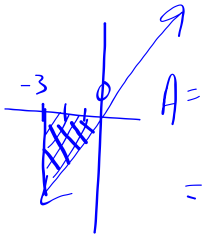
More Integrals & Area

What if part of the area is below the x-axis?

ex) Find the area bound by $y=2x$ and the x-axis from $x=-3$ to $x=0$.

Method 1 - Picture



$A = \frac{b \times h}{2}$

$= \frac{3 \times 6}{2}$?

$= 9$?

Method 2 - Anti-Deriv

$$\int_{-3}^{0} 2x \, dx$$

$$x^2 \Big|_{-3}^{0}$$

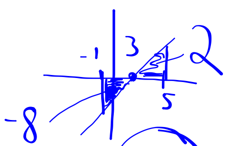
$$0^2 - (-3)^2 = -9$$

Method 3 - fnInt

fnInt(2x, x, -3, 0)

-9

ex) As above for $y=x-3$ from -1 to 5.



-6

$$\int_{-1}^{5} (x-3) \, dx$$

$$= \frac{1}{2}x^2 - 3x \Big|_{-1}^{5}$$

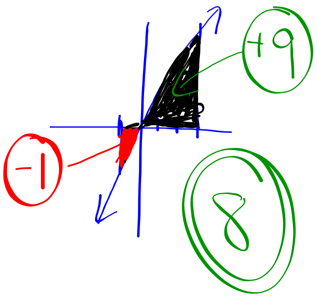
$$= \left(\frac{1}{2}(5)^2 - 3(5)\right) - \left(\frac{1}{2}(-1)^2 - 3(-1)\right)$$

$$12.5 - 15 - \frac{1}{2} - 3 = -6$$

fnInt...

-6

ex) As above for $y=2x$ from -1 to 3.



8

$$\int_{-1}^{3} 2x \, dx$$

8

fnInt

8

Rules for Integrals

1. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

2. $\int_a^a f(x)\,dx = 0$

3. $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$

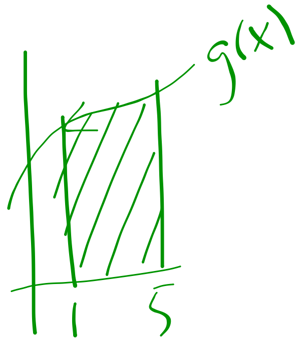
4. $\int_a^b \left(f(x) \pm g(x)\right)dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

ex) Given f(x) & g(x) are continuous, and:

$$\int_1^2 f(x)\,dx = -4 \qquad \int_1^5 f(x)\,dx = 6 \qquad \int_1^5 g(x)\,dx = 8$$

evaluate:



a) $\int_2^2 g(x)\,dx = 0$

b) $\int_5^1 g(x)\,dx = -8$

c) $\int_1^2 3f(x)\,dx = -12$

$$\int_1^5 = \int_1^2 + \int_2^5$$

$$6 = -4 + x$$

d) $\int_2^5 f(x)\,dx = 10$

e) $\int_1^5 [f(x) - g(x)]\,dx = -2$

f) $\int_1^5 [4f(x) - g(x)]\,dx = 16$

ex) Evaluate $\int_{-2}^{2}\sqrt{4-x^2}\,dx$

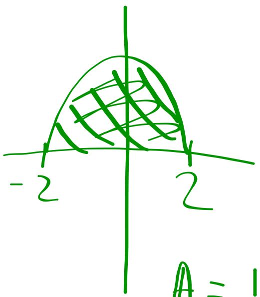
$\cancel{-\frac{1}{3}x}(4-x^2)^{3/2}$

$y^2 = \left(\sqrt{4-x^2}\right)^2$

$y^2 = 4-x^2$

$y^2+x^2=4$

$x^2+y^2=4 \qquad x^2+y^2=r^2$



$A=\frac{1}{2}\pi r^2$

$=\frac{1}{2}\pi(2)^2 = 2\pi = 6.28\ldots$

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