Right Side}\end{aligned}$$

24. a) It could be an identity as the graphs look the same.





b)
$$\begin{aligned}\text{Left Side} &= \frac{1 + \tan x}{\sec x} \\ &= \frac{1}{\sec x} + \frac{\tan x}{\sec x} \\ &= \cos x + \frac{\sin x}{\cos x} \div \frac{1}{\cos x} \\ &= \cos x + \sin x \\ &= \text{Right Side}\end{aligned}$$

25.
$$\begin{aligned}\text{Right Side} &= \frac{\cos 2\theta}{1 + \sin 2\theta} \\ &= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta} \\ &= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta + \sin\theta)(\cos\theta + \sin\theta)} \\ &= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} \\ &= \text{Left Side}\end{aligned}$$

26. a) $x = \frac{5\pi}{6} + \pi n, n \in I$, $x = \frac{\pi}{6} + \pi n, n \in I$

b) $x = \frac{\pi}{2} + \pi n, n \in \mathrm{I}, x = \frac{7\pi}{6} + 2\pi n, n \in \mathrm{I},$
$x = \frac{11\pi}{6} + 2\pi n, n \in \mathrm{I}$

27. a) This is an identity so all θ are a solution.
b) Yes, because the left side can be simplified to 1.

Unit 2 Test, pages 328 to 329

1. B **2.** D **3.** C **4.** C **5.** B **6.** D **7.** C **8.** A

9. $-\frac{\sqrt{3}}{2}$

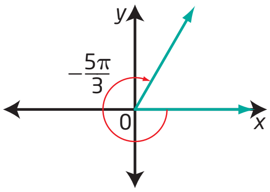
10. $-\frac{2}{3}, \frac{2}{3}$

11. $\frac{7}{13\sqrt{2}}$ or $\frac{7\sqrt{2}}{26}$

12. 1.5, 85.9°

13. $-\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$

14. a)

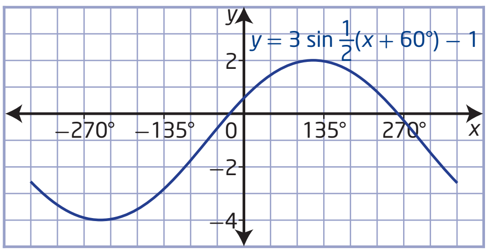


b) $-300°$
c) $-\frac{5\pi}{3} \pm 2\pi n, n \in \mathrm{N}$
d) No, following the equation above it is impossible to obtain $\frac{10\pi}{3}$.

15. $x = 0.412, 2.730, 4.712$

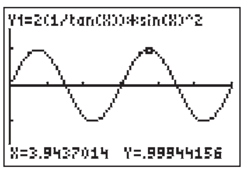
16. Sam is correct, there are four solutions in the given domain. Pat made an error when finding the square root. Pat forgot to solve for the positive and negative solutions.

17. a)



b) $-4 \le y \le 2$
c) amplitude 3, period 720°, phase shift 60° left, vertical displacement 1 unit down
d) $x \approx -21°, 261°$

18. a)



b) $g(\theta) = \sin 2\theta$
c) $f(\theta) = 2\cot\theta\sin^2\theta$
$= \frac{2\cos\theta\sin^2\theta}{\sin\theta}$
$= 2\cos\theta\sin\theta$
$= \sin 2\theta$
$= g(\theta)$

19. a) It is true: both sides have the same value.
b) $x \ne \frac{\pi n}{2}, n \in \mathrm{I}$
c) Left Side
$= \tan x + \frac{1}{\tan x}$
$= \frac{\tan^2 x + 1}{\tan x}$
$= \frac{\sec^2 x}{\tan x}$
$= \sec x\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)$
$= \frac{\sec x}{\sin x}$
= Right Side

20. a) 6.838 m **b)** 12.37 h **c)** 3.017 m

Chapter 7 Exponential Functions

7.1 Characteristics of Exponential Functions, pages 342 to 345

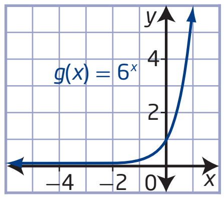
1. a) No, the variable is not the exponent.
b) Yes, the base is greater than 0 and the variable is the exponent.
c) No, the variable is not the exponent.
d) Yes, the base is greater than 0 and the variable is the exponent.

2. a) $f(x) = 4^x$ **b)** $g(x) = \left(\frac{1}{4}\right)^x$
c) $x = 0$, which is the y-intercept

3. a) B **b)** C **c)** A

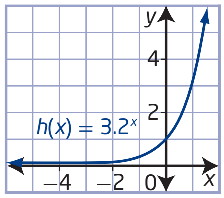
4. a) $f(x) = 3^x$ **b)** $f(x) = \left(\frac{1}{5}\right)^x$

5. a)



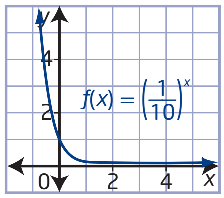
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function increasing, horizontal asymptote $y = 0$

b)



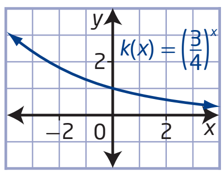
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function increasing, horizontal asymptote $y = 0$

c)



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

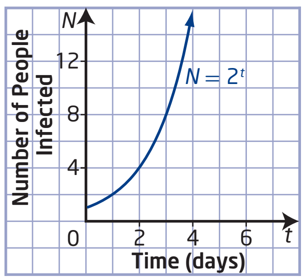
d)



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

6. a) $c > 1$; number of bacteria increases over time
b) $0 < c < 1$; amount of actinium-225 decreases over time
c) $0 < c < 1$; amount of light decreases with depth
d) $c > 1$; number of insects increases over time

7. a)

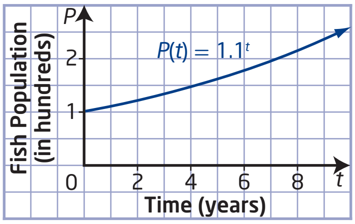


The function $N = 2^t$ is exponential since the base is greater than zero and the variable t is an exponent.

b) **i)** 1 person **ii)** 2 people
iii) 16 people **iv)** 1024 people

8. a) If the population increases by 10% each year, the population becomes 110% of the previous year's population. So, the growth rate is 110% or 1.1 written as a decimal.

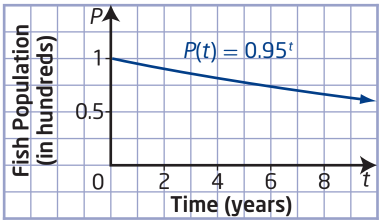
b)



domain $\{t \mid t \geq 0, t \in R\}$ and range $\{P \mid P \geq 100, P \in R\}$

c) The base of the exponent would become 100% − 5% or 95%, written as 0.95 in decimal form.

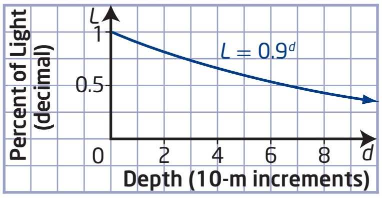
d)



domain $\{t \mid t \geq 0, t \in R\}$ and range $\{P \mid 0 < P \leq 100, P \in R\}$

9. a) $L = 0.9^d$

b)

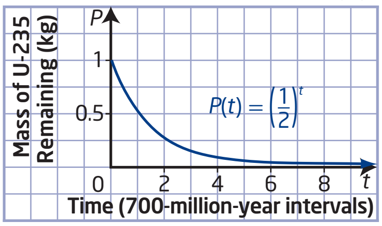


c) domain $\{d \mid d \geq 0, d \in R\}$ and range $\{L \mid 0 < L \leq 1, L \in R\}$

d) 76.8%

10. a) Let P represent the percent, as a decimal, of U-235 remaining. Let t represent time, in 700-million-year intervals. $P(t) = \left(\frac{1}{2}\right)^t$

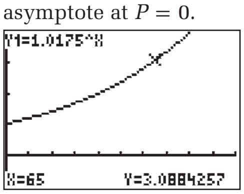
b)



c) 2.1×10^9 years

d) No, the sample of U-235 will never decay to 0 kg, since the graph of $P(t) = \left(\frac{1}{2}\right)^t$ has a horizontal asymptote at $P = 0$.

11. a)



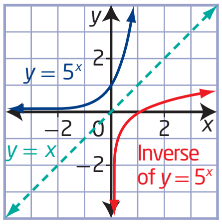
b) 64 years

c) No; since the amount invested triples, it does not matter what initial investment is made.

d) graph: 40 years; rule of 72: 41 years

12. 19.9 years

13. a)



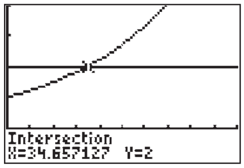
b) The x- and y-coordinates of any point and the domains and ranges are interchanged. The horizontal asymptote becomes a vertical asymptote.

c) $x = 5^y$

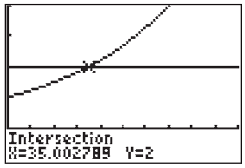
14. a) Another way to express $D = 2^{-\varphi}$ is as $D = \left(\frac{1}{2}\right)^{\varphi}$, which indicates a decreasing exponential function. Therefore, a negative value of φ represents a greater value of D.

b) The diameter of fine sand (0.125 mm) is $\frac{1}{256}$ the diameter of course gravel (32 mm).

15. a) 34.7 years

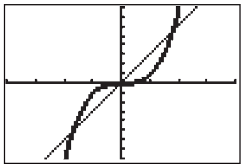


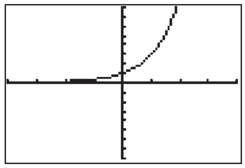
b) 35 years



c) The results are similar, but the continuous compounding function gives a shorter doubling period by approximately 0.3 years.

C1 a)





b)

Feature	$f(x) = 3x$	$g(x) = x^3$	$h(x) = 3^x$
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid y \in R\}$	$\{y \mid y \in R\}$	$\{y \mid y > 0, y \in R\}$
intercepts	x-intercept 0, y-intercept 0	x-intercept 0, y-intercept 0	no x-intercept, y-intercept 1
equations of asymptotes	none	none	$y = 0$

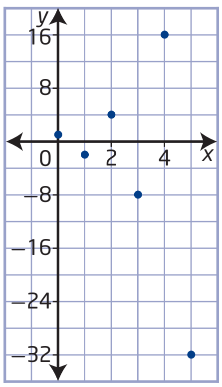
c) Example: All three functions have the same domain, and each of their graphs has a y-intercept. The functions $f(x)$ and $g(x)$ have all key features in common.

d) Example: The function $h(x)$ is the only function with an asymptote, which restricts its range and results in no x-intercept.

C2 a)

x	$f(x)$
0	1
1	−2
2	4
3	−8
4	16
5	−32

b)



c) No, the points do not form a smooth curve. The locations of the points alternate between above the x-axis and below the x-axis.

d) The values are undefined because they result in the square root of a negative number.

$f(x) = (-2)^x$ $\qquad$ $f(x) = (-2)^x$

$f\left(\frac{1}{2}\right) = (-2)^{\frac{1}{2}}$ $\qquad$ $f\left(\frac{5}{2}\right) = (-2)^{\frac{5}{2}}$

$f\left(\frac{1}{2}\right) = \sqrt{-2}$ $\qquad$ $f\left(\frac{5}{2}\right) = \sqrt{(-2)^5}$

e) Example: Exponential functions with positive bases result in smooth curves.

7.2 Transformations of Exponential Functions, pages 354 to 357

1. a) C **b)** D **c)** A **d)** B

2. a) D **b)** A **c)** B **d)** C

3. a) $a = 2$: vertical stretch by a factor of 2; $b = 1$: no horizontal stretch; $h = 0$: no horizontal translation; $k = -4$: vertical translation of 4 units down

b) $a = 1$: no vertical stretch; $b = 1$: no horizontal stretch; $h = 2$: horizontal translation of 2 units right; $k = 3$: vertical translation of 3 units up

c) $a = -4$: vertical stretch by a factor of 4 and a reflection in the x-axis; $b = 1$: no horizontal stretch; $h = -5$: horizontal translation of 5 units left; $k = 0$: no vertical translation

d) $a = 1$: no vertical stretch; $b = 3$: horizontal stretch by a factor of $\frac{1}{3}$; $h = 1$: horizontal translation of 1 unit right; $k = 0$: no vertical translation

e) $a = -\frac{1}{2}$: vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis; $b = 2$: horizontal stretch by a factor of $\frac{1}{2}$; $h = 4$: horizontal translation of 4 units right; $k = 3$: vertical translation of 3 units up

f) $a = -1$: reflection in the x-axis; $b = 2$: horizontal stretch by a factor of $\frac{1}{2}$; $h = 1$: horizontal translation of 1 unit right; $k = 0$: no vertical translation

g) $a = 1.5$: vertical stretch by a factor of 1.5; $b = \frac{1}{2}$: horizontal stretch by a factor of 2; $h = 4$: horizontal translation of 4 units right; $k = -\frac{5}{2}$: vertical translation of $\frac{5}{2}$ units down

4. a) C: reflection in the x-axis, $a < 0$ and $0 < c < 1$, and vertical translation of 2 units up, $k = 2$

b) A: horizontal translation of 1 unit right, $h = 1$, and vertical translation of 2 units down, $k = -2$

c) D: reflection in the x-axis, $a < 0$ and $c > 1$, and vertical translation of 2 units up, $k = 2$

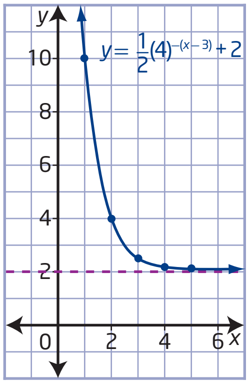
d) B: horizontal translation of 2 units right, $h = 2$, and vertical translation of 1 unit up, $k = 1$

5. a) $a = \frac{1}{2}$: vertical stretch by a factor of $\frac{1}{2}$; $b = -1$: reflection in the y-axis; $h = 3$: horizontal translation of 3 units right 3; $k = 2$: vertical translation of 2 units up

b)

$y = 4^x$	$y = 4^{-x}$	$y = \frac{1}{2}(4)^{-x}$	$y = \frac{1}{2}(4)^{-(x-3)} + 2$
$\left(-2, \frac{1}{16}\right)$	$\left(2, \frac{1}{16}\right)$	$\left(2, \frac{1}{32}\right)$	$\left(5, \frac{65}{32}\right)$
$\left(-1, \frac{1}{4}\right)$	$\left(1, \frac{1}{4}\right)$	$\left(1, \frac{1}{8}\right)$	$\left(4, \frac{17}{8}\right)$
(0, 1)	(0, 1)	$\left(0, \frac{1}{2}\right)$	$\left(3, \frac{5}{2}\right)$
(1, 4)	(−1, 4)	(−1,2)	(2, 4)
(2, 16)	(−2, 16)	(−2, 8)	(1, 10)

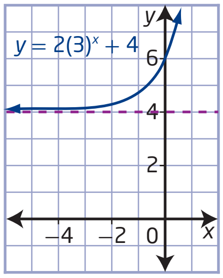
c)



d) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 2, y \in \mathrm{R}\}$, horizontal asymptote $y = 2$, y-intercept 34

6. a) i), ii) $a = 2$: vertical stretch by a factor of 2; $b = 1$: no horizontal stretch; $h = 0$: no horizontal translation; $k = 4$: vertical translation of 4 units up

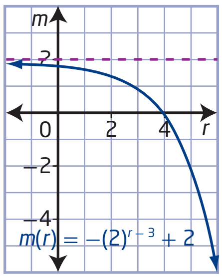
iii)



iv) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 4, y \in \mathrm{R}\}$, horizontal asymptote $y = 4$, y-intercept 6

b) i), ii) $a = -1$: reflection in the x-axis; $b = 1$: no horizontal stretch; $h = 3$: horizontal translation of 3 units right; $k = 2$: vertical translation of 2 units up

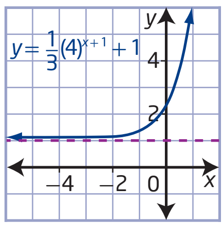
iii)



iv) domain $\{r \mid r \in \mathrm{R}\}$, range $\{m \mid m < 2, m \in \mathrm{R}\}$, horizontal asymptote $m = 2$, m-intercept $\frac{15}{8}$, r-intercept 4

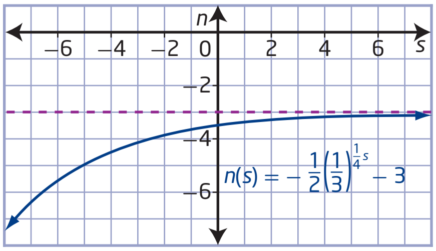
c) i), ii) $a = \frac{1}{3}$: vertical stretch by a factor of $\frac{1}{3}$; $b = 1$: no horizontal stretch; $h = -1$: horizontal translation of 1 unit left; $k = 1$: vertical translation of 1 unit up

iii)



iv) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 1, y \in \mathrm{R}\}$, horizontal asymptote $y = 1$, y-intercept $\frac{7}{3}$

d) i), ii) $a = -\frac{1}{2}$: vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis; $b = \frac{1}{4}$: horizontal stretch by a factor of 4; $h = 0$: no horizontal translation; $k = -3$: vertical translation of 3 units down

iii) 

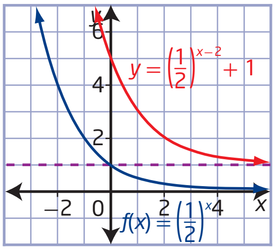
iv) domain $\{s \mid s \in \mathrm{R}\}$, range $\{n \mid n < -3, n \in \mathrm{R}\}$, horizontal asymptote $n = -3$, n-intercept $-\frac{7}{2}$

7. a) horizontal translation of 2 units right and vertical translation of 1 unit up; $y = \left(\frac{1}{2}\right)^{x-2} + 1$

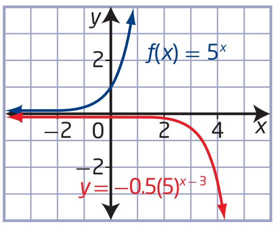
b) reflection in the x-axis, vertical stretch by a factor of 0.5, and horizontal translation of 3 units right; $y = -0.5(5)^{x-3}$

c) reflection in the x-axis, horizontal stretch by a factor of $\frac{1}{3}$, and vertical translation of 1 unit up; $y = -\left(\frac{1}{4}\right)^{3x} + 1$

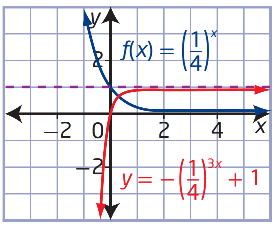
d) vertical stretch by a factor of 2, reflection in the y-axis, horizontal stretch by a factor of 3, horizontal translation of 1 unit right, and vertical translation of 5 units down; $y = 2(4)^{-\frac{1}{3}(x-1)} - 5$

8. a) 

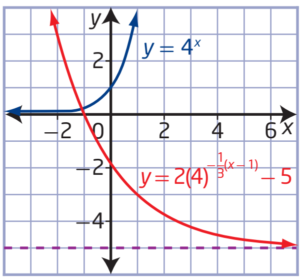
Map all points (x, y) on the graph of $f(x)$ to $(x + 2, y + 1)$.

b) 

Map all points (x, y) on the graph of $f(x)$ to $(x + 3, -0.5y)$.

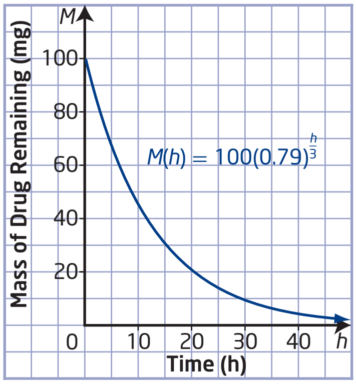
c) 

Map all points (x, y) on the graph of $f(x)$ to $\left(\frac{1}{3}x, -y + 1\right)$.

d) 

Map all points (x, y) on the graph of $f(x)$ to $(-3x + 1, 2y - 5)$.

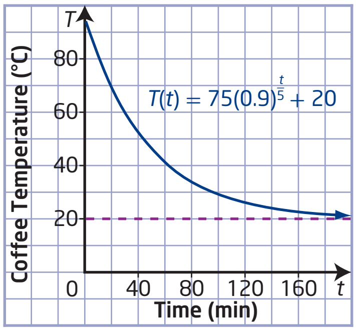
9. a) 0.79 represents the 79% of the drug remaining in exponential decay after $\frac{1}{3}$ h.

b) 

c) The M-intercept represents the drug dose taken.

d) domain $\{h \mid h \geq 0, h \in \mathrm{R}\}$, range $\{M \mid 0 < M \leq 100, M \in \mathrm{R}\}$

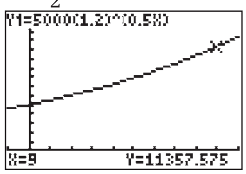
10. a) $a = 75$: vertical stretch by a factor of 75; $b = \frac{1}{5}$: horizontal stretch by a factor of 5; $h = 0$: no horizontal translation; $k = 20$: vertical translation of 20 units up

b) 

c) 29.1 °C **d)** final temperature of the coffee

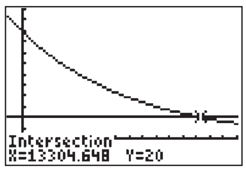
11. a) $P = 5000(1.2)^{\frac{1}{2}x}$

b) $a = 5000$: vertical stretch by a factor of 5000; $b = \frac{1}{2}$: horizontal stretch by a factor of 2

c) 

approximately 11 357 bacteria

12. a) $P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

b) 

approximately 13 305 years old

13. a) 527.8 cm^2 **b)** 555 h

14. a) 1637 foxes

b) Example: Disease or lack of food can change the rate of growth of the foxes. Exponential growth suggests that the population will grow without bound, and therefore the fox population will grow beyond the possible food sources, which is not good if not controlled.

