Applications of the FTC

ex) A population of e-coli bacteria grows at a rate given by $A'(t) = \sqrt[3]{t+2}$ where $A(t)$ is the amount of bacteria in mg and t is the time in hrs since the animal was first infected at 1:00pm.

a) Determine the amount of bacteria at 3:00pm

$$\int_0^2 \sqrt[3]{t+2}\,dt = \int_0^2 (t+2)^{1/3}\,dt$$

Math 9: fnInt

$$= \frac{3}{4}(t+2)^{4/3}\Big|_0^2$$

$$= \frac{3}{4}(2+2)^{4/3} - \frac{3}{4}(0+2)^{4/3}$$

$$= \frac{3}{4}\left(4^{4/3} - 2^{4/3}\right)$$

$$= 2.87 \text{ mg}$$

b) What would $\int_1^3 \sqrt[3]{t+2}\,dt$ represent?

The amount of bacteria that accumulated between 2pm & 4pm.

c) What would $2.87 + \int_2^6 \sqrt[3]{t+2}\,dt$ represent?

Total amount of bacteria at 7:00pm.

d) What would $\frac{1}{4}\int_1^5 \sqrt[3]{t+2}\,dt$ represent?

The average increase of bacteria per hour, from 2:00pm to 6:00pm.

ex) A population is growing at a rate of $P'(t) = 50\sin\left(\frac{\pi}{4}t\right)$ insects/month where t is the # of months since March 1st. If the population is 600 insects on April 1st, find the population on Aug 1st.

April 1st: $t=1$

$P(1) = 600$

$$600 + \int_1^5 50\sin\left(\frac{\pi}{4}t\right)dt$$

$$600 + \left(-\frac{200}{\pi}\cos\left(\frac{\pi}{4}t\right)\right)\Big|_1^5$$

$$600 + \left(-\frac{200}{\pi}\cos\frac{\pi}{4}(5) + \frac{200}{\pi}\cos\left(\frac{\pi}{4}(1)\right)\right)$$

$$600 - \frac{\overset{100}{\cancel{200}}}{\pi}\left(-\frac{\sqrt{2}}{\cancel{2}}\right) + \frac{\overset{100}{\cancel{200}}}{\pi}\left(\frac{\sqrt{2}}{\cancel{2}}\right)$$

$$600 + \frac{200}{\pi}(\sqrt{2}) \doteq 690 \text{ insects.}$$

$$\frac{600\pi + 200\sqrt{2}}{\pi}$$

ex) The average value of homes in Kings Co. is growing at a rate of $\sqrt{t+4}$ thousand dollars per year, where t is the # of years since 2005.

a) Find the average value of a home in 2015 if the average was \$90,000 in 2007.

$$90 + \int_2^{10} (t+4)^{1/2}\,dt$$

$$= 90 + \frac{2}{3}(t+4)^{3/2}\Big|_2^{10} \quad \text{etc...}$$

$$= \$115124.18$$

b) When will the average home value reach \$200000?

$$90 + \int_2^{x} (t+4)^{1/2}\,dt$$

$$200 = 90 + \frac{2}{3}(t+4)^{3/2}\Big|_2^{x}$$

$$110 = \frac{2}{3}(x+4)^{3/2} - \frac{2}{3}(2+4)^{3/2}$$

$$(179.69\ldots)^{2/3} = \left((x+4)^{3/2}\right)^{2/3}$$

$$-4 \qquad\qquad -4$$

$$27.8\text{yr} = x$$

$$2005 + 27.8 = 2032.8\ldots$$

2032 or 2033