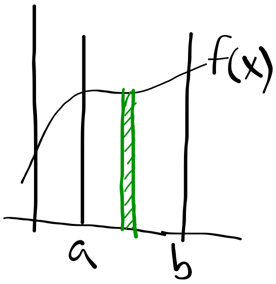
May 21, 2013

Using Integrals to Find Areas

Recall: To find areas using LRAM/RRAM we cut the area below the curve but above the x-axis into very thin rectangles, and then add up the area of all of the rectangles.

Area of each rectangle: $w \times l$

$(\Delta x) f(x)$



Total area is the sum of ∞ rectangles, with width $\Delta x \to 0$.

Notation: $\int_a^b f(x)\,dx$

upper bound

sum

lower bound

height of each rectangle

width of each rectangle

ex) To find area b/w $y = x^2$ & $y = 0$ from $x=0$ to $x=3$

$$A = \int_0^3 x^2\,dx = \frac{1}{3}x^3 \Big|_0^3 = \frac{1}{3}(3)^3 - \frac{1}{3}(0)^3$$

$\boxed{\text{MATH}}$ 9:fnInt(x^2,x,0,3)

$$= \frac{1}{3}(27) - \frac{1}{3}(0)$$

$$= 9$$

ex) Find area b/w $y = x^2 + 4$ and $y = 0$ from $x = 0$ to $x = 6$.

$$A = \int_0^6 (x^2 + 4)\,dx = \frac{1}{3}x^3 + 4x \Big|_0^6$$

$$= \left(\frac{1}{3}(6)^3 + 4(6)\right) - \left(\frac{1}{3}(0)^3 + 4(0)\right)$$

$$= 72 + 24 = \underline{\underline{96\,u^2}}$$

ex) Find the area between $y = x^2 + 4$ and $y = 0$ from -2 to 2.

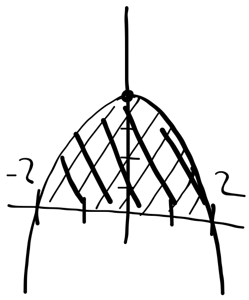
$$A = \int_{-2}^{2} (x^2 + 4)\,dx = \frac{1}{3}x^3 + 4x \Big|_{-2}^{2}$$

$$= \left(\frac{1}{3}(2)^3 + 4(2)\right) - \left(\frac{1}{3}(-2)^3 + 4(-2)\right)$$

$$= \left(\frac{8}{3} + 8\right) - \left(-\frac{8}{3} - 8\right)$$

$$= \frac{8}{3} + 8 + \frac{8}{3} + 8 = 21\tfrac{1}{3} \text{ or } \frac{64}{3}$$

ex) Find the area between $y=-x^2+4$ and the x-axis, between -2 & 2.



HIGH GUESS: 16

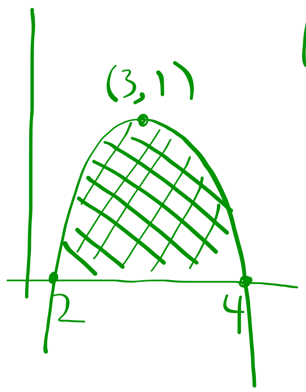
$$A=\int_{-2}^{2}(-x^2+4)\,dx$$

$$=-\frac{1}{3}x^3+4x\Big|_{-2}^{2}$$

$$=\left(-\frac{1}{3}(2)^3+4(2)\right)-\left(-\frac{1}{3}(-2)^3+4(-2)\right)$$

$$=-\frac{8}{3}+8-\frac{8}{3}+8=10\tfrac{2}{3}$$

ex) Find A under $f(x)=-x^2+6x-8$ that is above the x-axis.



max guess = 2×1 = 2

$$A=\int_{2}^{4}(-x^2+6x-8)\,dx$$

$$=-\frac{1}{3}x^3+3x^2-8x\Big|_{2}^{4}$$

$$=4/3$$

$f'(x)=-2x+6$

$0=-2x+6$

$\frac{-6}{-2}=x$