C1 Example: The graph of an exponential function of the form $y = c^x$ has a horizontal asymptote at $y = 0$. Since $y \neq 0$, the graph cannot have an x-intercept.

C2 a) Example: For a function of the form $y = a(c)^{b(x-h)} + k$, the parameters a and k can affect the x-intercept. If $a > 0$ and $k < 0$ or $a < 0$ and $k > 0$, then the graph of the exponential function will have an x-intercept.

b) Example: For a function of the form $y = a(c)^{b(x-h)} + k$, the parameters a, h, and k can affect the y-intercept. The point $(0, y)$ on the graph of $y = c^x$ gets mapped to $(h, ay + k)$.

7.3 Solving Exponential Equations, pages 364 to 365

1. a) 2^{12} **b)** 2^9 **c)** 2^{-6} **d)** 2^4

2. a) 2^3 and 2^4 **b)** 3^{2x} and 3^3

c) $\left(\frac{1}{2}\right)^{2x}$ and $\left(\frac{1}{2}\right)^{2x-2}$ **d)** 2^{-3x+6} and 2^{4x}

3. a) 4^2 **b)** $4^{\frac{2}{3}}$ **c)** 4^3 **d)** 4^3

4. a) $x = 3$ **b)** $x = -2$ **c)** $w = 3$ **d)** $m = \frac{7}{4}$

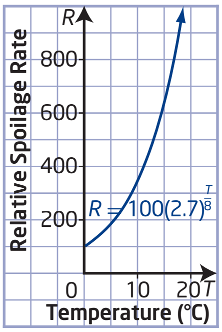
5. a) $x = -3$ **b)** $x = -4$ **c)** $y = \frac{11}{4}$ **d)** $k = 9$

6. a) 10.2 **b)** 11.5 **c)** −2.8 **d)** 18.9

7. a) 58.71 **b)** −1.66 **c)** −5.38 **d)** −8

e) 2.71 **f)** 14.43 **g)** −3.24 **h)** −1.88

8. a)



b) approximately 5.6 °C
c) approximately 643
d) approximately 13.0 °C

9. 3 h

10. 4 years

11. a) $A = 1000(1.02)^n$ **b)** \$1372.79 **c)** 9 years

12. a) $C = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$ **b)** $\frac{1}{32}$ of the original amount

c) 47.7 years

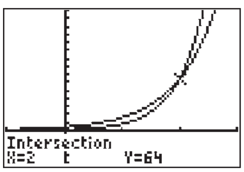
13. a) $A = 500(1.033)^n$ **b)** \$691.79

c) approximately 17 years

14. \$5796.65

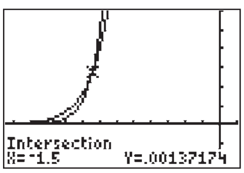
15. a) i) $x > 2$ **ii)** $x > -\frac{3}{2}$

b) i)



Since the graph of $y = 2^{3x}$ is greater than (above) the graph of $y = 4^{x+1}$ when $x > 2$, the solution is $x > 2$.

ii)



Since the graph of $y = 81^x$ is less than (below) the graph of $y = 27^{2x+1}$ when $x > -\frac{3}{2}$, the solution is $x > -\frac{3}{2}$.

c) Example: Solve the inequality $\left(\frac{1}{2}\right)^{x+3} > 2^{x-1}$.

Answer: $x < -1$

16. Yes. Rewrite the equation as $(4^x)^2 + 2(4^x) - 3 = 0$ and factor as $(4^x + 3)(4^x - 1) = 0$; $x = 0$

17. $(2^x)^x = \left(2^{\frac{5}{2}}\right)^{\frac{5}{2}} \approx 76.1$

18. 20 years

C1 a) You can express 16^2 with a base of 4 by writing 16 as 4^2 and simplifying.

$16^2 = (4^2)^2$
$16^2 = 4^4$

b) Example: You can express 16^2 with a base of 2 by writing 16 as 2^4 and simplifying.

$16^2 = (2^4)^2$
$16^2 = 2^8$

Or, you can express 16^2 with a base of $\frac{1}{4}$ by writing 16 as $\left(\frac{1}{4}\right)^{-2}$ and simplifying.

$$16^2 = \left(\left(\frac{1}{4}\right)^{-2}\right)^2$$
$$16^2 = \left(\frac{1}{4}\right)^{-4}$$

C2 a)

$16^{2x} = 8^{x-3}$
$(2^4)^{2x} = (2^3)^{x-3}$
$2^{8x} = 2^{3x-9}$
$8x = 3x - 9$
$5x = -9$
$x = -\frac{9}{5}$

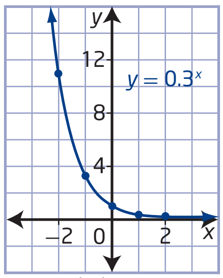
b) **Step 1:** Express the bases on both sides as powers of 2.
Step 2: Apply the power of a power law.
Step 3: Equate the exponents.
Step 4: Isolate the term containing x.
Step 5: Solve for x.

Chapter 7 Review, pages 366 to 367

1. a) B **b)** D **c)** A **d)** C

2. a)

x	y
−2	$11.\overline{1}$
−1	$3.\overline{3}$
0	1
1	0.3
2	0.09



b) domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

3. $y = \left(\frac{1}{4}\right)^x$

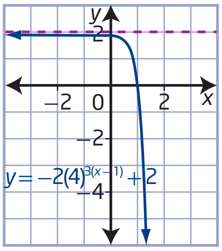
4. a) Since the interest rate is 3.25% per year, each year the investment grows by a factor of 103.25%, which, written as a decimal, is 1.0325.

b) \$1.38 **c)** 21.7 years

5. a) $a = -2$: vertical stretch by a factor of 2 and reflection in the x-axis; $b = 3$: horizontal stretch by a factor of $\frac{1}{3}$; $h = 1$: horizontal translation of 1 unit right; $k = 2$: vertical translation of 2 units up

b)

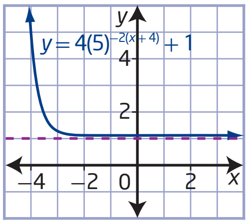
Transformation	Parameter Value	Function Equation
horizontal stretch	$b = 3$	$y = 4^{3x}$
vertical stretch	$a = -2$	$y = -2(4)^x$
translation left/right	$h = 1$	$y = (4)^{x-1}$
translation up/down	$k = 2$	$y = 4^x + 2$

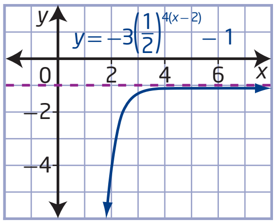
c) 

d) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y < 2, y \in \mathrm{R}\}$, horizontal asymptote $y = 2$, y-intercept $\frac{63}{32}$, x-intercept 1

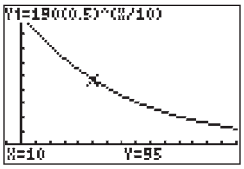
6. a) horizontal translation of 3 units right
b) vertical translation of 4 units down
c) reflection in the x-axis and a translation of 1 unit left and 2 units up

7. a) $y = 4(5)^{-2(x+4)} + 1$ **b)** $y = -3\left(\frac{1}{2}\right)^{4(x-2)} - 1$





8. a) $a = 190$: vertical stretch by a factor of 190; $b = \frac{1}{10}$: horizontal stretch by a factor of 10

b) 

c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{T \mid 0 < T \leq 190, T \in \mathrm{R}\}$
d) approximately 31.1 h

9. a) 6^2 **b)** 6^{-2} **c)** 6^5

10. a) $x = -\frac{3}{2}$ **b)** $x = \frac{12}{11}$

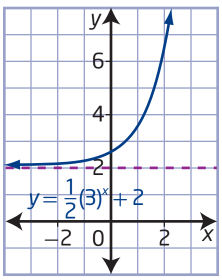
11. a) $x \approx -4.30$ **b)** $x \approx -6.13$

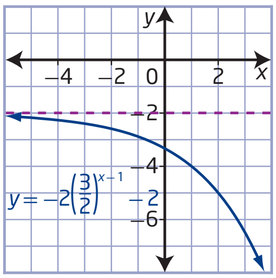
12. a) $N = \left(\frac{1}{2}\right)^{\frac{t}{2.5}}$ **b)** $\frac{1}{16}$ **c)** 25 h

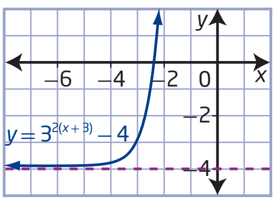
Chapter 7 Practice Test, pages 368 to 369

1. B **2.** C **3.** B **4.** A **5.** D

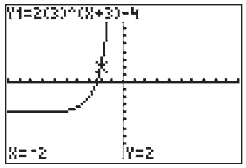
6. a) $y = 5^{x+3} + 2$ **b)** $y = -0.5(2)^{x-1} - 4$

7. a) 

b) 

c) 

8. a) $a = 2$: vertical stretch by a factor of 2; $h = -3$: horizontal translation of 3 units left; $k = -4$: vertical translation of 4 units down

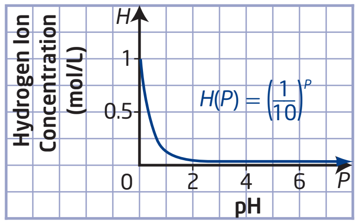
b) 

c) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > -4, y \in \mathrm{R}\}$, horizontal asymptote $y = -4$

9. a) $x = -4$ **b)** $x = 18$ **c)** $x = \frac{13}{8}$

10. a) $x \approx 9.7$ **b)** $x \approx 0.1$

11. a) 1.0277 **b)** $P = 100(1.0277)^t$
c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{P \mid P \geq 100, P \in \mathrm{R}\}$
d) approximately 8.2 years

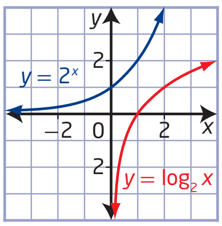
12. a) 

b) 1.0×10^{-7} [H$^+$]
c) 1.0×10^{-7} to 2.5×10^{-8} [H$^+$]

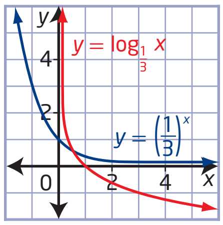
13. 4.5 years
14. 9.97 years

Chapter 8 Logarithmic Functions

8.1 Understanding Logarithms, pages 380 to 382

1. a) i) 

ii) $y = \log_2 x$
iii) domain $\{x \mid x > 0, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

b) i) 

ii) $y = \log_{\frac{1}{3}} x$
iii) domain $\{x \mid x > 0, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

2. a) $\log_{12} 144 = 2$ **b)** $\log_8 2 = \frac{1}{3}$
c) $\log_{10} 0.000\,01 = -5$ **d)** $\log_7 (y + 3) = 2x$

3. a) $5^2 = 25$ **b)** $8^{\frac{2}{3}} = 4$
c) $10^6 = 1\,000\,000$ **d)** $11^y = x + 3$

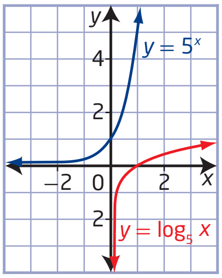
4. a) 3 **b)** 0 **c)** $\frac{1}{3}$ **d)** -3

5. $a = 4$; $b = 5$

6. a) $x > 1$ **b)** $0 < x < 1$ **c)** $x = 1$
d) Example: $x = 9$

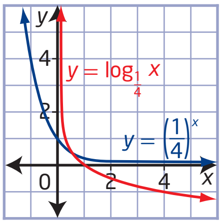
7. a) 0 raised to any non-zero power is 0.
b) 1 raised to any power is 1.
c) Exponential functions with a negative base are not continuous.

8. a) $y = \log_5 x$

b) 

domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y \in R\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

9. a) $g^{-1}(x) = \left(\frac{1}{4}\right)^x$

b) 

domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, no x-intercept, y-intercept 1, horizontal asymptote $y = 0$

10. They are reflections of each other in the line $y = x$.

11. a) They have the exact same shape.
b) One of them is increasing and the other is decreasing.

12. a) 216 **b)** 81 **c)** 64 **d)** 8

13. a) 7 **b)** 6

14. a) 0 **b)** 1

15. -1

16. 16

17. a) $t = \log_{1.1} N$ **b)** 145 days

18. The larger asteroid had a relative risk that was 1479 times as dangerous.

19. 1000 times as great

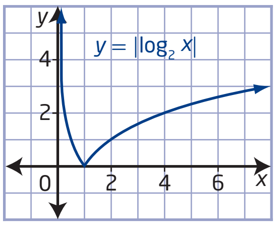
20. 5

21. $m = 14$, $n = 13$

22. $4n$

23. $y = 3^{2^x}$

24. $n = 8$; $m = 3$

C1 

The function has the same general shape, but instead of decreasing, after $x = 1$ the function increases without limit.

C2 Answers will vary.

C3 Step 1: a) $e = 2.718\ 281\ 828$ **b)** 10^{10}

Step 2: a) domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y \in R\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

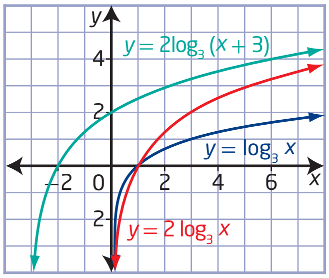
b) $y = \ln x$

Step 3: a) $r = 2.41$

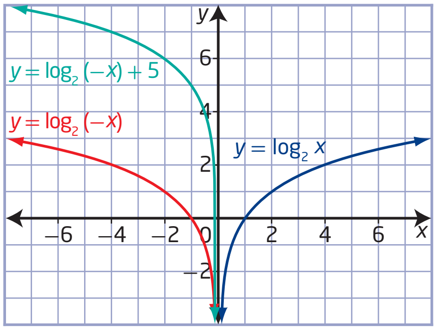
b) i) $\theta = \dfrac{\ln r}{0.14}$ **ii)** $\theta = 17.75$

8.2 Transformations of Logarithmic Functions, pages 389 to 391

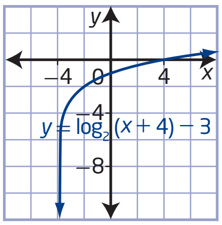
1. a) Translate 1 unit right and 6 units up.
b) Reflect in the x-axis, stretch vertically about the x-axis by a factor of 4, and stretch horizontally about the y-axis by a factor of $\frac{1}{3}$.
c) Reflect in the y-axis, stretch vertically about the x-axis by a factor of $\frac{1}{2}$, and translate 7 units up.

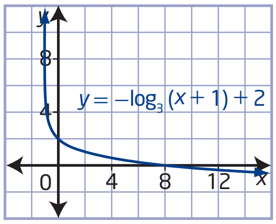
2. a) 

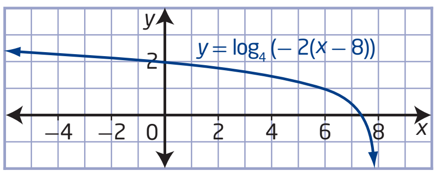
b) $y = 2 \log_3 (x + 3)$

3. a) 

b) $y = \log_2 (-x) + 5$

4. a) 

b) 

c) 

5. a) i) vertical asymptote $x = -3$
ii) domain $\{x \mid x > -3, x \in R\}$, range $\{y \mid y \in R\}$
iii) y-intercept -5 **iv)** x-intercept -2

b) i) vertical asymptote $x = -9$
ii) domain $\{x \mid x > -9, x \in R\}$, range $\{y \mid y \in R\}$
iii) y-intercept 2 **iv)** x-intercept -8.75

c) i) vertical asymptote $x = -3$
ii) domain $\{x \mid x > -3, x \in R\}$, range $\{y \mid y \in R\}$
iii) y-intercept -1.3 **iv)** x-intercept 22

d) i) vertical asymptote $x = -1$
ii) domain $\{x \mid x > -1, x \in R\}$, range $\{y \mid y \in R\}$
iii) y-intercept -6 **iv)** x-intercept $-\frac{3}{4}$

6. a) $y = 5 \log x$ **b)** $y = \log_8 2x$
c) $y = \frac{1}{3} \log_2 x$ **d)** $y = \log_4 \left(\frac{x}{2}\right)$

7. a) stretch horizontally about the y-axis by a factor of $\frac{1}{4}$; translate 5 units left and 6 units up
b) stretch horizontally about the y-axis by a factor of 3; stretch vertically about the x-axis by a factor of 2; reflect in the y-axis; translate 1 unit right and 4 units down

8. a) $a = -1$, $b = 1$, $h = -6$, $k = 3$; $y = -\log_3 (x + 6) + 3$
b) $a = 5$, $b = 3$, $h = 0$, $k = 0$; $y = 5 \log_3 3x$
c) $a = 0.75$, $b = -0.25$, $h = 2$, $k = -5$; $y = \frac{3}{4} \log_3 \left(-\frac{1}{4}(x - 2)\right) - 5$

9. a) Reflect in the y-axis, stretch vertically about the x-axis by a factor of 5, stretch horizontally about the y-axis by a factor of $\frac{1}{4}$, and translate 3 units right and 2 units down.
b) Reflect in the x-axis, reflect in the y-axis, stretch vertically about the x-axis by a factor of $\frac{1}{4}$, translate 6 units right and 1 unit up.

10. a) $y = \log_3 x - 6$ **b)** $y = \log_2 \left(\frac{x}{4}\right)$

11. Stretch vertically about the x-axis by a factor of 3 and translate 4 units right and 2 units down.

12. a) Stretch vertically about the x-axis by a factor of 0.67, stretch horizontally about the y-axis by a factor of $\frac{25}{9}$ or approximately 2.78, and translate 1.46 units up.
b) 515 649 043 kWh

13. a) 0.8 µL **b)** 78 mmHg

14. a) 172 cm **b)** 40 kg

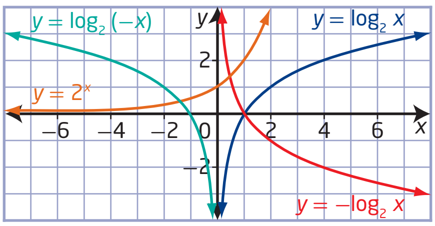
15. $a = \frac{1}{3}$

16. a) $y = -2 \log_5 x + 13$ **b)** $y = \log 2x$

17. $a = \frac{1}{2}$, $k = -8$

C1 $a = \frac{1}{4}$, $b = \frac{1}{3}$, $h = 4$, $k = -1$; $g(x) = 0.25 \log_5 \left(\frac{1}{3}\right)(x - 4) - 1$

C2 a) $y = -\log_2 x$, $y = \log_2 (-x)$, $y = 2^x$
b) Reflect in the x-axis, reflect in the y-axis, and reflect in the line $y = x$.



C3 a) $y = \frac{1}{2} \log_7 \frac{(x - 5)}{3} + \frac{1}{2}$ **b)** $y = 3^{\frac{x-8}{2}} + 1$

C4 Answers will vary.

8.3 Laws of Logarithms, pages 400 to 403

1. a) $\log_7 x + 3 \log_7 y + \frac{1}{2} \log_7 z$
b) $8(\log_5 x + \log_5 y + \log_5 z)$
c) $2 \log x - \log y - \frac{1}{3} \log z$
d) $y = \log_3 x + \left(\frac{1}{2}\right)(\log_3 y - \log_3 z)$

2. a) 2 **b)** 3 **c)** 3.5 **d)** 3

3. a) $\log_9 \left(\frac{xz^4}{y}\right)$ **b)** $y = \log_3 \frac{\sqrt{x}}{y^2}$
c) $\log_6 \left(\frac{x}{\sqrt[5]{xy^2}}\right)$ **d)** $\log \sqrt[3]{xy}$

4. a) 1.728 **b)** 1.44 **c)** 1.2

5. a) 27 **b)** 49

6. a) Stretch horizontally about the y-axis by a factor of $\frac{1}{8}$.
b) Translate 3 units up.

