Evaluation 5: Integration and the Fundamental Theorem

You should be able to:

- Interpret the integrals
- Evaluate both definite integrals
- Apply the Fundamental Theorem of Calculus

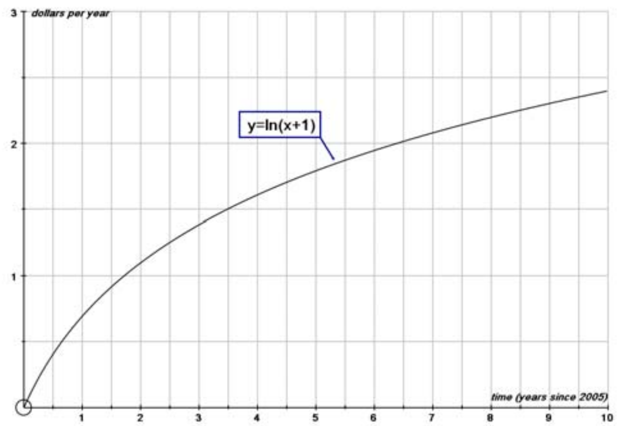
Practice Questions:

1. Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 s. The maximum rate of air flow into the lungs is about 0.5L/s. This explains, in part, why the function

   $$f(t) = \tfrac{1}{2}\sin(2\pi t/5)$$

   has often been used to model the rate of airflow into the lungs.
   Use this model to find the ***volume of inhaled air*** in the lungs when the individual has finished the inhale part of the breathing cycle .

2. If shares for stock A were worth \$1200 at the beginning of 1997 (t = 0) and the rate of change of these same shares is shown below as R(t) = ln(t+1), in dollars per year and t is in years,
   a) State an integral that would describe the value of the stock shares at the beginning of the year 2005.
   b) Use the graph to the right to estimate their value at the beginning of 2005.



3. Evaluate each of the following *without* the use of the fnInt function on your calculator.

   a) $\int_1^3 (x^2 - 1)\,dx$ b) $\int_{-1}^4 (6x - 2)\,dx$ c) $\int_{-\pi}^{3\pi} 2\sin x\,dx$

   d) $\int_0^2 e^{3x}\,dx$ e) $\int_0^{\frac{\pi}{3}} \sec^2(\frac{1}{2}x)\,dx$ f) $\int_1^3 \frac{2}{4p-1}\,dp$

   g) $\int_1^3 6(2^{3x})\,dx$ h) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} 2\cos(4x)\,dx$ i) $\int_4^9 (2\sqrt{x} - \frac{6}{x})dx$

4. The rate of change in the air temperature during a period of 12 hours is given by the model $T(t) = 53t + 2.4t^2 - t^3$, $0 \le t \le 12$ where t is measured in hours and T in degrees Fahrenheit per hour. Determine the value and meaning of $\int_0^{12} T(t)\,dt$

5. Suppose that gasoline is increasing in price according to the equation $p(t) = 1 + 0.1t + 0.2t^2$ where $p(t)$ is the price per litre per year and $t = 0$ represents the year January 2005.
   a) Determine the cost for a litre of gas in June 2008 if the price in January 2007 was $4
   b) Predict the increase in the price of a litre of gas from January 2005 to January 2009.

6. The rate at which water is flowing into a tank is $r(t) = 20e^{0.02t}$ gallons per minute with t representing time in minutes since 12 noon. If the tank contained 2500 gallons of water when it was measured at 12:30 determine the amount of water in the tank at 2:00.

