

More Integration

ex) Evaluate $\int_{-\pi}^0 \frac{\sin x}{\sqrt{5+2\cos x}} dx \rightarrow \int_3^7 \frac{-1/2}{\sqrt{u}} du$

let $u = 5 + 2\cos x$
 $du = -2\sin x dx$

$$= \int_3^7 \frac{1}{2} u^{-1/2} du$$

$$= -u^{1/2} \Big|_3^7$$

$$= -7^{1/2} + 3^{1/2}$$

$$= -\sqrt{7} + \sqrt{3}$$

$$\begin{aligned}
 \text{ex) } \int \frac{2\sqrt{x}}{\sqrt{5+x^{1.5}}} dx &\rightarrow \int \frac{4/3}{\sqrt{u}} du && \frac{1}{2}(\cdot) = \frac{4}{3} \\
 &&& \frac{1}{2} \quad \frac{1}{2} \\
 \text{let } u = 5+x^{1.5} & && \\
 du = 1.5x^{0.5} dx & && \cdot \quad ? = \frac{4}{3} \times 2 \\
 \frac{4}{3} (du = \frac{3}{2} x^{1/2} dx) & && \cdot \quad ? = \frac{8}{3} \\
 \frac{4}{3} du = \underline{2\sqrt{x} dx} & && \\
 & = \int \frac{4}{3} u^{-1/2} du && \\
 & = \frac{8}{3} u^{1/2} + C && \\
 & = \frac{8}{3} (5+x^{1.5})^{1/2} + C && \\
 & = \frac{8}{3} \sqrt{5+x^{3/2}} + C &&
 \end{aligned}$$

$$\text{ex) } \int_e^{e^2} \frac{2}{x \ln x^2} dx$$

$$\int_e^{e^2} \frac{2}{x(2 \ln x)} dx \rightarrow \int_1^2 \frac{1}{u} du$$

$$\begin{aligned} \text{let } u &= \ln x \\ du &= \frac{1}{x} dx \end{aligned}$$

$$\begin{aligned} &= \ln u \Big|_1^2 = \ln 2 - \ln 1 \\ &= \ln 2 \end{aligned}$$

$$\begin{aligned} \ln u \Big|_2^4 &= \ln 4 \\ &= \ln 2 \\ &= \ln 2 \end{aligned}$$

$$\text{ex) } \int \sqrt{x} (1+2x) dx$$

$$\int x^{1/2} (1+2x) dx$$

$$\int x^{1/2} + 2x^{3/2} dx \rightarrow \frac{2}{3} x^{3/2} + \frac{4}{5} x^{5/2} + C$$

$$\int x^{1/2} dx + \int 2x^{3/2} dx \rightarrow \frac{2}{3} x^{3/2} + C_1 + \frac{4}{5} x^{5/2} + C_2$$

$$\int (1+2x)^2 dx$$

$$\int (1+2x)(1+2x) dx$$