7. a) False; the division must take place inside the logarithm.
b) False; it must be a multiplication inside the logarithm.
c) True
d) False; the power must be inside the logarithm.
e) True

8. a) $P - Q$ **b)** $P + Q$ **c)** $P + \frac{Q}{2}$ **d)** $2Q - 2P$

9. a) $6K$ **b)** $1 + K$ **c)** $2K + 2$ **d)** $\frac{K}{5} - 3$

10. a) $\frac{1}{2} \log_5 x$, $x > 0$ **b)** $\frac{2}{3} \log_{11} x$, $x > 0$

11. a) $\log_2 \left(\frac{x + 5}{3}\right)$, $x < -5$ or $x > 5$
b) $\log_7 \left(\frac{x + 4}{x + 2}\right)$, $x < -4$ or $x > 4$
c) $\log_8 \left(\frac{x + 3}{x - 2}\right)$, $x > 2$

12. a)
$$\begin{aligned} \text{Left Side} &= \log_c 48 - (\log_c 3 + \log_c 2) \\ &= \log_c 48 - \log_c 6 \\ &= \log_c 8 \\ &= \text{Right Side} \end{aligned}$$
b)
$$\begin{aligned} \text{Left Side} &= 7 \log_c 4 \\ &= 7 \log_c 2^2 \\ &= 2(7) \log_c 2 \\ &= 14 \log_c 2 \\ &= \text{Right Side} \end{aligned}$$
c)
$$\begin{aligned} \text{Left Side} &= \tfrac{1}{2}(\log_c 2 + \log_c 6) \\ &= \tfrac{1}{2}(\log_c 2 + \log_c 3 + \log_c 2) \\ &= \tfrac{1}{2}(2 \log_c 2) + \tfrac{1}{2} \log_c 3 \\ &= \log_c 2 + \log_c \sqrt{3} \\ &= \text{Right Side} \end{aligned}$$
d)
$$\begin{aligned} \text{Left Side} &= \log_c (5c)^2 \\ &= 2 \log_c 5c \\ &= 2 (\log_c 5 + \log_c c) \\ &= 2 (\log_c 5 + 1) \\ &= \text{Right Side} \end{aligned}$$

13. a) 70 dB **b)** approximately 1995 times as loud
c) approximately 98 dB

14. Decibels must be changed to intensity to gauge loudness. The function that maps the change is not linear.

15. 3.2 V

16. a) 10^{-7} mol/L **b)** 12.6 times as acidic **c)** 3.4

17. 0.18 km/s

18. a) The graphs are the same for $x > 0$. However, the graph of $y = \log x^2$ has a second branch for $x < 0$, which is the reflection in the y-axis of the branch for $x > 0$.
b) The domains are different. The function $y = \log x^2$ is defined for all values of x except 0, while the function $y = 2 \log x$ is defined only for $x > 0$.
c) $x > 0$

19. a)
$$\begin{aligned} y &= \log_c x \\ c^y &= x \\ \log_d c^y &= \log_d x \\ y \log_d c &= \log_d x \\ y &= \frac{\log_d x}{\log_d c} \end{aligned}$$

b) 3.2479

c) $\varphi = -\dfrac{\log D}{\log 2}$

d) 207.9 times larger

20. a) Left Side
$$\begin{aligned} &= \log_{q^3} p^3 \\ &= \frac{\log_q p^3}{\log_q q^3} \\ &= \frac{3 \log_q p}{3 \log_q q} \\ &= \frac{\log_q p}{1} \\ &= \text{Right Side} \end{aligned}$$

b) Left Side
$$\begin{aligned} &= \frac{1}{\log_p 2} - \frac{1}{\log_q 2} \\ &= \frac{1}{\frac{\log_2 2}{\log_2 p}} - \frac{1}{\frac{\log_2 2}{\log_2 q}} \\ &= \frac{\log_2 p}{\log_2 2} - \frac{\log_2 q}{\log_2 2} \\ &= \frac{\log_2 p - \log_2 q}{\log_2 2} \\ &= \log_2 \frac{p}{q} \\ &= \text{Right Side} \end{aligned}$$

c) Left Side
$$\begin{aligned} &= \frac{1}{\log_q p} + \frac{1}{\log_q p} \\ &= \frac{1}{\frac{\log p}{\log q}} + \frac{1}{\frac{\log q}{\log p}} \\ &= \frac{\log q}{\log p} + \frac{\log q}{\log p} \\ &= \frac{2 \log q}{\log p} \end{aligned}$$

Right Side
$$\begin{aligned} &= \frac{1}{\log_{q^2} p} \\ &= \frac{1}{\frac{\log p}{\log q^2}} \\ &= \frac{\log q^2}{\log p} \\ &= \frac{2 \log q}{\log p} \end{aligned}$$

Left Side = Right Side

d) Left Side $= \log_{\frac{1}{q}} p$
$$\begin{aligned} &= \frac{\log_q p}{\log_q q^{-1}} \\ &= -\log_q p \\ &= \log_q \frac{1}{p} \\ &= \text{Right Side} \end{aligned}$$

C1 a) Stretch vertically about the x-axis by a factor of 3.

b) Stretch vertically about the x-axis by a factor of 5 and translate 2 units left.

c) Reflect in the x-axis.

d) Reflect in the x-axis, stretch vertically about the x-axis by a factor of $\frac{1}{2}$, and translate 6 units right.

C2 -1

C3 a) log 2 **b)** 15 log 2

C4 Answers will vary.

8.4 Logarithmic and Exponential Equations, pages 412 to 415

1. a) 1000 **b)** 14 **c)** 3 **d)** 108

2. a) 1.61 **b)** 10.38 **c)** 4.13 **d)** 0.94

3. No, since $\log_3 (x - 8)$ and $\log_3 (x - 6)$ are not defined when $x = 5$.

4. a) $x = 0$ is extraneous.

b) Both roots are extraneous.

c) $x = -6$ is extraneous.

d) $x = 1$ is extraneous.

5. a) $x = 8$ **b)** $x = 25$ **c)** $x = 96$ **d)** $x = 9$

6. a) Rubina subtracted the contents of the log when she should have divided them. The solution should be
$$\begin{aligned} \log_6 \left(\frac{2x + 1}{x - 1}\right) &= \log_6 5 \\ 2x + 1 &= 5(x - 1) \\ 1 + 5 &= 5x - 2x \\ 6 &= 3x \\ x &= 2 \end{aligned}$$

b) Ahmed incorrectly concluded that there was no solution. The solution is $x = 0$.

c) Jennifer incorrectly eliminated the log in the third line. The solution, from the third line on, should be
$$\begin{aligned} x(x + 2) &= 2^3 \\ x^2 + 2x - 8 &= 0 \\ (x - 2)(x + 4) &= 0 \end{aligned}$$
So, $x = 2$ or $x = -4$.
Since $x > 0$, the solution is $x = 2$.

7. a) 0.65 **b)** -0.43 **c)** 81.37 **d)** 4.85

8. a) no solution ($x = -3$ not possible)

b) $x = 10$ **c)** $x = 4$ **d)** $x = 2$ **e)** $x = -8, 4$

9. a) about 2.64 pc **b)** about 8.61 light years

10. 64 kg

11. a) 10 000 **b)** 3.5%

c) approximately 20.1 years

12. a) 248 Earth years **b)** 228 million kilometres

13. a) 2 years **b)** 44 days **c)** 20.5 years

14. 30 years

15. approximately 9550 years

16. 8 days

17. 34.0 m

18. $x = 4.5$, $y = 0.5$

19. a) The first line is not true.

b) To go from line 4 to line 5, you are dividing by a negative quantity, so the inequality sign must change direction.

20. a) $x = 100$ **b)** $x = \frac{1}{100}, 100$ **c)** $x = 1, 100$

21. a) $x = 16$ **b)** $x = 9$

22. $x = -5, 2, 4$

C1 a)
$$\begin{aligned} \log 8 + \log 2^x &= \log 512 \\ x \log 2 &= \log 512 - \log 8 \\ x \log 2 &= \log 64 \\ x &= 6 \end{aligned}$$

b) She could have divided by 8 as the first step.

c) Answers will vary.

C2 12

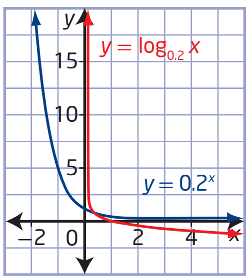
C3 14

C4 a) $x = \frac{\pi}{4}, \frac{7\pi}{4}$ **b)** $x = \frac{\pi}{2}$

C5 Answers will vary.

Chapter 8 Review, pages 416 to 418

1. a)



b) i) domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y \in R\}$

ii) x-intercept 1

iii) no y-intercept

iv) vertical asymptote $x = 0$

c) $y = \log_{0.2} x$

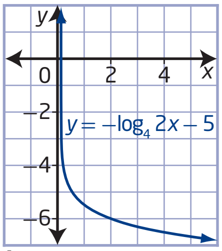
2. $c = 4$

3. $2^4 = 16$ and $2^5 = 32$, so the answer must be between 4 and 5.

4. a) 25 **b)** -2 **c)** 3.5 **d)** 16 **e)** 0.01

5. 40 times as great

6. a)



b) $a = -1$, $b = 2$, $c = 4$, $h = 0$, $k = -5$

7. $y = \log_2 4x$

8. a) Reflect in the x-axis, stretch horizontally about the y-axis by a factor of $\frac{1}{3}$, and translate 12 units right and 2 units up.

b) Reflect in the y-axis, stretch vertically about the x-axis by a factor of $\frac{1}{4}$, and translate 6 units right and 7 units down.

9. a) $x = -8$

b) domain $\{x \mid x > -8, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

c) y-intercept 15 **d)** x-intercept -7.75

10. a) Transform by stretching the graph horizontally about the y-axis by a factor of 440 and stretching vertically about the x-axis by a factor of 12.

b) 5 notes above **c)** 698.46 Hz

11. a) $5 \log_5 x - \log_5 y - \frac{1}{3} \log_5 z$

b) $\frac{1}{2}(\log x + 2 \log y - \log z)$

12. a) $\log \frac{xz^{\frac{2}{3}}}{y^3}$ **b)** $\log_7 \frac{x}{y^{\frac{1}{2}}z^{\frac{3}{2}}}$

13. a) $\log \sqrt{x}$, $x > 0$ **b)** $\log \frac{x-5}{x+5}$, $x < -5$ or $x > 5$

14. a) 2 **b)** 0.5

15. 6.3 times as acidic

16. 398 107 times as bright

17. 93 dB

18. a) 1.46 **b)** 4.03

19. a) 5 **b)** 10 **c)** $\frac{5}{3}$ **d)** -4, 25

20. 6.5 years

21. 35 kg

22. 2.5 h

23. a) 14 years **b)** 25.75 years

Chapter 8 Practice Test, pages 419 to 420

1. D **2.** A **3.** B **4.** A **5.** C **6.** B

7. a) $\frac{1}{81}$ **b)** 25 **c)** 5 **d)** 3 **e)** $\frac{13}{3}$

8. $m = 2.5$, $n = 0.5$

9. Example: Stretch vertically about the x-axis by a factor of 5, stretch horizontally about the y-axis by a factor of $\frac{1}{8}$, reflect in the x-axis, and translate 1 unit right.

10. a) $x = -5$

b) domain $\{x \mid x > -5, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

c) y-intercept 8 **d)** $-4\frac{124}{125}$

11. a) no solution **b)** $x = 6$ **c)** $x = -2, 4$

12. a) 1.46 **b)** 21.09

13. 33 years

14. 875 times as great

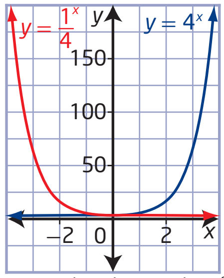
15. She should not be worried: adding another refrigerator will only increase the decibels to 48 dB.

16. 4.8 h

17. 2029

Cumulative Review, Chapters 7–8, pages 422 to 423

1. a)



b) The two functions have the same domain, $x \in \mathrm{R}$; the same range, $y > 0$; the same y-intercept, 1; and the same horizontal asymptote, $y = 0$.

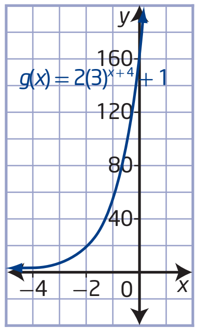
c) $y = 4^x$ is a increasing function: as x increases, the corresponding values of y also increase. $y = \left(\frac{1}{4}\right)^x$ is a decreasing function: as x increases, the corresponding values of y decrease.

2. a) B **b)** D **c)** A **d)** C

3. a) 1000 **b)** 3 h **c)** 256 000 **d)** 21 h

4. a) a vertical stretch by a factor of 2 about the x-axis, a horizontal translation of 4 units left, and a vertical translation of 1 unit up

b)



c) The domain remains the same: $x \in \mathrm{R}$; the range changes from $y > 0$ to $y > 1$ due to the vertical translation; the equation of the horizontal asymptote changes from $y = 0$ to $y = 1$ due to the vertical translation; the y-intercept changes from 1 to 163 due to the vertical stretch and the vertical translation.

5. a) 2^{3x+6} and 2^{3x-15} or 8^{x+2} and 8^{x-5}

b) 3^{12-3x} and 3^{-4x} or $\left(\frac{1}{3}\right)^{3x-12}$ and $\left(\frac{1}{3}\right)^{4x}$

6. a) -1 **b)** $\frac{1}{8}$

7. a) -0.72 **b)** 0.63

8. a) 39% **b)** 3.7 s

9. a) $\log_3 y = x$ **b)** $\log_2 m = a + 1$

10. a) $x^4 = 3$ **b)** $a^b = x + 5$

11. a) -4 **b)** 4.5 **c)** -1 **d)** 49

12. a) 2 **b)** 32 **c)** $\frac{1}{125}$ **d)** $\frac{243}{32}$

13. a vertical stretch by a factor of $\frac{1}{3}$ about the x-axis, a horizontal stretch by a factor of $\frac{1}{2}$ about the y-axis, a horizontal translation of 4 units right and a vertical translation of 5 units up

14. a) $y = 3 \log (x + 5)$ **b)** $y = -\log 2x - 2$

15. a) 1.6×10^{-8} mol/L to 6.3×10^{-7} mol/L

b) yes

16. a) $\log \frac{m^2}{\sqrt{n}p^3}$, $m > 0$, $n > 0$, $p > 0$

b) $\log_a 3x^{\frac{13}{6}}$, $x > 0$ **c)** $\log (x + 1)$, $x > 1$

d) $\log_2 3^{2x}$, $x \in \mathrm{R}$

17. In the last step, Zack incorrectly factored the quadratic equation; $x = -5$ and 13.

18. a) 0.53 **b)** 9 **c)** 3 **d)** 2

19. a) $E = 10^{10}$ J and $E = 10^{11.4}$ J

b) approximately 25.1 times

20. 54.25 years

Unit 3 Test, pages 424 to 425

1. D **2.** B **3.** A **4.** C **5.** A **6.** A **7.** D

8. $y = -2\left(\frac{1}{4}\right)^{x-3}$

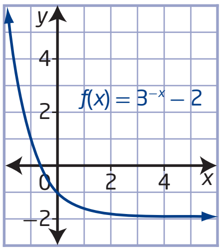
9. 3^{-1}

10. $(2, -2)$

11. 0.001

12. -2

13. a)



b) domain $\{x \mid x \in R\}$, range $\{y \mid y > -2, y \in R\}$

c) $x = -0.6$

14. a) -7 **b)** 2

15. a) domain $\{x \mid x > 2, x \in R\}$, range $\{y \mid y \in R\}$, asymptote $x = 2$

b) $y = 10^{1-x} + 2$ **c)** 12

16. a) $\frac{1}{3}$, 4 **b)** 7

17. Giovanni multiplied the base by 2, which is not correct. The second line should be $3^x = 4$. Giovanni also incorrectly applied the quotient law of logarithms in the sixth line. This line should be deleted. This leads to the solution $x = 1.26$.

18. 5.0

19. a) $P(t) = 6(1.013^t)$, where t is the number of years since 2000

b) year 2040

20. 12 deposits

Chapter 9 Rational Functions

9.1 Exploring Rational Functions Using Transformations, pages 442 to 445

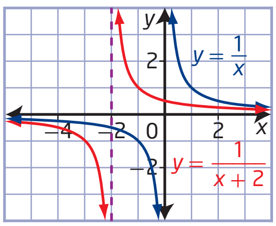
1. a) Since the graph has a vertical asymptote at $x = -1$, it has been translated 1 unit left; $B(x) = \frac{2}{x + 1}$.

b) Since the graph has a horizontal asymptote at $y = -1$, it has been translated 1 unit down; $A(x) = \frac{2}{x} - 1$.

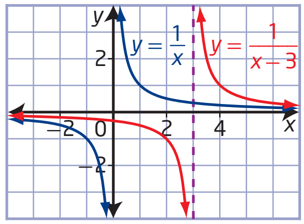
c) Since the graph has a horizontal asymptote at $y = 1$, it has been translated 1 unit up; $D(x) = \frac{2}{x} + 1$.

d) Since the graph has a vertical asymptote at $x = 1$, it has been translated 1 unit right; $C(x) = \frac{2}{x - 1}$.

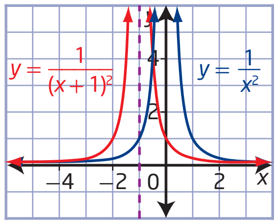
2. a) Base function $y = \frac{1}{x}$; vertical asymptote $x = -2$, horizontal asymptote $y = 0$



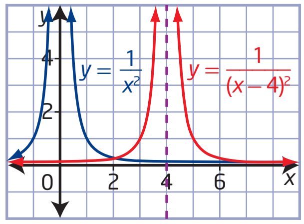
b) Base function $y = \frac{1}{x}$; vertical asymptote $x = 3$, horizontal asymptote $y = 0$



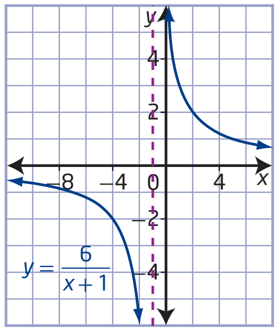
c) Base function $y = \frac{1}{x^2}$; vertical asymptote $x = -1$, horizontal asymptote $y = 0$



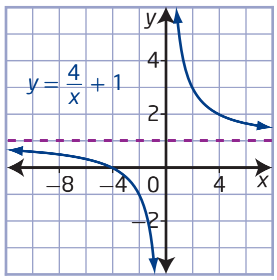
d) Base function $y = \frac{1}{x^2}$; vertical asymptote $x = 4$, horizontal asymptote $y = 0$



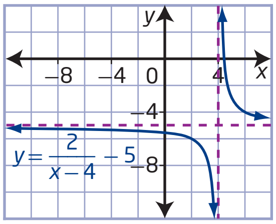
3. a) Apply a vertical stretch by a factor of 6, and then a translation of 1 unit left to the graph of $y = \frac{1}{x}$. domain $\{x \mid x \neq -1, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$, no x-intercept, y-intercept 6, horizontal asymptote $y = 0$, vertical asymptote $x = -1$



b) Apply a vertical stretch by a factor of 4, and then a translation of 1 unit up to the graph of $y = \frac{1}{x}$. domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 1, y \in R\}$, x-intercept -4, no y-intercept, horizontal asymptote $y = 1$, vertical asymptote $x = 0$

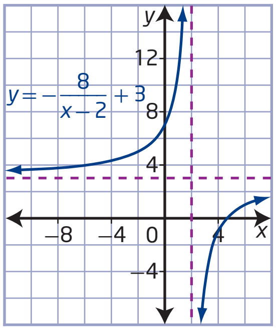


c) Apply a vertical stretch by a factor of 2, and then a translation of 4 units right and 5 units down to the graph of $y = \frac{1}{x}$.



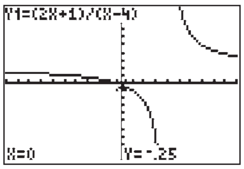
domain $\{x \mid x \neq 4, x \in \mathrm{R}\}$, range $\{y \mid y \neq -5, y \in \mathrm{R}\}$, x-intercept 4.4, y-intercept −5.5, horizontal asymptote $y = -5$, vertical asymptote $x = 4$

d) Apply a vertical stretch by a factor of 8 and a reflection in the x-axis, and then a translation of 2 units right and 3 units up to the graph of $y = \frac{1}{x}$.



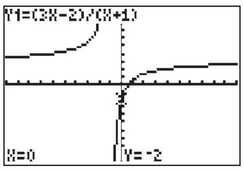
domain $\{x \mid x \neq 2, x \in \mathrm{R}\}$, range $\{y \mid y \neq 3, y \in \mathrm{R}\}$, x-intercept $\frac{14}{3}$, y-intercept 7, horizontal asymptote $y = 3$, vertical asymptote $x = 2$

4. a)



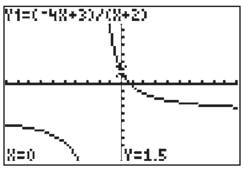
horizontal asymptote $y = 2$, vertical asymptote $x = 4$, x-intercept −0.5, y-intercept −0.25

b)



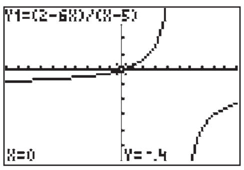
horizontal asymptote $y = 3$, vertical asymptote $x = -1$, x-intercept 0.67, y-intercept −2

c)



horizontal asymptote $y = -4$, vertical asymptote $x = -2$, x-intercept 0.75, y-intercept 1.5

d)



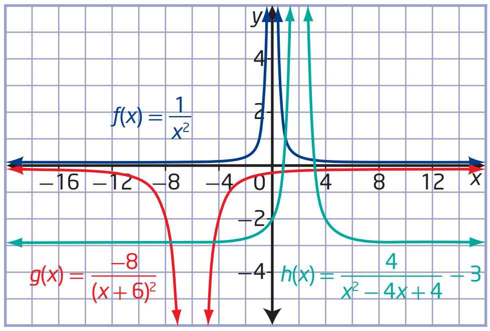
horizontal asymptote $y = -6$, vertical asymptote $x = 5$, x-intercept 0.33, y-intercept −0.4

5. a) $y = \frac{12}{x} + 11$; horizontal asymptote $y = 11$, vertical asymptote $x = 0$, x-intercept −1.09, no y-intercept

b) $y = -\frac{8}{x+8} + 1$; horizontal asymptote $y = 1$, vertical asymptote $x = -8$, x-intercept $x = 0$, y-intercept $y = 0$

c) $y = \frac{4}{x+6} - 1$; horizontal asymptote $y = -1$, vertical asymptote $x = -6$, x-intercept −2, y-intercept −0.33

6.



For $f(x) = \frac{1}{x^2}$:

- Non-permissible value: $x = 0$
- Behaviour near non-permissible value: As x approaches 0, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, y approaches 0.
- Domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$
- Asymptotes: $x = 0$, $y = 0$

For $g(x) = \frac{-8}{(x+6)^2}$:

- Non-permissible value: $x = -6$
- Behaviour near non-permissible value: As x approaches −6, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, y approaches 0.
- Domain $\{x \mid x \neq -6, x \in \mathrm{R}\}$, range $\{y \mid y < 0, y \in \mathrm{R}\}$
- Asymptotes: $x = -6$, $y = 0$

For $h(x) = \frac{4}{x^2 - 4x + 4} - 3$:

- Non-permissible value: $x = 2$
- Behaviour near non-permissible value: As x approaches 2, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, y approaches −3.
- Domain $\{x \mid x \neq 2, x \in \mathrm{R}\}$, range $\{y \mid y > -3, y \in \mathrm{R}\}$
- Asymptotes: $x = 2$, $y = -3$

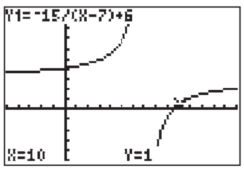
Each function has a single non-permissible value, a vertical asymptote, and a horizontal asymptote. The domain of each function consists of all real numbers except for a single value. The range of each function consists of a restricted set of the real numbers. $|y|$ becomes very large for each function when the values of x approach the non-permissible value for the function.

7. a) $y = -\frac{4}{x}$ **b)** $y = \frac{1}{x+3}$

c) $y = \frac{8}{x-2} + 4$ **d)** $y = \frac{-4}{x-1} - 6$

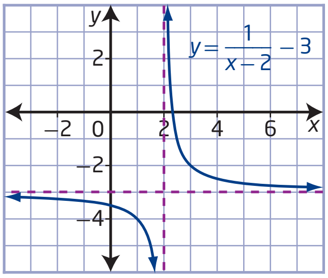
8. a) $a = -15$, $k = 6$

b)



9. a) $y = \frac{1}{x - 2} - 3$

b)



domain $\{x \mid x \neq 2, x \in \mathrm{R}\}$, range $\{y \mid y \neq -3, y \in \mathrm{R}\}$

c) No, there are many functions with different values of a for which the asymptotes are the same.

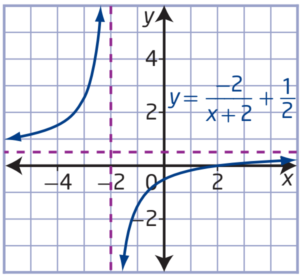
10. a) When factoring the 3 out of the numerator, Mira forgot to change the sign of the 21.

$$y = \frac{-3x + 21 - 21 + 2}{x - 7}$$
$$y = \frac{-3(x - 7) - 19}{x - 7}$$
$$y = \frac{-19}{x - 7} - 3$$

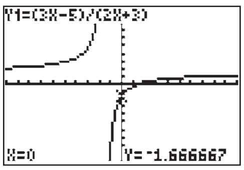
b) She could try sample points without technology. With technology, she could check if the asymptotes are the same.

11. a) $y = \frac{-2}{x + 2} + \frac{1}{2}$

b)



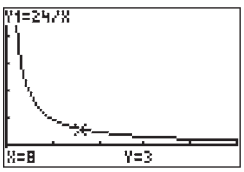
12.



x-intercept $\frac{5}{3}$, y-intercept $-\frac{5}{3}$, horizontal asymptote $y = 1.5$, vertical asymptote $x = -1.5$

13. As p increases, N decreases, and vice versa. This shows that as the average price of a home increases, the number of buyers looking for a house decreases.

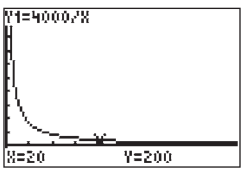
14. a) $l = \frac{24}{w}$



b) As the width increases, the length decreases to maintain the same area.

15. a) $y = \frac{4000}{x}$

b)

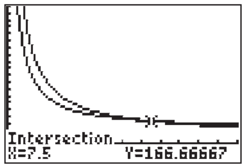


c) If 4000 students contribute, they will only need to donate \$1 each to reach their goal.

d) $y = \frac{4000}{x} + 1000$; This amounts to a vertical translation of 1000 units up.

16. a) $y = \frac{100x + 500}{x}$, $y = \frac{60x + 800}{x}$

b)



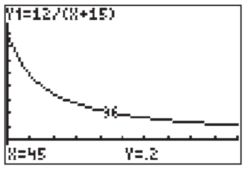
c) The graph shows that the more years you run the machine, the less the average cost per year is. One of the machines is cheaper to run for a short amount of time, while the other is cheaper if you run it for a longer period of time.

d) If Hanna wants to run the machine for more than 7.5 years, she should choose the second model. Otherwise, she is better off with the first one.

17. a) $I = \frac{12}{x + 15}$

b) Domain $\{x \mid 0 \leq x \leq 100, x \in \mathrm{R}\}$; the graph does not have a vertical asymptote for this domain.

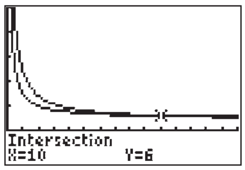
c)



A setting of 45 Ω is needed for 0.2 A.

d) In this case, there would be an asymptote at $x = 0$.

18. a) $y = \frac{4x + 20}{x}$, $y = \frac{5x + 10}{x}$

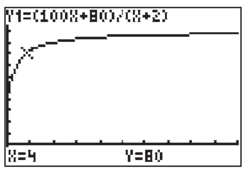


b) The graph shows that for a longer rental the average price goes down.

c) No. For rentals of less than 10 h, the second store is cheaper. For any rental over 10 h, the first store is cheaper.

19. a) $v = \frac{100t + 80}{t + 2}$

b)



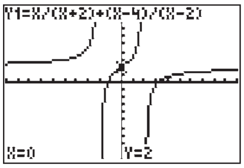
c) Horizontal asymptote $y = 100$; the horizontal asymptote demonstrates that the average speed gets closer and closer to 100 km/h but never reaches it. Vertical asymptote $t = -2$; the vertical asymptote does not mean anything in this context, since time cannot be negative.

d) 4 h after the construction zone

e) Example: Showing the average speed is a good indication of your fuel economy.

20. $y = \frac{-4x - 4}{x - 6}$

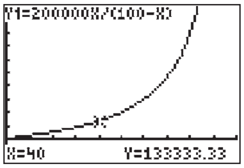
21. a) $y = \frac{-x - 3}{x - 1}$ **b)** $y = \frac{5(x - 4)}{x - 6}$

22. 

This rational function has two vertical asymptotes ($x = -2$ and $x = 2$) and appears to have a horizontal asymptote ($y = 2$) for values of x less than -2 and greater than 2.

C1 Answers may vary.

C2 a) Domain $\{p \mid 0 \leq p < 100, p \in \mathrm{R}\}$; you can nearly eliminate 100% of emissions.

b) 

The shape of the graph indicates that as the percent of emissions eliminated increases, so does the cost.

c) It costs almost 6 times as much. This is not a linear function, so doubling the value of p does not correspond to a doubling of the value of C.

d) No it is not possible. There is a vertical asymptote at $p = 100$.

C3 Example: Both functions are vertically stretched by a factor of 2, and then translated 3 units right and 4 units up. In the case of the rational function, the values of the parameters h and k represent the locations of asymptotes. For the square root function, the point (h, k) gives the location of the endpoint of the graph.

9.2 Analysing Rational Functions, pages 451 to 456

1. a)

Characteristic	$y = \dfrac{x - 4}{x^2 - 6x + 8}$
Non-permissible value(s)	$x = 2$, $x = 4$
Feature exhibited at each non-permissible value	vertical asymptote, point of discontinuity
Behaviour near each non-permissible value	As x approaches 2, $\lvert y \rvert$ becomes very large. As x approaches 4, y approaches 0.5.
Domain	$\{x \mid x \neq 2, 4, x \in \mathrm{R}\}$
Range	$\{y \mid y \neq 0, 0.5, y \in \mathrm{R}\}$

b) There is an asymptote at $x = 2$ because 2 is a zero of the denominator only. There is a point of discontinuity at (4, 0.5) because $x - 4$ is a factor of both the numerator and the denominator.

2. a)

x	y
−1.5	−4.5
−1.0	−4.0
−0.5	−3.5
0.5	−2.5
1.0	−2.0
1.5	−1.5

Since the function does not increase or decrease drastically as x approaches the non-permissible value, it must be a point of discontinuity.

b)

x	y
1.7	40.7
1.8	60.8
1.9	120.9
2.1	−118.9
2.2	−58.8
2.3	−38.7

Since the function changes sign at the non-permissible value and $\lvert y \rvert$ increases, it must be a vertical asymptote.

c)

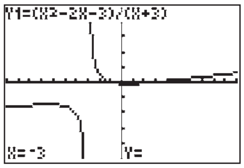
x	y
−3.7	74.23
−3.8	120.6
−3.9	260.3
−4.1	−300.3
−4.2	−160.6
−4.3	−114.23

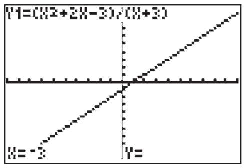
Since the function changes sign at the non-permissible value and $\lvert y \rvert$ increases, it must be a vertical asymptote.

d)

x	y
0.17	1.17
0.18	1.18
0.19	1.19
0.21	1.21
0.22	1.22
0.23	1.23

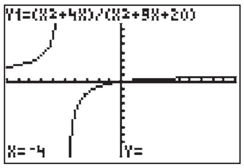
Since the function does not increase or decrease drastically as x approaches the non-permissible value, it must be a point of discontinuity.

3. a) 

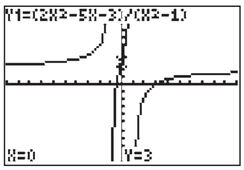


Both of the functions have a non-permissible value of −3. However, the graph of $f(x)$ has a vertical asymptote, while the graph of $g(x)$ has a point of discontinuity.

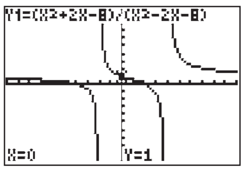
b) The graph of $f(x)$ has a vertical asymptote at $x = -3$ because $x + 3$ is a factor of the denominator only. The graph of $g(x)$ has a point of discontinuity at $(-3, -4)$ because $x + 3$ is a factor of both the numerator and the denominator.

4. a) 

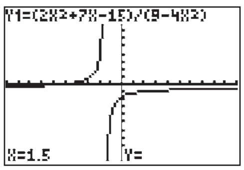
Vertical asymptote $x = -5$; point of discontinuity $(-4, -4)$; x-intercept 0; y-intercept 0

b) 

Vertical asymptotes $x = \pm 1$; no points of discontinuity; x-intercepts −0.5, 3; y-intercept 3

c) 

Vertical asymptotes $x = -2, 4$; no points of discontinuity; x-intercepts −4, 2; y-intercept 1

d) 

Vertical asymptote $x = -1.5$; point of discontinuity $(1.5, -1.083)$; x-intercept −5; y-intercept −1.67

5. a) The graph of $A(x) = \dfrac{x(x + 2)}{x^2 + 4}$ has no vertical asymptotes or points of discontinuity and x-intercepts of 0 and −2; C.

b) The graph of $B(x) = \dfrac{x - 2}{x(x - 2)}$ has a vertical asymptote at $x = 0$, a point of discontinuity at (2, 0.5), and no x-intercept; A.

c) The graph of $C(x) = \dfrac{x + 2}{(x - 2)(x + 2)}$ has a vertical asymptote at $x = 2$, a point of discontinuity at $(-2, -0.25)$, and no x-intercept; D.

d) The graph of $D(x) = \dfrac{2x}{x(x + 2)}$ has a vertical asymptote at $x = -2$, a point of discontinuity at (0, 1), and no x-intercept; B.

6. a) Since the graph has vertical asymptotes at $x = 1$ and $x = 4$, the equation of the function has factors $x - 1$ and $x - 4$ in the denominator only; the x-intercepts of 2 and 3 mean that the factors $x - 2$ and $x - 3$ are in the numerator; C.

b) Since the graph has vertical asymptotes at $x = -1$ and $x = 2$, the equation of the function has factors $x + 1$ and $x - 2$ in the denominator only; the x-intercepts of 1 and 4 mean that the factors $x - 1$ and $x - 4$ are in the numerator; B.

c) Since the graph has vertical asymptotes at $x = -2$ and $x = 5$, the equation of the function has factors $x + 2$ and $x - 5$ in the denominator only; the x-intercepts of -4 and 3 mean that the factors $x + 4$ and $x - 3$ are in the numerator; D.

d) Since the graph has vertical asymptotes at $x = -5$ and $x = 4$, the equation of the function has factors $x + 5$ and $x - 4$ in the denominator only; the x-intercepts of -2 and 1 mean that the factors $x + 2$ and $x - 1$ are in the numerator; A.

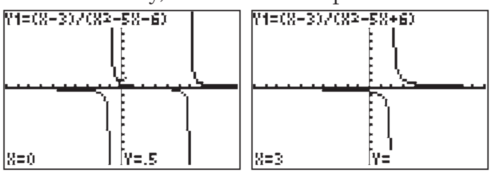
7. a) $y = \dfrac{x^2 + 6x}{x^2 + 2x}$ **b)** $y = \dfrac{x^2 - 4x - 21}{x^2 + 2x - 3}$

8. a) $y = \dfrac{(x + 10)(x - 4)}{(x + 5)(x - 5)}$ **b)** $y = \dfrac{(2x + 11)(x - 8)}{(x + 4)(2x + 11)}$

c) $y = \dfrac{(x + 2)(x + 1)}{(x - 3)(x + 2)}$ **d)** $y = \dfrac{x(4x + 1)}{(x - 3)(7x - 6)}$

9. a) Example: The graphs will be different. Factoring the denominators shows that the graph of $f(x)$ will have two vertical asymptotes, no points of discontinuity, and an x-intercept, while the graph of $g(x)$ will have one vertical asymptote, one point of discontinuity, and no x-intercept.

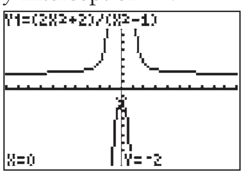
b)



10. $y = -\dfrac{3(x - 2)(x + 3)}{(x - 2)(x + 3)}$

11. a) The function will have two vertical asymptotes at $x = -1$ and $x = 1$, no x-intercept, and a y-intercept of -2.

b)

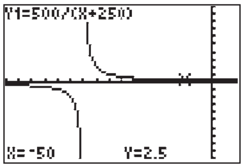


c) **i)** The graph will be a line at $y = 2$, but with points of discontinuity at $(-1, 2)$ and $(1, 2)$.

ii) The graph will be a line at $y = 2$.

12. a) $t = \dfrac{500}{w + 250}$, $w \neq -250$

b)

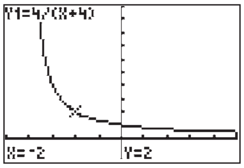


c) When the headwind reaches the speed of the aircraft, theoretically it will come to a standstill, so it will take an infinite amount of time for the aircraft to reach its destination.

d) Example: The realistic part of the graph would be in the range of normal wind speeds for whichever area the aircraft is in.

13. a) $t = \dfrac{4}{w + 4}$; $\{w \mid -4 < w \le 4, w \in \mathrm{R}\}$

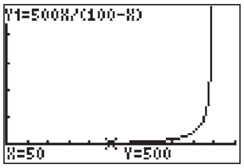
b)



c) As the current increases against the kayakers, in other words as the current reaches -4 km/h, the time it takes them to paddle 4 km approaches infinity.

14. a) The non-permissible value will result in a vertical asymptote. It corresponds to a factor of the denominator only.

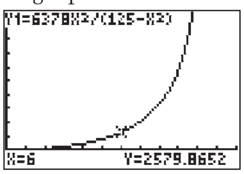
b)



It is not possible to vaccinate 100% of the population.

c) Yes, the vaccination process will get harder after you have already reached the major urban centres. It will be much more costly to find every single person.

15. a)



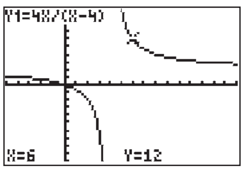
The only parts of the graph that are applicable are when $0 \le x < \sqrt{125}$.

b) As the initial velocity increases, the maximum height also increases but at a greater rate.

c) The non-permissible value represents the vertical asymptote of the graph; this models the escape velocity since when the initial velocity reaches the escape velocity the object will leave Earth and never return.

16. $y = \dfrac{-(x + 6)(x - 2)}{2(x + 2)(x - 3)}$

17. a)



b) The image distance decreases while the object distance is still less than the focal length. The image distance starts to increase once the object distance is more than the focal length.

c) The non-permissible value results in a vertical asymptote. As the object distance approaches the focal length, it gets harder to resolve the image.

18. a) Example: Functions $f(x)$ and $h(x)$ will have similar graphs since they are the same except for a point of discontinuity in the graph of $h(x)$.

b) All three graphs have a vertical asymptote at $x = -b$, since $x + b$ is a factor of only the denominators. All three graphs will also have an x-intercept of $-a$, since $x + a$ is a factor of only the numerators.

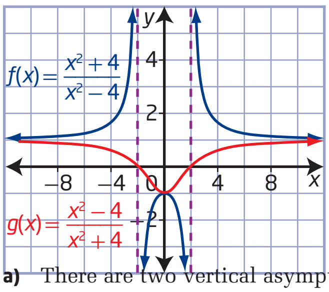
19. The x-intercept is 3 and the vertical asymptote is at $x = \frac{3}{4}$.

20. $y = \dfrac{x^2 - 4x + 3}{2x^2 - 18x - 20}$

21. a) $y = \dfrac{(x + 4)(x - 2)(3x + 4)}{4(x + 4)(x - 2)}$

b) $y = \dfrac{(x - 1)(x + 2)^2(x - 2)}{(x - 1)(x + 2)}$

22.



They are reciprocals since when one of them approaches infinity the other approaches 0.

23. a) There are two vertical asymptotes at $x = \pm 2$.

b) There is a point of discontinuity at $\left(5, \frac{65}{9}\right)$ and a vertical asymptote at $x = -4$.

C1 Examples:

a) No. Some rational functions have no points of discontinuity or asymptotes.

b) A rational function is a function that has a polynomial in the numerator and/or in the denominator.

C2 Example: True. It is possible to express a polynomial function as a rational function with a denominator of 1.

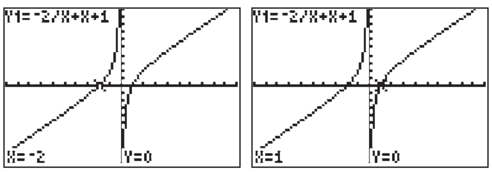
C3 Answers may vary.

9.3 Connecting Graphs and Rational Equations, pages 465 to 467

1. a) B **b)** D **c)** A **d)** C

2. a) $x = -2, x = 1$

b) $x = -2, x = 1$



c) The value of the function is 0 when the value of x is -2 or 1. The x-intercepts of the graph of the corresponding function are the same as the roots of the equation.

3. a) $x = -\frac{7}{4}$ **b)** $x = 4$ **c)** $x = \frac{3}{2}$ **d)** $x = -\frac{6}{5}$

4. a) $x = -8, x = 1$ **b)** $x = 0, x = 3$

c) $x = 4$ **d)** $x = 1, x = \frac{5}{3}$

5. a) $x \approx -0.14, x \approx 3.64$ **b)** $x \approx -2.30, x \approx 0.80$

c) $x \approx -2.41, x \approx 0.41$ **d)** $x \approx -5.74, x \approx -0.26$

6. a) $x = -\frac{2}{5}$ **b)** $x = 1$

c) $x = -5$ **d)** $x = -\frac{1}{3}$

7. Example: Her approach is correct but there is a point of discontinuity at (1, 4). Multiplying by $(x - 1)$ assumes that $x \neq 1$.

8. $x = -1, x = -\frac{2}{7}$

9. No solutions

10. 2.82 m

11. 20.6 h

12. 15 min

13. a) $y = \dfrac{0.5x + 2}{x + 28}$

b) After she takes 32 shots, she will have a 30% shooting percentage.

14. a) 200.4 K **b)** 209.3 K

15. a) $C(x) = \dfrac{0.01x + 10}{x + 200}$ **b)** 415 mL

16. $x \approx 1.48$

17. a) $x \leq -\frac{13}{4}$ or $x > 1$ **b)** $-8 \leq x < -6, 2 < x \leq 4$

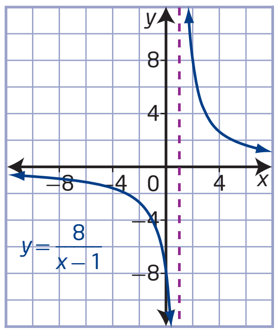
C1 Example: No, this is incorrect. For example, $\frac{1}{x} = 0$ has no solution.

C2 Example: The extraneous root in the radical equation occurs because there is a restriction that the radicand be positive. This same principle of restricted domain is the reason why the rational equation has an extraneous root.

C3 Answers may vary.

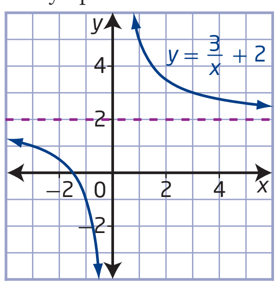
Chapter 9 Review, pages 468 to 469

1. a) Apply a vertical stretch by a factor of 8, and then a translation of 1 unit right to the graph of $y = \frac{1}{x}$.



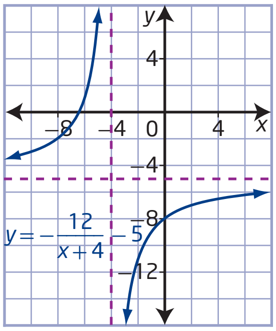
domain $\{x \mid x \neq 1, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$, no x-intercept, y-intercept -8, horizontal asymptote $y = 0$, vertical asymptote $x = 1$

b) Apply a vertical stretch by a factor of 3 and then a translation of 2 units up to the graph of $y = \frac{1}{x}$.



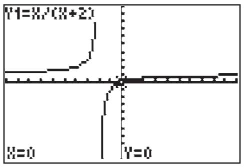
domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 2, y \in \mathrm{R}\}$, x-intercept -1.5, no y-intercept, horizontal asymptote $y = 2$, vertical asymptote $x = 0$

c) Apply a vertical stretch by a factor of 12 and a reflection in the x-axis, and then a translation of 4 units left and 5 units down to the graph of $y = \frac{1}{x}$.



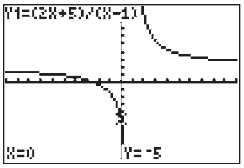
domain $\{x \mid x \neq -4, x \in \mathrm{R}\}$, range $\{y \mid y \neq -5, y \in \mathrm{R}\}$, x-intercept -6.4, y-intercept -8, horizontal asymptote $y = -5$, vertical asymptote $x = -4$

2. a)



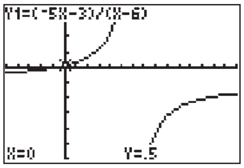
Horizontal asymptote $y = 1$, vertical asymptote $x = -2$, x-intercept 0, y-intercept 0

b)



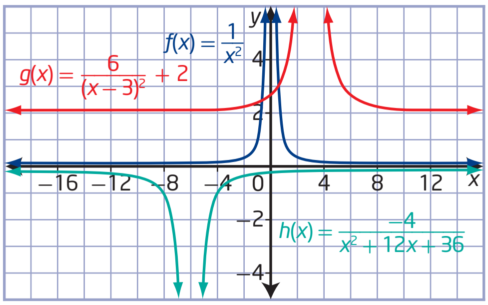
Horizontal asymptote $y = 2$, vertical asymptote $x = 1$, x-intercept -2.5, y-intercept -5

c)



Horizontal asymptote $y = -5$, vertical asymptote $x = 6$, x-intercept -0.6, y-intercept 0.5

3.



For $f(x) = \frac{1}{x^2}$:

- Non-permissible value: $x = 0$
- Behaviour near non-permissible value: As x approaches 0, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, y approaches 0.
- Domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$
- Asymptotes: $x = 0$, $y = 0$

For $g(x) = \frac{6}{(x-3)^2} + 2$:

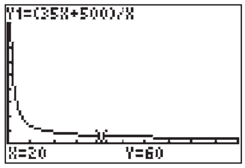
- Non-permissible value: $x = 3$
- Behaviour near non-permissible value: As x approaches 3, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, y approaches 2.
- Domain $\{x \mid x \neq 3, x \in \mathrm{R}\}$, range $\{y \mid y > 2, y \in \mathrm{R}\}$
- Asymptotes: $x = 3$, $y = 2$

For $h(x) = \frac{-4}{x^2 + 12x + 36}$:

- Non-permissible value: $x = -6$
- Behaviour near non-permissible value: As x approaches -6, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, y approaches 0.
- Domain $\{x \mid x \neq -6, x \in \mathrm{R}\}$, range $\{y \mid y < 0, y \in \mathrm{R}\}$
- Asymptotes: $x = -6$, $y = 0$

Each function has a single non-permissible value, a vertical asymptote, and a horizontal asymptote. The domain of each function consist of all real numbers except for a single value. The range of each function is a restricted set of real numbers. $|y|$ becomes very large for each function when the values of x approach the non-permissible value for the function.

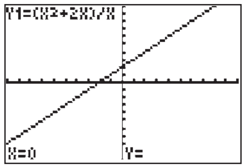
4. a) $y = \frac{35x + 500}{x}$



b) The more uniforms that are bought, the less expensive their average cost.

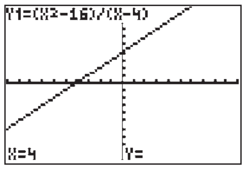
c) They will need to buy 100 uniforms.

5. a)



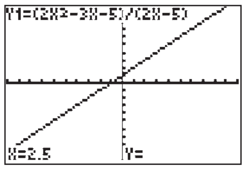
linear with a point of discontinuity at (0, 2)

b)



linear with a point of discontinuity at (4, 8)

c)



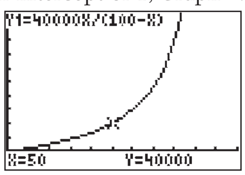
linear with a point of discontinuity at (2.5, 3.5)

6. The graph of $A(x) = \frac{x-4}{(x-4)(x-1)}$ has a vertical asymptote at $x = 1$, a point of discontinuity at $\left(4, \frac{1}{3}\right)$, and no x-intercept; Graph 3.

The graph of $B(x) = \frac{(x+4)(x+1)}{x^2+1}$ has no vertical asymptotes or points of discontinuity and x-intercepts of -4 and -1; Graph 1.

The graph of $C(x) = \frac{x-1}{(x-2)(x+2)}$ has vertical asymptotes at $x = \pm 2$, no points of discontinuity, and an x-intercept of 1; Graph 2.

7. a)

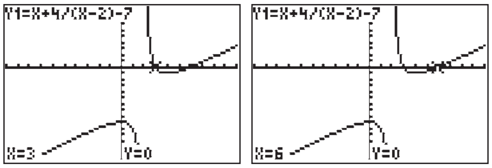


b) As the percent of the spill cleaned up approaches 100, the cost approaches infinity.

c) No, since there is a vertical asymptote at $p = 100$.

8. a) $x = 3, x = 6$

b)



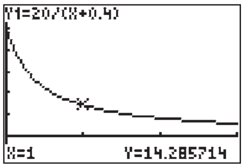
c) The value of the function is 0 when the value of x is 3 or 6. The x-intercepts of the graph of the corresponding function are the same as the roots of the equation.

9. a) $x = -3, x = 11$ **b)** $x = 4, x = 6$
c) $x = -1, x = 5$ **d)** $x = -2, x = 4.5$

10. a) $x \approx 2.71$ **b)** $x \approx -6.15, x \approx 3.54$
c) $x \approx \pm 0.82$ **d)** $x \approx 2.67$

11. a) $\{d \mid -0.4 \leq d \leq 2.6, d \in R\}$

b) As the distance along the lever increases, less mass can be lifted.



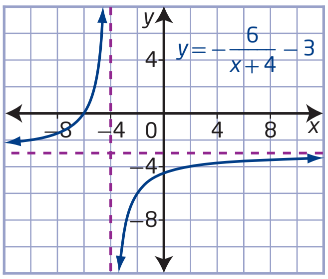
c) The non-permissible value corresponds to the fulcrum point ($d = -0.4$), which does not move when the lever is moved. As the mass gets closer to the fulcrum, it is possible to move a much heavier mass, but when the mass is on the fulcrum, it cannot be moved.

d) 0.74 m

Chapter 9 Practice Test, pages 470 to 471

1. C **2.** D **3.** C **4.** B **5.** D **6.** C **7.** $x = -\frac{6}{5}$

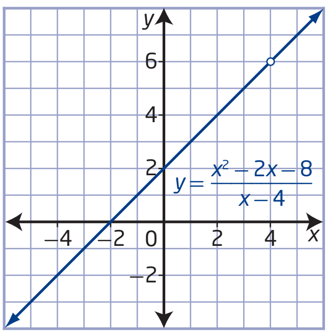
8. a)



b) domain $\{x \mid x \neq -4, x \in R\}$, range $\{y \mid y \neq -3, y \in R\}$, horizontal asymptote $y = -3$, vertical asymptote $x = -4$, x-intercept -6, y-intercept $-\frac{9}{2}$

9. $x \approx -2.47, x \approx -0.73$

10. a)



b) As x approaches 4, the function approaches 6.

11. vertical asymptote $x = 3$, point of discontinuity $\left(-4, \frac{9}{7}\right)$, x-intercept 0.5, y-intercept $\frac{1}{3}$

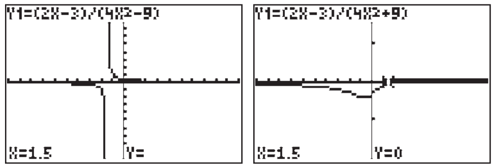
12. a) The graph of $A(x) = \frac{x(x-9)}{x}$ has no vertical asymptote, a point of discontinuity at $(0, -9)$, and an x-intercept of 9; D.

b) The graph of $B(x) = \frac{x^2}{(x-3)(x+3)}$ has vertical asymptotes at $x = \pm 3$, no points of discontinuity, and an x-intercept of 0; A.

c) The graph of $C(x) = \frac{(x-3)(x+3)}{x^2}$ has a vertical asymptote at $x = 0$, no points of discontinuity, and x-intercepts of ± 3; B.

d) The graph of $D(x) = \frac{x^2}{x(x-9)}$ has a vertical asymptote at $x = 9$, a point of discontinuity at $(0, 0)$, and no x-intercept; C.

13.



The main difference is that the second function has no non-permissible values since the denominator cannot be factored.

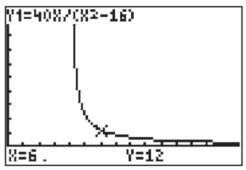
14. a) $x = 3$; Alex forgot to take into account the restricted domain.

b) Using graphical methods, it is easier to see true solutions.

15. a) $A = \frac{0.5x + 10}{x + 31}$ **b)** an additional 24 putts

16. a) $\{v \mid v > 4, v \in R\}$; speed must be positive and the function is undefined when $v = 4$.

b)



As the boat's speed increases, the total time for the round trip decreases.

c) As the boat's speed approaches 4 km/h, the time it takes for a round trip approaches infinity. The water flows at 4 km/h. If the boat's speed is less, the boat will never make the return trip, which is why there is an asymptote at $x = 4$.

d) approximately 27.25 km/h

Chapter 10 Function Operations

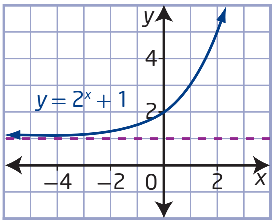
10.1 Sums and Differences of Functions, pages 483 to 487

1. a) $h(x) = |x - 3| + 4$ **b)** $h(x) = 2x - 3$
c) $h(x) = 2x^2 + 3x + 2$ **d)** $h(x) = x^2 + 5x + 4$

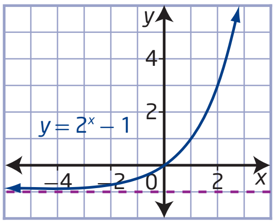
2. a) $h(x) = 5x + 2$ **b)** $h(x) = -3x^2 - 4x + 9$
c) $h(x) = -x^2 - 3x + 12$ **d)** $h(x) = \cos x - 4$

3. a) $h(x) = x^2 - 6x + 1$; $h(2) = -7$
b) $m(x) = -x^2 - 6x + 1$; $m(1) = -6$
c) $p(x) = x^2 + 6x - 1$; $p(1) = 6$

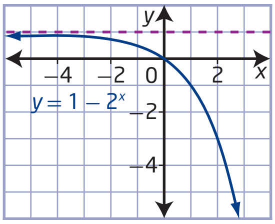
4. a) $y = 3x^2 + 2 + \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in R\}$
b) $y = 4x - 2 - \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in R\}$
c) $y = \sqrt{x + 4} - 4x + 2$; domain $\{x \mid x \geq -4, x \in R\}$
d) $y = 3x^2 + 4x$; domain $\{x \mid x \in R\}$

5. a) 

domain
$\{x \mid x \in \mathrm{R}\}$,
range
$\{y \mid y > 1, y \in \mathrm{R}\}$

b) 

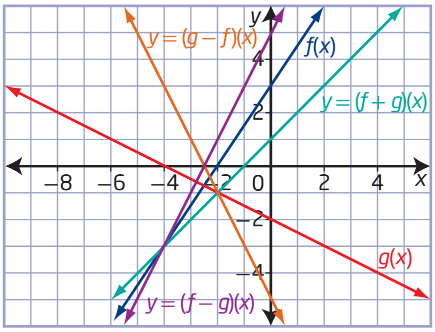
domain
$\{x \mid x \in \mathrm{R}\}$,
range
$\{y \mid y > -1, y \in \mathrm{R}\}$

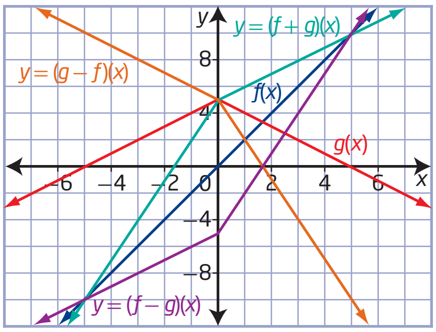
c) 

domain
$\{x \mid x \in \mathrm{R}\}$,
range
$\{y \mid y < 1, y \in \mathrm{R}\}$

6. a) 8 **b)** 6 **c)** 7
d) not in the domain

7. a) B **b)** C **c)** A

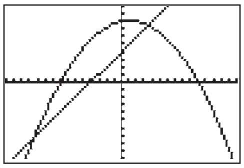
8. a) 

b) 

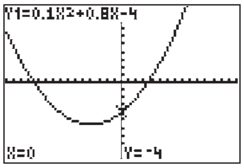
9. a) $y = 3x^2 + 11x + 1$ **b)** $y = 3x^2 - 3x + 3$
c) $y = 3x^2 + 3x + 1$ **d)** $y = 3x^2 - 11x + 3$

10. a) $g(x) = x^2$ **b)** $g(x) = \sqrt{x + 7}$
c) $g(x) = -3x + 1$ **d)** $g(x) = 3x^2 - x - 4$

11. a) $g(x) = x^2 - 1$ **b)** $g(x) = -\sqrt{x - 4}$
c) $g(x) = 8x - 9$ **d)** $g(x) = 2x^2 - 11x - 6$

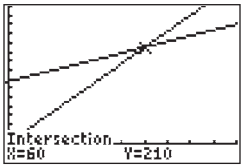
12. a) 

The points of intersection represent where the supply equals the demand. The intersection point in quadrant III should not be considered since the price cannot be negative.

b) 

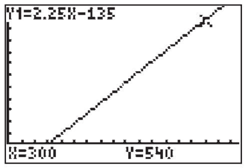
It represents the excess supply as a function of cost.

13. a) $C(n) = 1.25n + 135$, $R(n) = 3.5n$

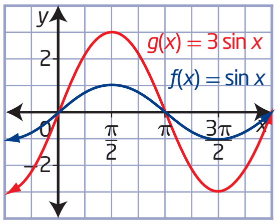
b) 

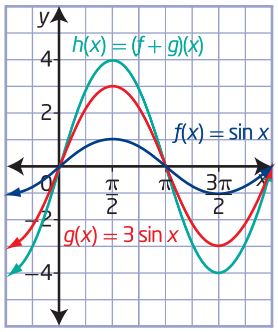
c) (60, 210)

d) $P(n) = 2.25n - 135$

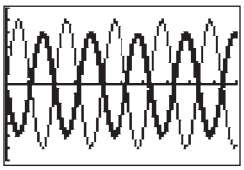


e) $540

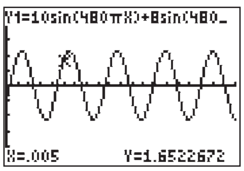
14. a) 

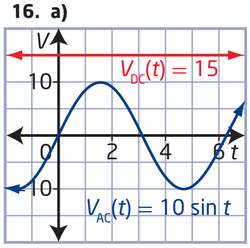
b) 

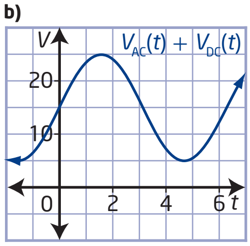
c) 4 cm

15. a) 

b) The maxima and minima are located at the same x-coordinates. This will result in destructive interference.

c) 

16. a) 

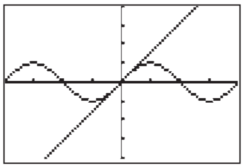
b) 

c) domain $\{t \mid t \in \mathrm{R}\}$, range $\{V \mid 5 \le V \le 25,\ V \in \mathrm{R}\}$

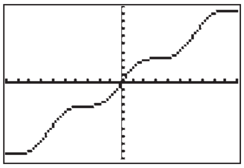
d) **i)** 5 V **ii)** 25 V

17. $h(t) = 5t^2 - 20t - 20$

18. a)



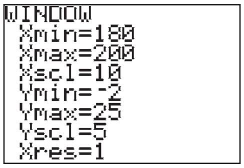
b) It will be a sinusoidal function on a diagonal according to $y = x$.

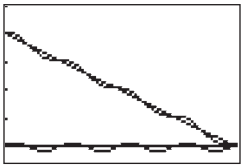


19. a) $d = 200 - t$

b) $h(t) = 200 - t + 0.75 \sin 1.26t$

c)





20. Example: Replace all x with $-x$ and then simplify. If the new function is equal to the original, then it is even. If it is the negative of the original, then it is odd. Answers may vary.

21. The graph shows the sum of an exponential function and a constant function.

22. a) $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \ge -9,\ y \in \mathrm{R}\}$; $g(x)$: domain $\{x \mid x \ne 0,\ x \in \mathrm{R}\}$, range $\{y \mid y \ne 0,\ y \in \mathrm{R}\}$

b) $h(x) = x^2 - 9 + \frac{1}{x}$

c) Example: The domain and range of $f(x)$ are different from the domain and range of $h(x)$. The domain and range of $g(x)$ are the same as that of $h(x)$.

C1 a) Yes, addition is commutative.

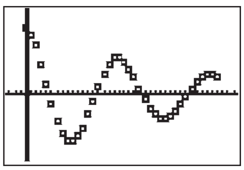
b) No, subtraction is not commutative.

C2 a) $y_3 = x^3 + 4$

b) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

C3 Example:

Step 1:



The graph exhibits sinusoidal features in its shape and the fact that it is periodic.

Step 2: The graph exhibits exponential features in that it is decreasing and approaching 0 with asymptote $y = 0$.

Step 3: $h = \cos 0.35t$

Step 4: $h = 100(0.5)^{0.05t}$

Step 5: $h = (100 \cos 0.35t)((0.5)^{0.05t})$

Step 6: 15.5 m

10.2 Products and Quotients of Functions, pages 496 to 498

1. a) $h(x) = x^2 - 49$, $k(x) = \dfrac{x + 7}{x - 7}$, $x \ne 7$

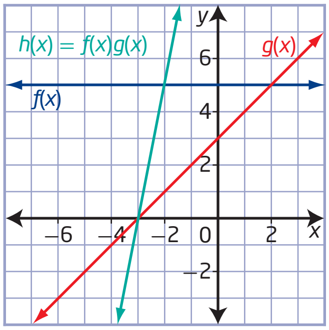
b) $h(x) = 6x^2 + 5x - 4$, $k(x) = \dfrac{2x - 1}{3x + 4}$, $x \ne -\frac{4}{3}$

c) $h(x) = (x + 2)\sqrt{x + 5}$, $k(x) = \dfrac{\sqrt{x + 5}}{x + 2}$, $x \ge -5$, $x \ne -2$

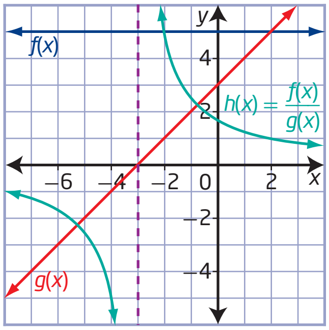
d) $h(x) = \sqrt{-x^2 + 7x - 6}$, $k(x) = \dfrac{\sqrt{x - 1}}{\sqrt{6 - x}}$, $1 \le x < 6$

2. a) -3 **b)** 0 **c)** -1 **d)** 0

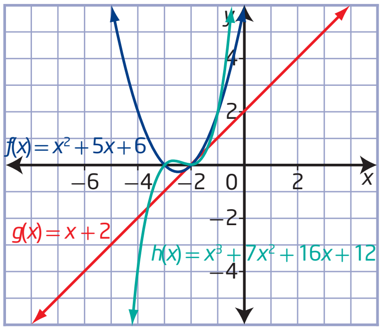
3. a)



b)

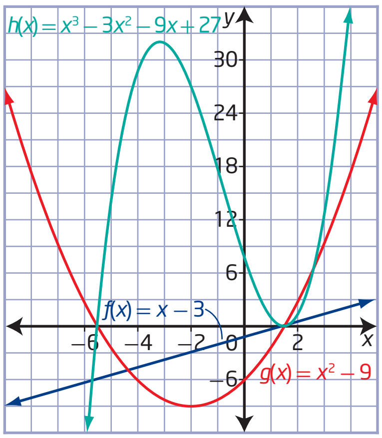


4. a) $h(x) = x^3 + 7x^2 + 16x + 12$



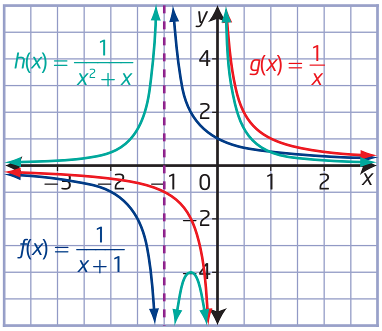
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

b) $h(x) = x^3 - 3x^2 - 9x + 27$



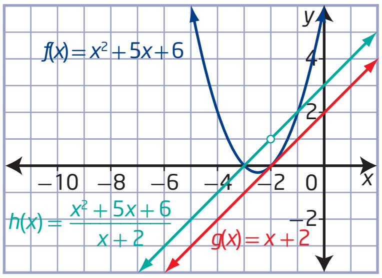
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

c) $h(x) = \dfrac{1}{x^2 + x}$



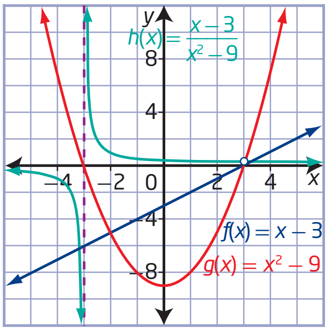
domain $\{x \mid x \neq 0, -1, x \in \mathrm{R}\}$,
range $\{y \mid y \leq -4 \text{ or } y > 0, y \in \mathrm{R}\}$

5. a) $h(x) = x + 3, x \neq -2$



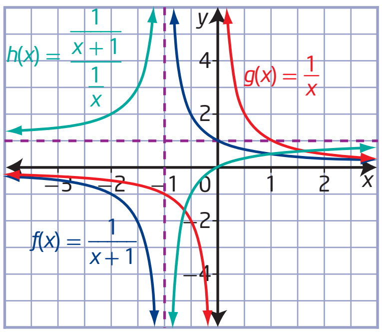
domain $\{x \mid x \neq -2, x \in \mathrm{R}\}$,
range $\{y \mid y \neq 1, y \in \mathrm{R}\}$

b) $h(x) = \dfrac{1}{x + 3}, x \neq \pm 3$



domain $\{x \mid x \neq \pm 3, x \in \mathrm{R}\}$,
range $\left\{y \mid y \neq 0, \dfrac{1}{6}, y \in \mathrm{R}\right\}$

c) $h(x) = \dfrac{x}{x + 1}, x \neq -1, 0$



domain $\{x \mid x \neq -1, 0, x \in \mathrm{R}\}$,
range $\{y \mid y \neq 0, 1, y \in \mathrm{R}\}$

6. a) $y = x^3 + 3x^2 - 10x - 24$

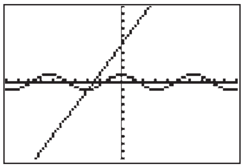
b) $y = \dfrac{x^2 - x - 6}{x + 4}, x \neq -4$ **c)** $y = \dfrac{2x - 1}{x + 4}, x \neq -4$

d) $y = \dfrac{x^2 - x - 6}{x^2 + 8x + 16}, x \neq -4$

7. a) $g(x) = 3$ **b)** $g(x) = -x$
c) $g(x) = \sqrt{x}$ **d)** $g(x) = 5x - 6$

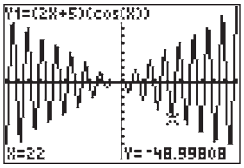
8. a) $g(x) = x + 7$ **b)** $g(x) = \sqrt{x + 6}$
c) $g(x) = 2$ **d)** $g(x) = 3x^2 + 26x - 9$

9. a)



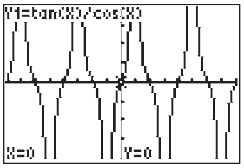
$f(x)$:
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$,
range
$\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$

b)



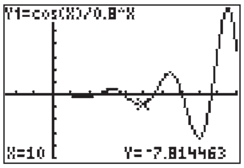
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

10. a)



domain
$\{x \mid x \neq (2n - 1)\frac{\pi}{2}$,
$n \in \mathrm{I}, x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

b)

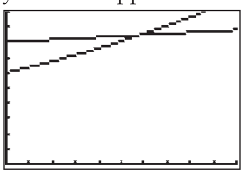


domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

11. a) $y = \dfrac{f(x)}{g(x)}$ **b)** $y = f(x)f(x)$

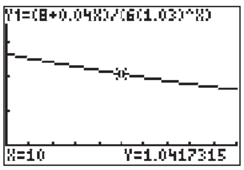
c) The graphs of $y = \dfrac{\sin x}{\cos x}$ and $y = \tan x$ appear to be the same. The graphs of $y = 1 - \cos^2 x$ and $y = \sin^2 x$ appear to be the same.

12. a)



Both graphs are increasing over time. However, the graph of $P(t)$ increases more rapidly and overtakes the graph of $F(t)$.

b)

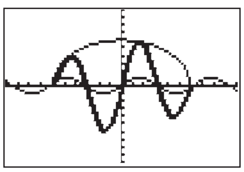


Yes; negative values of t should not be considered.

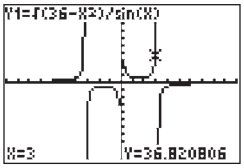
c) $t = 0$

d) In approximately 11.6 years, there will be less than 1 unit of food per fish; determine the point of intersection for the graphs of $y = \dfrac{F(t)}{P(t)}$ and $y = 1$.

13. a)



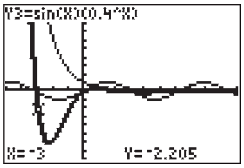
b) domain $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$, range $\{y \mid -5.8 \le y \le 5.8, y \in \mathrm{R}\}$

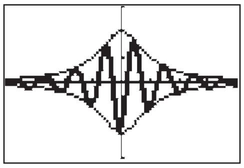
c) 

domain $\{x \mid -6 \le x \le 6, x \ne n\pi, n \in \mathrm{I}, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

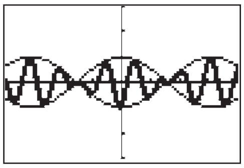
d) The domain in part d) is restricted to $-6 < x < 6$ but has no non-permissible values. In part c), the domain is restricted to to $-6 \le x \le 6$ with non-permissible values. The ranges in parts c) and d) are the same.

14. a), b) $f(t) = A \sin kt$, $g(t) = 0.4^{ct}$



15. a) 

b) Yes



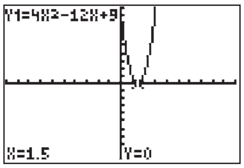
16. The price per tonne decreases.

17. $A = 4x\sqrt{r^2 - x^2}$

C1 Yes; multiplication is commutative. Examples may vary.

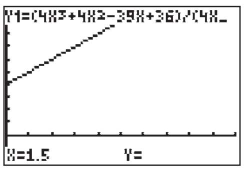
C2 Example: Multiplication generally increases the range and domain, although this is not always true. Quotients generally produce asymptotes and points of discontinuity, although this is not always true.

C3 a) $A(x) = 4x^2 - 12x + 9$

b) 

domain $\{x \mid x \ge 1.5, x \in \mathrm{R}\}$, range $\{A \mid A \ge 0, A \in \mathrm{R}\}$

c) $h(x) = x + 4, x \ne \frac{3}{2}$; this represents the height of the box.

d) 

domain $\{x \mid x > 1.5, x \in \mathrm{R}\}$, range $\{h \mid h > 5.5, h \in \mathrm{R}\}$

10.3 Composite Functions, pages 507 to 509

1. a) 3 **b)** 0 **c)** 2 **d)** -1

2. a) 2 **b)** 2 **c)** -4 **d)** -5

3. a) 10 **b)** -8 **c)** -2 **d)** 28

4. a) $f(g(a)) = 3a^2 + 1$ **b)** $g(f(a)) = 9a^2 + 24a + 15$

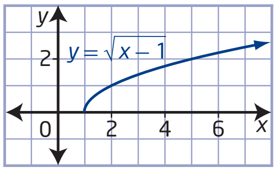
c) $f(g(x)) = 3x^2 + 1$ **d)** $g(f(x)) = 9x^2 + 24x + 15$

e) $f(f(x)) = 9x + 16$ **f)** $g(g(x)) = x^4 - 2x^2$

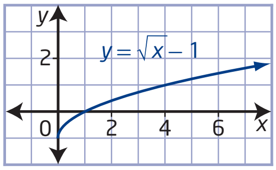
5. a) $f(g(x)) = x^4 + 2x^3 + 2x^2 + x$, $g(f(x)) = x^4 + 2x^3 + 2x^2 + x$

b) $f(g(x)) = \sqrt{x^4 + 2}$, $g(f(x)) = x^2 + 2$

c) $f(g(x)) = x^2$, $g(f(x)) = x^2$

6. a) 

domain $\{x \mid x \ge 1, x \in \mathrm{R}\}$, range $\{y \mid y \ge 0, y \in \mathrm{R}\}$

b) 

domain $\{x \mid x \ge 0, x \in \mathrm{R}\}$, range $\{y \mid y \ge -1, y \in \mathrm{R}\}$

7. a) $g(x) = 2x - 5$ **b)** $g(x) = 5x + 1$

8. Christine is right. Ron forgot to replace all x's with the other function in the first step.

9. Yes. $k(j(x)) = j(k(x)) = x^6$; using the power law: $2(3) = 6$ and $3(2) = 6$.

10. No. $s(t(x)) = x^2 - 6x + 10$ and $t(s(x)) = x^2 - 2$.

11. a) $W(C(t)) = 3\sqrt{100 + 35t}$

b) domain $\{t \mid t \ge 0, t \in \mathrm{R}\}$, range $\{W \mid W \ge 30, W \in \mathrm{W}\}$

12. a) $s(p) = 0.75p$ **b)** $t(s) = 1.05s$

c) $t(s(p)) = 0.7875p$; \$70.87

13. a) $g(d) = 0.06d$ **b)** $c(g) = 1.23g$

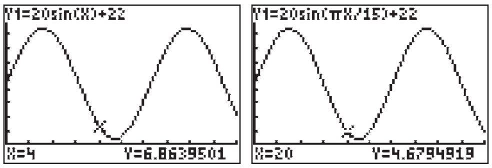
c) $c(g(d)) = 0.0738d$; \$14.76

d) $d(c) = 13.55c$; 542 km

14. a) $3x^2 - 21$ **b)** $3x^2 - 7$

c) $3x^2 - 42x + 147$ **d)** $9x^2 - 42x + 49$

15. a) $h(\theta(t)) = 20 \sin \frac{\pi t}{15} + 22$

b) 

The period of the combined functions is much greater.

16. a) $C(P(t)) = 14.375(2)^{\frac{t}{10}} + 53.12$

b) approximately 17.1 years

17. a) $f(x) = 2x - 1$, $g(x) = x^2$

b) $f(x) = \frac{2}{3 - x}$, $g(x) = x^2$

c) $f(x) = |x|$, $g(x) = x^2 - 4x + 5$

18. a) $g(f(x)) = \frac{1 - x}{1 - 1 + x} = \frac{1 - x}{x} = \frac{1}{g(x)}$

b) $f(g(x)) = 1 - \frac{x}{1 - x} = \frac{1 - 2x}{1 - x} \ne \frac{1}{f(x)}$

No, they are not the same.

19. a) $m = \dfrac{m_0}{\sqrt{1 - \frac{t^6}{c^2}}}$ **b)** $\frac{2}{\sqrt{3}}m_0$

20. a) The functions $f(x) = 5x + 10$ and $g(x) = \frac{1}{5}x - 2$ are inverses of each other since $f(g(x)) = x$ and $g(f(x)) = x$.

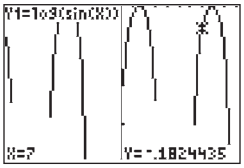
b) The functions $f(x) = \frac{x - 1}{2}$ and $g(x) = 2x + 1$ are inverses of each other since $f(g(x)) = x$ and $g(f(x)) = x$.

c) The functions $f(x) = \sqrt[3]{x + 1}$ and $g(x) = x^3 - 1$ are inverses of each other since $f(g(x)) = x$ and $g(f(x)) = x$.

d) The functions $f(x) = 5^x$ and $g(x) = \log_5 x$ are inverses of each other since $f(g(x)) = x$ and $g(f(x)) = x$.

21. a) $\{x \mid x > 0, x \in \mathrm{R}\}$ **b)** $f(g(x)) = \log(\sin x)$

c)



d) domain $\{x \mid 2n\pi < x < (2n + 1)\pi, n \in \mathrm{I}, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

22. $f(g(x)) = \dfrac{x + 2}{x + 3}, x \neq -3, -2, -1$

23. a) **i)** $y = \dfrac{1}{1 - x}, x \neq 1$ **ii)** $y = -\dfrac{x}{1 - x}, x \neq 1$

iii) $y = \dfrac{1}{x}, x \neq 0$ **iv)** $y = \dfrac{1}{x}, x \neq 0$

b) $f_2(f_3(x))$

C1 No. One is a composite function, $f(g(x))$, and the other is the product of functions, $(f \cdot g)(x)$. Examples may vary.

C2 a) Example: Since $f(1) = 5$ and $g(5) = 10$, $g(f(1)) = 10$.

b) Example: Since $f(3) = 7$ and $g(7) = 0$, $g(f(3)) = 0$.

C3 Yes, the functions are inverses of each other.

C4 Step 1: a) $f(x + h) = 2x + 2h + 3$

b) $\dfrac{f(x + h) - f(x)}{h} = 2$

Step 2: a) $f(x + h) = -3x - 3h - 5$

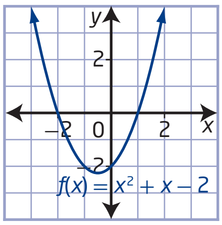
b) $\dfrac{f(x + h) - f(x)}{h} = -3$

Step 3: $\dfrac{f(x + h) - f(x)}{h} = \dfrac{3}{4}$; Each value is the slope of the linear function.

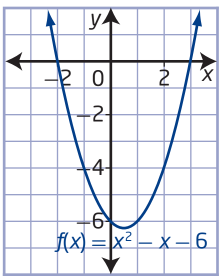
Chapter 10 Review, pages 510 to 511

1. a) 26 **b)** 1 **c)** −5 **d)** 13

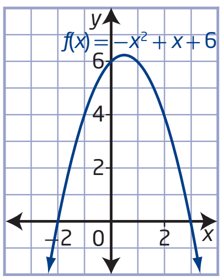
2. a) **i)** $f(x) = x^2 + x - 2$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$



ii) $f(x) = x^2 - x - 6$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

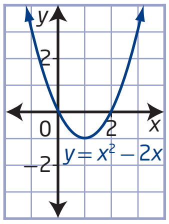


iii) $f(x) = -x^2 + x + 6$ domain $\{x \mid x \in \mathrm{R}\}$ range $\{y \mid y \in \mathrm{R}\}$



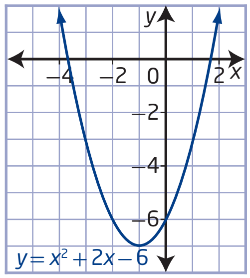
b) **i)** 4 **ii)** −4 **iii)** 4

3. a) $y = x^2 - 2x$



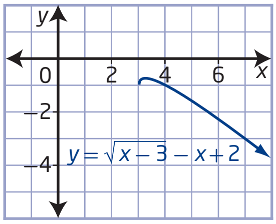
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -1, y \in \mathrm{R}\}$

$y = x^2 + 2x - 6$



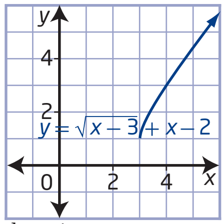
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -7, y \in \mathrm{R}\}$

b) $y = \sqrt{x - 3} - x + 2$



domain $\{x \mid x \geq 3, x \in \mathrm{R}\}$, range $\{y \mid y \leq -0.75, y \in \mathrm{R}\}$

$y = \sqrt{x - 3} + x - 2$



domain $\{x \mid x \geq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$

4. a) $y = \dfrac{1}{x - 1} + \sqrt{x}$; domain $\{x \mid x \geq 0, x \neq 1, x \in \mathrm{R}\}$, range $\{y \mid y \leq -0.7886 \text{ or } y \geq 2.2287, y \in \mathrm{R}\}$

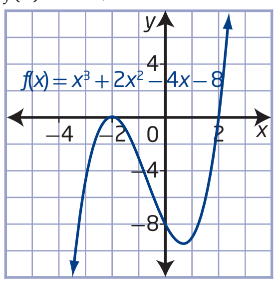
b) $y = \dfrac{1}{x - 1} - \sqrt{x}$; domain $\{x \mid x \geq 0, x \neq 1, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

5. a) $P = 2x - 6$

b) The net change will continue to increase, going from a negative value to a positive value in year 3.

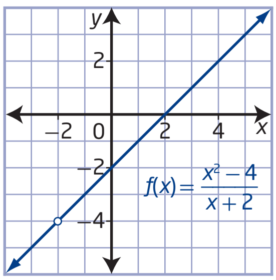
c) after year 3

6. a) $f(x) = x^3 + 2x^2 - 4x - 8$



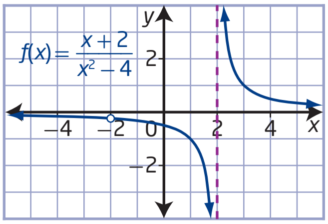
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$, no asymptotes

b) $f(x) = x - 2, x \neq -2$



domain $\{x \mid x \neq -2, x \in \mathrm{R}\}$, range $\{y \mid y \neq -4, y \in \mathrm{R}\}$, no asymptotes

c) $f(x) = \frac{1}{x - 2}, x \neq -2, 2$



domain $\{x \mid x \neq -2, 2, x \in \mathrm{R}\}$,
range $\left\{y \mid y \neq -\frac{1}{4}, 0, y \in \mathrm{R}\right\}$, horizontal asymptote $y = 0$, vertical asymptote $x = 2$

7. a) 0 **b)** does not exist

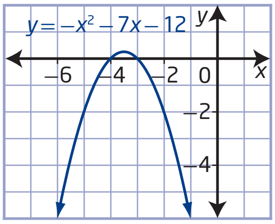
c) does not exist

8. a) $f(x) = \frac{1}{x^3 + 4x^2 - 16x - 64}, x \neq \pm 4$
domain $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$,
range $\{y \mid y \neq 0, y \in \mathrm{R}\}$

b) $f(x) = x - 4, x \neq \pm 4$
domain $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$,
range $\{y \mid y \neq -8, 0, y \in \mathrm{R}\}$

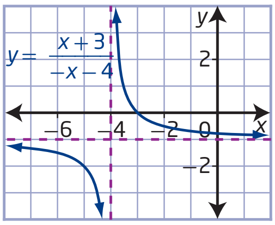
c) $f(x) = \frac{1}{x - 4}, x \neq \pm 4$
domain $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$,
range $\left\{y \mid y \neq -\frac{1}{8}, 0, y \in \mathrm{R}\right\}$

9. a) $y = -x^2 - 7x - 12$



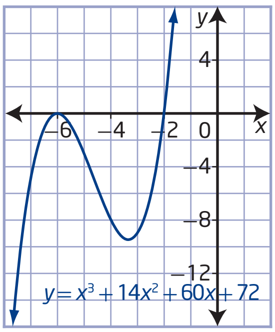
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \leq 0.25, y \in \mathrm{R}\}$

$y = \frac{x + 3}{-x - 4}, x \neq -4$



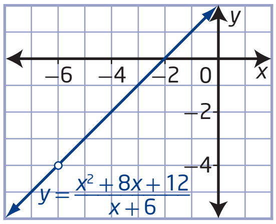
domain $\{x \mid x \neq -4, x \in \mathrm{R}\}$, range $\{y \mid y \neq -1, y \in \mathrm{R}\}$

b) $y = x^3 + 14x^2 + 60x + 72$



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

$y = x + 2, x \neq -6$



domain $\{x \mid x \neq -6, x \in \mathrm{R}\}$, range $\{y \mid y \neq -4, y \in \mathrm{R}\}$

10. a) 1 **b)** 5

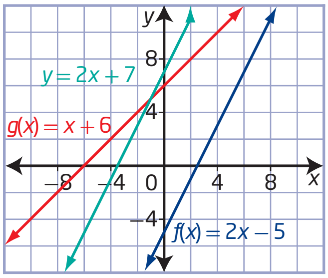
11. a) $y = \frac{32}{x^2}; x \neq 0$ **b)** $y = \frac{2}{x^2}; x \neq 0$

c) 0.5

12. a) $y = -\frac{2}{\sqrt{x}}, x > 0$

b) domain $\{x \mid x > 0, x \in \mathrm{R}\}$, range $\{y \mid y < 0, y \in \mathrm{R}\}$

13.



14. $T = 0.05t + 20$

15. a) $d(x) = 0.75x$; $c(x) = x - 10$

b) $c(d(x)) = 0.75x - 10$; this represents using the coupon after the discount.

c) $d(c(x)) = 0.75x - 7.5$; this represents applying the coupon before the discount.

d) Using the coupon after the discount results in a lower price of $290.

Chapter 10 Practice Test, pages 512 to 513

1. B **2.** D **3.** A **4.** C **5.** A

6. a) $h(x) = \sin x + 2x^2$ **b)** $h(x) = \sin x - 2x^2$

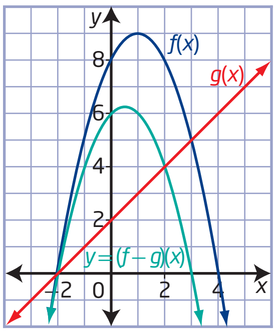
c) $h(x) = 2x^2 \sin x$ **d)** $h(x) = \frac{\sin x}{2x^2}, x \neq 0$

7.

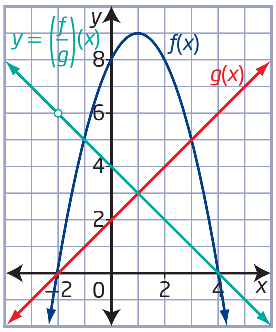
	$g(x)$	$f(x)$	$(f + g)(x)$	$(f \circ g)(x)$
a)	$x - 8$	$\sqrt{x}$	$\sqrt{x} + x - 8$	$\sqrt{x - 8}$
b)	$x + 3$	$4x$	$5x + 3$	$4x + 12$
c)	x^2	$\sqrt{x - 4}$	$\sqrt{x - 4} + x^2$	$\sqrt{x^2 - 4}$
d)	$\frac{1}{x}$	$\frac{1}{x}$	$\frac{2}{x}$	x

8. $y = \frac{1}{2x^2 + 5x + 3}, x \neq -\frac{3}{2}, -1$
domain $\left\{x \mid x \neq -\frac{3}{2}, -1, x \in \mathrm{R}\right\}$

9. a)



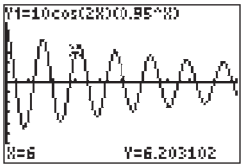
b)



10. a) $y = |6 - x|$; domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$
b) $y = 4^x + 1$; domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$
c) $y = x^2$; domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

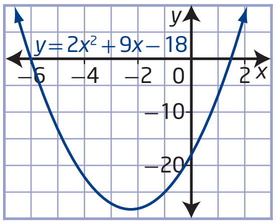
11. a) $r(x) = x - 200$; $t(x) = 0.72x$
b) $t(r(x)) = 0.72x - 144$; this represents applying federal taxes after deducting from her paycheque for her retirement.
c) \$1800 **d)** \$1744
e) The order changes the final amount. If you tax the income after subtracting \$200, you are left with more money.

12. a)

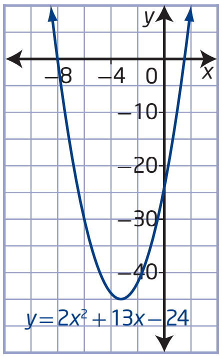


b) The function $f(t) = 10 \cos 2t$ is responsible for the periodic motion. The function $g(t) = 0.95^t$ is responsible for the exponential decay of the amplitude.

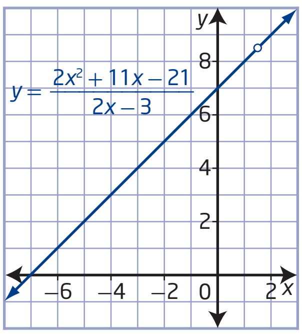
13. a) $y = 2x^2 + 9x - 18$



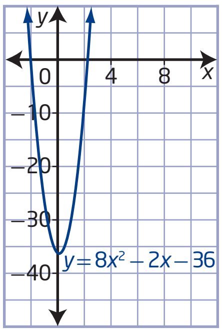
b) $y = 2x^2 + 13x - 24$



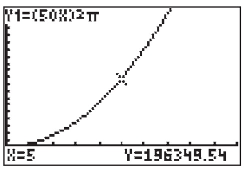
c) $y = x + 7, x \neq \frac{3}{2}$



d) $y = 8x^2 - 2x - 36$



14. a) $A(t) = 2500\pi t^2$
b)



c) approximately 196 350 cm^2
d) Example: No. In 30 s, the radius would be 1500 cm. Most likely the circular ripples would no longer be visible on the surface of the water due to turbulence.

Chapter 11 Permutations, Combinations, and the Binomial Theorem

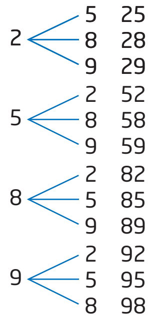
11.1 Permutations, pages 524 to 527

1. a)

Position 1	Position 2	Position 3
Jo	Amy	Mike
Jo	Mike	Amy
Amy	Jo	Mike
Amy	Mike	Jo
Mike	Jo	Amy
Mike	Amy	Jo

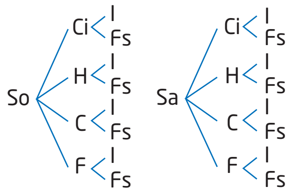
6 different arrangements

b)



12 different two-digit numbers

c) Use abbreviations: Soup (So), Salad (Sa), Chili (Ci), Hamburger (H), Chicken (C), Fish (F), Ice Cream (I) and Fruit Salad (Fs).



16 different meals

2. a) 56 **b)** 2520 **c)** 720 **d)** 4

3. Left Side $= 4! + 3!$ $= 4(3!) + 3!$ $= 5(3!)$ Right Side $= (4 + 3)!$ $= 7!$
Left Side $\neq$ Right Side

4. a) $9! = (9)(8)(7)(6)(5)(4)(3)(2)(1)$
$= 362\ 880$
b) $\frac{9!}{5!4!} = \frac{(9)(8)(7)(6)(5!)}{(5!)(4)(3)(2)(1)}$
$= 126$
c) $(5!)(3!) = (5)(4)(3)(2)(1)(3)(2)(1)$
$= 720$
d) $6(4!) = 6(4)(3)(2)(1)$
$= 144$
e) $\frac{102!}{100!2!} = \frac{(102)(101)(100!)}{100!(2)(1)}$
$= (51)(101)$
$= 5151$
f) $7! - 5! = (7)(6)(5!) - 5!$
$= 41(5!)$
$= 4920$

5. a) 360 **b)** 420 **c)** 138 600
d) 20 **e)** 20 **f)** 10 080

6. 24 ways

7. a) $n = 6$ **b)** $n = 11$ **c)** $r = 2$
d) $n = 6$

8. a) 6 **b)** 35 **c)** 10

9. a) Case 1: first digit is 3 or 5; Case 2: first digit is 2 or 4
b) Case 1: first letter is a B; Case 2: first letter is an E

10. a) 48 **b)** 240 **c)** 48

11. a) 5040 **b)** 2520 **c)** 1440 **d)** 576

12. 720 total arrangements; 288 arrangements begin and end with a consonant.

13. No. The organization has 25 300 members but there are only 18 000 arrangements that begin with a letter other than O followed by three different digits.

14. 20

15. $266\frac{2}{3}$ h

16. a) 5040 b) 1440 c) 3600

17. a) 3360 b) 360

18. a) AABBS b) Example: TEETH

19. 3645 integers contain no 7s

20. a) 17 576 000
 b) Example: Yes, Canada will eventually exceed 17.5 million postal communities.

21. a) 10^{14}
 b) Yes, $10^{14} = 100\ 000\ 000\ 000\ 000$, which is 100 million million.

22. a) $r = 3$ b) $r = 7$ c) $n = 4$ d) $n = 42$

23. ${}_nP_n = \dfrac{n!}{(n-n)!} = \dfrac{n!}{0!}$ and ${}_nP_n = n!$, so $0! = 1$.

24. The number of items to be arranged is less than the number of items in each set of arrangements.

25. 63 26. 84 27. 737 28. 15 29. 10

30. Example: Use the numbers 1 to 9 to represent the different students.

Day 1	Day 2	Day 3	Day 4
1 2 3	1 4 7	1 4 9	1 6 8
4 5 6	2 5 8	2 6 7	2 4 9
7 8 9	3 6 9	3 5 8	3 5 7

31. 24 zeros; Determine how many factors of 5 there are in 100!. Each multiple of 5 has one factor of 5 except 25, 50, 75, and 100, which have two factors of 5. So, there are 24 factors of 5 in 100!. There are more than enough factors of 2 to match up with the 5s to make factors of 10, so there are 24 zeros.

32. a) EDACB or BCADE b) 2
 c) None. Since F only knows A, then F must stand next to A. However, in both arrangements from part a), A must stand between C and D, but F does not know either C or D and therefore cannot stand next to either of them. Therefore, no possible arrangement satisfies the conditions.

C1 a) ${}_aP_b = \dfrac{a!}{(a-b)!}$ is the formula for calculating the number of ways that b objects can be selected from a group of a objects, if order is important; for example, if you have a group of 20 students and you want to choose a team of 3 arranged from tallest to shortest.
 b) $b \le a$

C2 By the fundamental counting principle, if the n objects are distinct, they can be arranged in $n!$ ways. However, if a of the objects are the same and b of the remaining objects are the same, then the number of different arrangements is reduced to $\dfrac{n!}{a!b!}$ to eliminate duplicates.

C3 a) $\dfrac{(n+2)(n+1)n}{4}$ b) $\dfrac{7+20r}{r(r+1)}$

C5 a) 362 880 b) 5.559 763… c) 6.559 763
 d) Example: The answer to part c) is 1 more than the answer to part b). This is because $10! = 10(9!)$ and $\log 10! = \log 10 + \log 9! = 1 + \log 9!$.

11.2 Combinations, pages 534 to 536

1. a) Combination, because the order that you shake hands is not important.
 b) Permutation, because the order of digits is important.
 c) Combination, since the order that the cars are purchased is not important.
 d) Combination, because the order that players are selected to ride in the van is not important.

2. ${}_5P_3$ is a permutation representing the number of ways of arranging 3 objects taken from a group of 5 objects. ${}_5C_3$ is a combination representing the number of ways of choosing any 3 objects from a group of 5 objects. ${}_5P_3 = 60$ and ${}_5C_3 = 10$.

3. a) ${}_6P_4 = 360$ b) ${}_7C_3 = 35$
 c) ${}_5C_2 = 10$ d) ${}_{10}C_7 = 120$

4. a) 210 b) 5040

5. a) AB, AC, AD, BC, BD, CD
 b) AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, DC
 c) The number of permutations is 2! times the number of combinations.

6. a) $n = 10$ b) $n = 7$ c) $n = 4$ d) $n = 5$

7. a) Case 1: one-digit numbers, Case 2: two-digit numbers, Case 3: three-digit numbers
 b) Cases of grouping the 4 members of the 5-member team from either grade: Case 1: four grade 12s, Case 2: three grade 12s and one grade 11, Case 3: two grade 12s and two grade 11s, Case 4: one grade 12 and three grade 11s, Case 5: four grade 11s

8. Left Side $= {}_{11}C_3$ Right Side $= {}_{11}C_8$

$$= \frac{11!}{(11-3)!3!} \qquad = \frac{11!}{(11-8)!8!}$$
$$= \frac{11!}{8!3!} \qquad = \frac{11!}{3!8!}$$

Left Side = Right Side

9. a) ${}_5C_5 = 1$
 b) ${}_5C_0 = 1$; there is only one way to choose 5 objects from a group of 5 objects and only one way to choose 0 objects from a group of 5 objects.

10. a) 4 b) 10

11. a) 15 b) 22

12. Left Side

$$= {}_nC_{r-1} + {}_nC_r$$
$$= \frac{n!}{(n-(r-1))!(r-1)!} + \frac{n!}{(n-r)!r!}$$
$$= \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r)!r!}$$
$$= \frac{[n!(n-r)!r!] + [n!(n-r+1)!(r-1)!]}{(n-r+1)!(r-1)!(n-r)!r!}$$
$$= \frac{n!(n-r)!r(r-1)! + n!(n-r+1)(n-r)!(r-1)!}{(n-r+1)!(r-1)!(n-r)!r!}$$
$$= \frac{n!(n-r)!(r-1)![r + (n-r+1)]}{(n-r+1)!(r-1)!(n-r)!r!}$$
$$= \frac{n!\overset{1}{\cancel{(n-r)!}}\overset{1}{\cancel{(r-1)!}}(n+1)}{(n-r+1)!\underset{1}{\cancel{(r-1)!}}\underset{1}{\cancel{(n-r)!}}r!}$$
$$= \frac{n!(n+1)}{(n-r+1)!r!}$$
$$= \frac{(n+1)!}{(n-r+1)!r!}$$

Right Side $= {}_{n+1}C_r$
$$= \frac{(n+1)!}{(n+1-r)!r!}$$

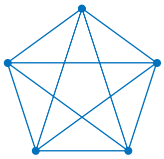
Left Side = Right Side

13. 20 different burgers; this is a combination because the order the ingredients is put on the burger is not important.

14. a) 210

b) combination, because the order of toppings on a pizza is not important

15. a) Method 1: Use a diagram.
Method 2: Use combinations.
${}_5C_2 = 10$, the same as the number of combinations of 5 people shaking hands.



b) 10

c) The number of triangles is given by ${}_{10}C_3 = \dfrac{10!}{(10-3)!3!} = \dfrac{10!}{7!3!}$. The number of lines is given by ${}_{10}C_2 = \dfrac{10!}{(10-2)!2!} = \dfrac{10!}{8!2!}$.
The number of triangles is determined by the number of selections with choosing 3 points from 10 non-collinear points, whereas the number of lines is determined by the number of selections with choosing 2 points from the 10 non-collinear points.

16. Left Side $= {}_nC_r$

$$= \frac{n!}{(n-r)!r!}$$

Right Side $= {}_nC_{n-r}$

$$= \frac{n!}{(n-(n-r))!(n-r)!}$$
$$= \frac{n!}{(n-n+r)!(n-r)!}$$
$$= \frac{n!}{r!(n-r)!}$$

Left Side = Right Side

17. a) 125 970 **b)** 44 352 **c)** 1945

18. a) 2 598 960 **b)** 211 926 **c)** 388 700

19. a) 525 **b)** 576

20. a) $\dfrac{40!}{20!20!}$ **b)** 116 280

21. a) $\dfrac{52!}{39!13!} \times \dfrac{39!}{26!13!} \times \dfrac{26!}{13!13!} \times \dfrac{13!}{0!13!}$

b) $\dfrac{52!}{13!13!13!13!} = \dfrac{52!}{(13!)^4}$ **c)** $5.364\ldots \times 10^{28}$

22. 90

23. a) 36 **b)** 1296

24. a) ${}_5C_2 = 10$, $10 \div 3 = 3$ Remainder 1. ${}_{15}C_6 = 5005$, and $5005 \div 3 = 1668$ Remainder 1.

b) yes, remainder 3 **c)** 7; 0, 1, 2, 3, 4, 5, 6

d) Example: First, I would try a few more cases to try to find a counterexample. Since the statement seems to be true, I would write a computer program to test many cases in an organized way.

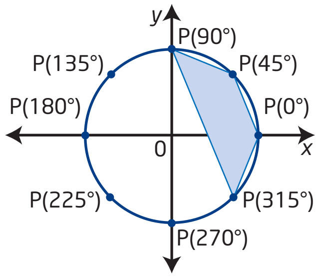
C1 No. The order of the numbers matters, so a combination lock would be better called a permutations lock.

C2 a) ${}_aC_b = \dfrac{a!}{(a-b)!b!}$ is the formula for calculating the number of ways that b objects can be selected from a group of a objects, if order is not important; for example, if you have a group of 20 students and you want to choose a team of any 3 people.

b) $a \geq b$ **c)** $b \geq 0$

C3 Example: Assuming that the rooms are the same and so any patient can be assigned to any of the six rooms, this is a combinations situation. Beth is correct.

C4 Step 1: Example:



Step 2: Number of each type of quadrilateral:
Squares: 2
Rectangles: 4
Parallelograms: 0
Isosceles trapezoids: 24

Step 3: Example: In the case drawn in Step 1, because of the symmetry of the given points on the unit circle, many of the possible quadrilaterals are the same. In general, there will be ${}_8C_4$ or 70 possible quadrilaterals.

11.3 The Binomial Theorem, pages 542 to 545

1. a) 1 4 6 4 1 **b)** 1 8 28 56 70 56 28 8 1

c) 1 11 55 165 330 462 462 330 165 55 11 1

2. a) ${}_2C_0$ ${}_2C_1$ ${}_2C_2$ **b)** ${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$

c) ${}_7C_0$ ${}_7C_1$ ${}_7C_2$ ${}_7C_3$ ${}_7C_4$ ${}_7C_5$ ${}_7C_6$ ${}_7C_7$

3. a) $\dfrac{3!}{2!1!}$ **b)** $\dfrac{6!}{3!3!}$ **c)** $\dfrac{1!}{0!1!}$

4. a) 5 **b)** 8 **c)** $q + 1$

5. a) $1x^2 + 2xy + 1y^2$ **b)** $1a^3 + 3a^2 + 3a + 1$

c) $1 - 4p + 6p^2 - 4p^3 + 1p^4$

6. a) $1a^3 + 9a^2b + 27ab^2 + 27b^3$

b) $243a^5 - 810a^4b + 1080a^3b^2 - 720a^2b^2 + 240ab^4 - 32b^5$

c) $16x^4 - 160x^3 + 600x^2 - 1000x + 625$

7. a) $126a^4b^5$ **b)** $-540x^3y^3$ **c)** $192\ 192t^6$

d) $96x^2y^2$ **e)** $3072w^2$

8. All outside numbers of Pascal's triangle are 1's; the middle values are determined by adding the two numbers to the left and right in the row above.

9. a) 1, 2, 4, 8, 16

b) 2^8 or 256

c) 2^{n-1}, where n is the row number

10. a) The sum of the numbers on the handle equals the number on the blade of each hockey stick.

b) No; the hockey stick handle must begin with 1 from the outside of the triangle and move diagonally down the triangle with each value being in a different row. The number of the blade must be diagonally below the last number on the handle of the hockey stick.

11. a) 13 **b)** $220x^9y^3$ **c)** $r = 6$, ${}_{12}C_6 = 924$

12. a) $(x + y)^4$ **b)** $(1 - y)^5$

13. a) No. While $11^0 = 1$, $11^1 = 11$, $11^2 = 121$, $11^3 = 1331$, and $11^4 = 14\ 641$, this pattern only works for the first five rows of Pascal's triangle.

b) m represents the row number minus 1, $m \leq 4$.

14. a) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$, $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$; the signs for the second and fourth terms are negative in the expansion of $(x - y)^3$

b) $(x + y)^3 + (x - y)^3$
$= x^3 + 3x^2y + 3xy^2 + y^3 + x^3 - 3x^2y + 3xy^2 - y^3$
$= 2x^3 + 6xy^2$
$= 2x(x^2 + 3y^2)$

c) $2y(3x^2 + y^2)$; the expansion of $(x + y)^3 - (x - y)^3$ has coefficients for x^2 and y^2 that are reversed from the expansion of $(x + y)^3 + (x - y)^3$, as well as the common factors $2x$ and $2y$ being reversed.

15. a) Case 1: no one attends, case 2: one person attends, case 3: two people attend, case 4: three people attend, case 5: four people attend, case 6: all five people attend

b) 32 or 2^5

c) The answer is the sum of the terms of the sixth row of Pascal's triangle.

16. a)

```
      H < H  HHH
   H <    T  HHT
      T < H  HTH
          T  HTT
      H < H  THH
   T <    T  THT
      T < H  TTH
          T  TTT
```

b) HHH + HHT + HTH + HTT + THH + THT + TTH + TTT
$= H^3 + 3H^2T + 3HT^2 + T^3$

c) H^3 represents the first term of the expansion of $(H + T)^3$ and $3H^2T$ represents the second term of the expansion of $(H + T)^3$.

17. a) $\frac{a^3}{b^3} + 6\left(\frac{a^2}{b^2}\right) + 12\left(\frac{a}{b}\right) + 8$ or $\frac{a^3}{b^3} + \frac{6a^2}{b^2} + \frac{12a}{b} + 8$

b) $\frac{a^4}{b^4} - 4\left(\frac{a^4}{b^3}\right) + 6\left(\frac{a^4}{b^2}\right) - 4\left(\frac{a^4}{b}\right) + a^4$
$= a^4\left(\frac{1}{b^4} - \frac{4}{b^3} + \frac{6}{b^2} - \frac{4}{b} + 1\right)$

c) $1 - 3x + \frac{15}{4}x^2 - \frac{5}{2}x^3 + \frac{15}{16}x^4 - \frac{3}{16}x^5 + \frac{1}{64}x^6$

d) $16x^8 - 32x^5 + 24x^2 - 8x^{-1} + x^{-4}$

18. a) $5670a^4b^{12}$ **b)** the fourth term; it is $-120x^{11}$

19. a) 126 720 **b)** the fifth term; its value is 495

20. $m = 3y$

21. Examples:
Step 1: The numerators start with the second value, 4, and decrease by ones, while the denominators start at 1 and increase by ones to 4.
For the sixth row:
$1 \times 5 = 5$, $5 \times \frac{4}{2} = 10$, $10 \times \frac{3}{3} = 10$, $10 \times \frac{2}{4} = 5$, $5 \times \frac{1}{5} = 1$.
Step 2: The second element in the row is equal to the row number minus 1.
Step 3: The first 2 terms in the 21st row are 1 and 20.
$\times \frac{20}{1}$; $\times \frac{19}{2}$, $\times \frac{18}{3}$, and so on to $\times \frac{3}{18}$, $\times \frac{2}{19}$, $\times \frac{1}{20}$

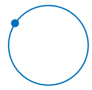
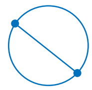
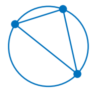
22. a) Each entry is the sum of the two values in the row below and slightly to the left and the right.

b) $\frac{1}{6}$ $\frac{1}{30}$ $\frac{1}{60}$ $\frac{1}{60}$ $\frac{1}{30}$ $\frac{1}{6}$
$\frac{1}{7}$ $\frac{1}{42}$ $\frac{1}{105}$ $\frac{1}{140}$ $\frac{1}{105}$ $\frac{1}{42}$ $\frac{1}{7}$

c) Examples: Outside values are the reciprocal of the row number. The product of two consecutive outside row values gives the value of the second term in the lower row.

23. Consider $a + b = x$ and $c = y$, and substitute in $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.
$(a + b + c)^3$
$= (a + b)^3 + 3(a + b)^2c + 3(a + b)c^2 + c^3$
$= a^3 + 3a^2b + 3ab^2 + b^3 + 3(a^2 + 2ab + b^2)c + 3ac^2 + 3bc^2 + c^3$
$= a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3$

24. a)

Diagram	Points	Line Segments	Triangles	Quadrilaterals	Pentagons	Hexagons
	1					
	2	1				
	3	3	1			
	4	6	4	1		
	5	10	10	5	1	
	6	15	20	15	6	1

b) The numbers are values from row 1 to row 6 of Pascal's triangle with the exception of the first term.

c) The numbers will be values from the 8th row of Pascal's triangle with the exception of the first term: 8 28 56 70 56 28 8 1.

25. a) 2.7083…

b) The value of e becomes more precise for the 7th and 8th terms. The more terms used, the more accurate the approximation.

c) Example: 2.718 281 828

d) $15! = \left(\frac{15}{e}\right)^{15}\sqrt{2\pi(15)} \approx 1.300 \times 10^{12}$;
on a calculator $15! \approx 1.3077 \times 10^{12}$

e) Using the formula from part d),
$50! = \left(\frac{50}{e}\right)^{50}\sqrt{2\pi(50)}$
$\approx 3.036\ 344\ 594 \times 10^{64}$;
using the formula from part e),
$50! = \left(\frac{50}{e}\right)^{50}\sqrt{2\pi(50)}\left(1 + \frac{1}{12(50)}\right)$
$\approx 3.041\ 405\ 168 \times 10^{64}$; using a calculator $50! = 3.041\ 409\ 32 \times 10^{64}$, so the formula in part e) seems to give a more accurate approximation.

C1 The coefficients of the terms in the expansion of $(x + y)^n$ are the same as the numbers in row $n + 1$ of Pascal's triangle. Examples: $(x + y)^2 = x^2 + 2xy + y^2$ and row 3 of Pascal's triangle is 1 2 1; $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ and row 4 of Pascal's triangle is 1 3 3 1.

C2 Examples:

a) Permutation: In how many different ways can four different chocolate bars be given to two people? Combination: Steve has two Canadian quarters and two U.S. quarters in his pocket. In how many different ways can he draw out two coins? Binomial expansion: What is the coefficient of the middle term in the expansion of $(a + b)^4$?

b) All three problems have the same answer, 6, but they answer different questions.

C3 Examples:

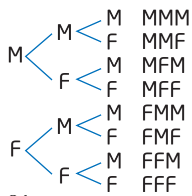
a) For small values of n, it is easier to use Pascal's triangle, but for large values of n it is easier to use combinations to determine the coefficients in the expansion of $(a + b)^n$.

b) If you have a large version of Pascal's triangle available, then that will immediately give a correct coefficient. If you have to work from scratch, both methods can be error prone.

C4 Answers will vary.

Chapter 11 Review, pages 546 to 547

1. a)



b) 3

2. a) 81 **b)** 32

3. a) 24 **b)** 360 **c)** 60 **d)** $\dfrac{12!}{2!3!2!}$

4. a) 48 **b)** 24 **c)** 72

5. a) 5040 **b)** 288 **c)** 144

6. a) 1 160 016 **b)** $8.513\,718\,8 \times 10^{11}$

c) about 270 000 years

7. a) $\dfrac{n + 1}{n - 1}$ **b)** $\dfrac{x^2 + x + 1}{x}$

8. a) 210 **b)** 63

9. a) 120 **b)** 5040 **c)** 200 **d)** 163 800

10. a) 15

b) amounts all in cents: 1, 5, 10, 25, 6, 11, 26, 15, 30, 35, 16, 31, 36, 40, 41

11. a) $n = 8$, ${}_8C_2 = 28$

b) $n = 26$, ${}_{26}C_3 = 2600$ and $4({}_{26}P_2) = 4(650) = 2600$

12. 2520

13. a) Example: Permutation: How many arrangements of the letters AAABB are possible? Combination: How many ways can you choose 3 students from a group of 5?

b) Yes, ${}_5C_2 = \dfrac{5!}{(5 - 2)!2!} = \dfrac{5!}{3!2!}$ and ${}_5C_3 = \dfrac{5!}{(5 - 3)!3!} = \dfrac{5!}{2!3!}$.

14. a) 1 3 3 1

b) 1 9 36 84 126 126 84 36 9 1

15. Examples: Multiplication: expand, collect like terms, and write the answer in descending order of the exponent of x.

$$(x + y)^3 = (x + y)(x + y)(x + y)$$
$$= x^3 + 3x^2y + 3xy^2 + y^3$$

Pascal's triangle: Coefficients are the terms from row $n + 1$ of Pascal's triangle. For $(x + y)^3$, row 4 is 1 3 3 1.

Combination: coefficients correspond to the combinations as shown:

$$(x + y)^3 = {}_3C_0x^3y^0 + {}_3C_1x^2y^1 + {}_3C_2x^1y^2 + {}_3C_3x^0y^3$$

16. a) $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

b) $x^3 - 9x^2 + 27x - 27$

c) $16x^8 - 32x^4 + 24 - \dfrac{8}{x^4} + \dfrac{1}{x^8}$

17. a) $36a^7b^2$ **b)** $-192xy^5$ **c)** $-160x^3$

18. a)

1	5	15	35	70	126	B
1	4	10	20	35	56	
1	3	6	10	15	21	
1	2	3	4	5	6	
1 A	1	1	1	1	1	

b) Pascal's triangle values are shown with the top of the triangle at point A and the rows appearing up and right of point A.

c) 126

d) There are 4 identical moves up and 5 identical moves right, so the number of possible pathways is $\dfrac{9!}{4!5!} = 126$.

19. a) 45 moves

b) 2 counters: 1 move; 3 counters: $1 + 2 = 3$ moves; 4 counters: $1 + 2 + 3 = 6$ moves; and so on up to 12 counters: $1 + 2 + 3 + \cdots + 10 + 11 = 66$ moves

c) 300 moves

Chapter 11 Practice Test, page 548

1. C **2.** D **3.** C **4.** B **5.** A **6.** C

7. a) 180

b) AACBDB, ABCADB, ABCBDA, BACBDA, BACADB, BBCADA

8. No, n must be a whole number, so n cannot equal -8.

9. a) 10 **b)** $\dfrac{5!}{2!3!}\left(\dfrac{4!}{2!2!}\right) = 60$

10. 69

11. Permutations determine the number of arrangements of n items chosen r at a time, when order is important. For example, the number of arrangements of 5 people chosen 2 at a time to ride on a motorcycle is ${}_5P_2 = 20$. A combination determines the number of different selections of n objects chosen r at a time when order is not important. For example, the number of selections of 5 objects chosen 2 at a time, when order is not important, is ${}_5C_2 = 10$.

12. $672x^9$

13. a) 420 **b)** 120

14. a) $n = 6$ **b)** $n = 9$

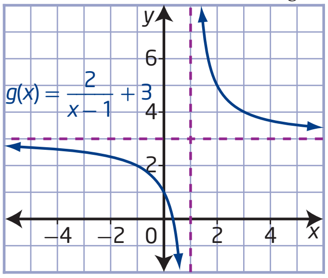
15. $y^5 - 10y^2 + 40y^{-1} - 80y^{-4} + 80y^{-7} - 32y^{-10}$

16. a) 24 **b)** 36 **c)** 18

Cumulative Review, Chapters 9–11, pages 550 to 551

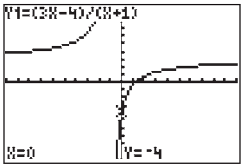
1. a) a vertical stretch by a factor of 2 about the x-axis and a translation of 1 unit right and 3 units up

b)



c) domain $\{x \mid x \neq 1, x \in \mathrm{R}\}$, range $\{y \mid y \neq 3, y \in \mathrm{R}\}$, x-intercept $\frac{1}{3}$, y-intercept 1, horizontal asymptote $y = 3$, vertical asymptote $x = 1$

2. a)



b) domain $\{x \mid x \neq -1, x \in \mathrm{R}\}$, range $\{y \mid y \neq 3, y \in \mathrm{R}\}$, x-intercept $\frac{4}{3}$, y-intercept -4, horizontal asymptote $y = 3$, vertical asymptote $x = -1$

3. a) The graph of $y = \frac{x^2 - 3x}{x^2 - 9}$ has a vertical asymptote at $x = -3$, a point of discontinuity at (3, 0.5), and an x-intercept of 0; C.

b) The graph of $y = \frac{x^2 - 1}{x + 1}$ has no vertical asymptote, a point of discontinuity at $(-1, -2)$, and an x-intercept of 1; A.

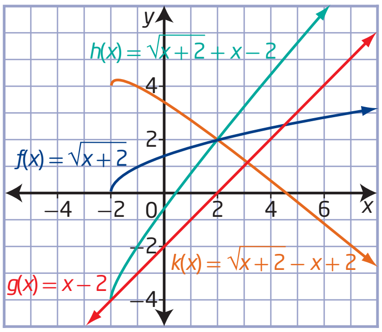
c) The graph of $y = \frac{x^2 + 4x + 3}{x^2 + 1}$ has no vertical asymptote, no point of discontinuity, and x-intercepts of -3 and -1; B.

4. a) 2 **b)** $-1, 9$ **c)** 0

5. a) $-0.71, 0.71$ **b)** $0.15, 5.52$

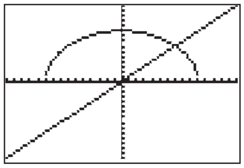
6. a) $h(x) = \sqrt{x + 2} + x - 2$, $k(x) = \sqrt{x + 2} - x + 2$

b)



c) $f(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$h(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \leq 4.25, y \in \mathrm{R}\}$

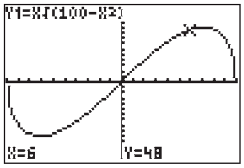
7. a)



$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid -10 \leq x \leq 10, x \in \mathrm{R}\}$, range $\{y \mid 0 \leq y \leq 10, y \in \mathrm{R}\}$

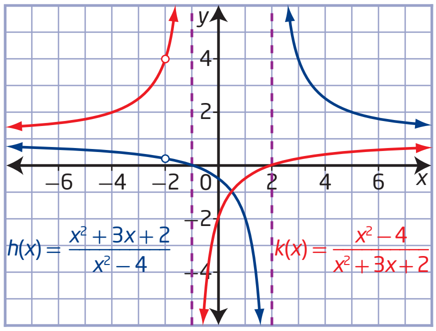
b) $h(x) = x\sqrt{100 - x^2}$

c)



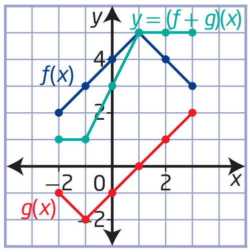
domain $\{x \mid -10 \leq x \leq 10, x \in \mathrm{R}\}$, range $\{y \mid -50 \leq y \leq 50, y \in \mathrm{R}\}$

8. a) $h(x) = \frac{x + 1}{x - 2}$, $x \neq -2, 2$; $k(x) = \frac{x - 2}{x + 1}$, $x \neq -2, -1$



b) The two functions have different domains but the same range; $h(x)$: domain $\{x \mid x \neq -2, 2, x \in \mathrm{R}\}$, range $\{y \mid y \neq 1, y \in \mathrm{R}\}$, $k(x)$: domain $\{x \mid x \neq -2, -1, x \in \mathrm{R}\}$, range $\{y \mid y \neq 1, y \in \mathrm{R}\}$

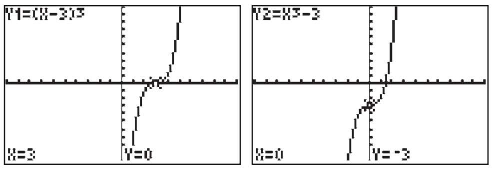
9.



10. a) -3 **b)** $\frac{1}{2}$

11. a) $(f \circ g)(x) = (x - 3)^3$ and $(g \circ f)(x) = x^3 - 3$

b)



c) The graph of $(f \circ g)(x) = (x - 3)^3$ is a translation of 3 units right of the graph of $f(x)$. The graph of $(g \circ f)(x) = x^3 - 3$ is a translation of 3 units down of the graph of $f(x)$.

12. a) $f(g(x)) = x$; domain $\{x \mid x \in \mathrm{R}\}$

b) $g(f(x)) = \csc x$; domain $\{x \mid x \neq \pi n, n \in \mathrm{I}, x \in \mathrm{R}\}$

c) $f(g(x)) = \frac{1}{x^2 - 1}$; domain $\{x \mid x \neq \pm 1, x \in \mathrm{R}\}$

13. 96 meals

14. 480 ways

15. 55

16. 525 ways

17. a) 103 680 **b)** 725 760

18. a) 3 **b)** 6 **c)** 5

19. Examples: Pascal's triangle:
$(x + y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$; the coefficients are values from the fifth row of Pascal's triangle.
$(x + y)^6 = 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$; the coefficients are values from the seventh row of Pascal's triangle.
Combinations: $(x + y)^4 = {}_4C_0x^4y^0 + {}_4C_1x^3y^1 + {}_4C_2x^2y^2 + {}_4C_3x^1y^3 + {}_4C_4x^0y^4$; the coefficients ${}_4C_0$, ${}_4C_1$, ${}_4C_2$, ${}_4C_3$, ${}_4C_4$ have the same values as in the fifth row of Pascal's triangle.
$(x + y)^6 = {}_6C_0x^6y^0 + {}_6C_1x^5y^1 + {}_6C_2x^4y^2 + {}_6C_3x^3y^3 + {}_6C_4x^2y^4 + {}_6C_5x^1y^5 + {}_6C_6x^0y^6$; the coefficients ${}_6C_0$, ${}_6C_1$, ${}_6C_2$, ${}_6C_3$, ${}_6C_4$, ${}_6C_5$, ${}_6C_6$ have the same values as the seventh row of Pascal's triangle.

20. a) $81x^4 - 540x^3 + 1350x^2 - 1500x + 625$
b) $\frac{1}{x^5} - \frac{10}{x^3} + \frac{40}{x} - 80x + 80x^3 - 32x^5$

21. a) 250 **b)** −56

22. a) ${}_{25}C_4$ **b)** 26 **c)** ${}_{25}C_3 = {}_{24}C_2 + {}_{24}C_3$

Unit 4 Test, pages 552 to 553

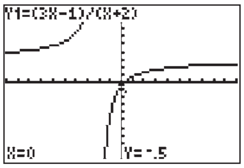
1. D **2.** B **3.** A **4.** B **5.** B **6.** D **7.** C

8. $\left(3, \frac{1}{7}\right)$

9. 0, 3.73, 0.27 **10.** $600x^2y^4$ **11.** −1

12. a) vertical stretch by a factor of 2 and translation of 1 unit left and 3 units down
b) $x = -1$ and $y = -3$
c) as x approaches −1, $|y|$ becomes very large

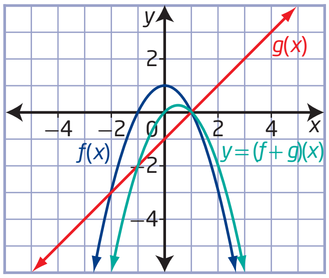
13. a)



b) domain $\{x \mid x \neq -2, x \in \mathrm{R}\}$, range $\{y \mid y \neq 3, y \in \mathrm{R}\}$, x-intercept $\frac{1}{3}$, y-intercept $-\frac{1}{2}$
c) $x = \frac{1}{3}$
d) The x-intercept of the graph of the function $y = \frac{3x - 1}{x + 2}$ is the root of the equation $0 = \frac{3x - 1}{x + 2}$.

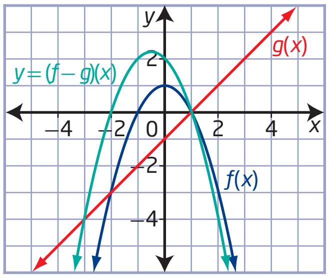
14. a) The graph of $f(x) = \frac{x - 4}{(x + 2)(x - 4)}$ has a vertical asymptote at $x = -2$, a point of discontinuity at $\left(4, \frac{1}{6}\right)$, y-intercept of 0.5, and no x-intercept.
b) The graph of $f(x) = \frac{(x + 3)(x - 2)}{(x + 3)(x - 1)}$ has a vertical asymptote at $x = 1$, a point of discontinuity at (−3, 1.25), y-intercept of 2, and an x-intercept of 2.
c) The graph of $f(x) = \frac{x(x - 5)}{(x - 3)(x + 1)}$ has vertical asymptotes at $x = -1$ and $x = 3$, no points of discontinuity, y-intercept of 0, and x-intercepts of 0 and 5.

15. a)



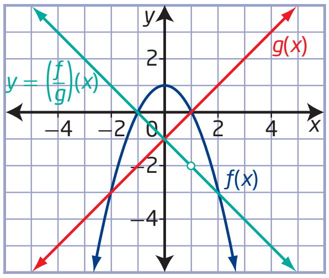
$y = x - x^2$
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \leq 0.25, y \in \mathrm{R}\}$

b)



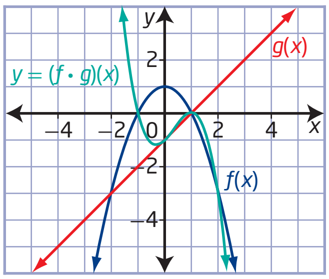
$y = 2 - x - x^2$
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \leq 2.25, y \in \mathrm{R}\}$

c)



$y = \frac{1 - x^2}{x - 1}$
domain $\{x \mid x \neq 1, x \in \mathrm{R}\}$, range $\{y \mid y \neq -2, y \in \mathrm{R}\}$

d)



$y = x - x^3 - 1 + x^2$
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

16. a) $h(x) = x - 3 + \sqrt{x - 1}$; $x \geq 1$
b) $h(x) = x - 3 - \sqrt{x - 1}$; $x \geq 1$
c) $h(x) = \frac{x - 3}{\sqrt{x - 1}}$; $x > 1$
d) $h(x) = (x - 3)\sqrt{x - 1}$; $x \geq 1$

17. a) 1 **b)** 1
c) $f(g(x)) = x^2 - 3$ **d)** $g(f(x)) = |x^2 - 3|$

18. a) $f(x) = 2^x$ and $g(x) = 3x + 2$
b) $f(x) = \sqrt{x}$ and $g(x) = \sin x + 2$

19. a) 21 **b)** 13 **c)** 10

20. a) 24 **b)** 232 more
c) There are fewer ways. Because the letter C is repeated, half of the arrangements will be repeats.

21. a) 60 **b)** 81

22. 4, −4