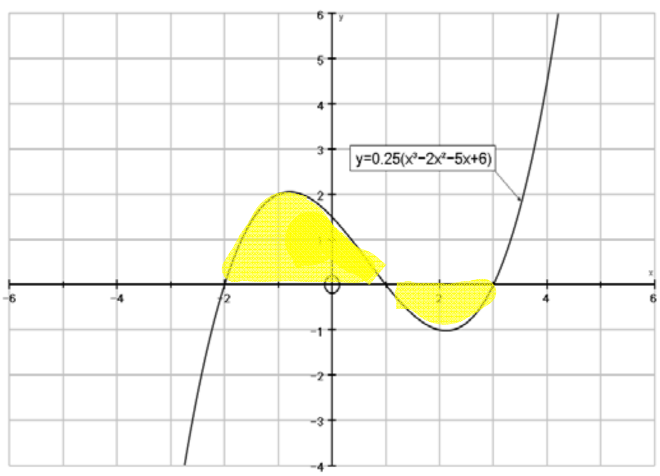
7. In 1997 the average per capita income in Canada was $31 000. Suppose that the average per capita income is increasing at a rate in dollars per year given by: $r(t) = 480(1.024)^t$ where t is the number of years since 1997. Determine the average per capita income in Canada in 2008.

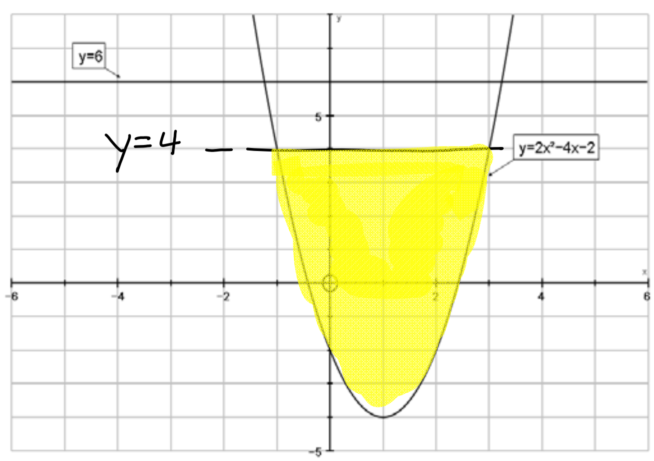
8. A rod has length 2 meters. At a distance x meters from its left end, the density of the rod is given by: $\rho(x) = 2 + 6x$ in grams per meter. Determine the mass of the rod.

9. The density of cars (in cars per kilometer) down a 20 km stretch of the 401 outside of Toronto can be approximated by: $\rho(x) = 600 + 300\sin\sqrt{16x + 0.6}$ where x is the distance from the city limits. Determine the total number of cars on this 20 km stretch of highway. *[Use your calculator]*

10. The thickness of ice formed on a lake is decreasing at a rate of $0.15t^{\frac{1}{3}}$ cm/day where t represents the time since March 2nd when it was measured to be 20 cm thick.
    a) Determine the thickness of the ice on March 12th.
    b) If it is no longer safe for recreational sports when it reaches a thickness less than 13 cm , how many more days can it be expected to be able to be used safely?

11. Determine the total area of the shaded regions shown below:

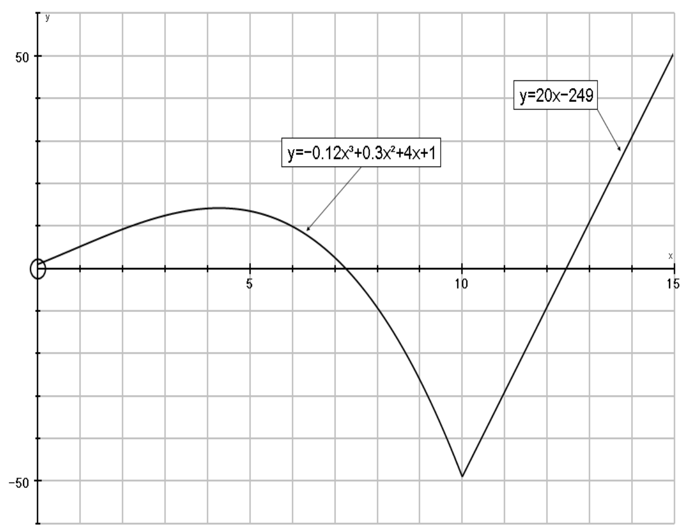




12. The velocity of a robotic chicken toddling along in a straight hallway is given by $v(t) = 0.2\sqrt{t}$ where t represents the time that has elapsed since said robotic chicken was set down at a location 5 m to the left of the supervisor's doorway.
    a) How far is the chicken from the supervisor's doorway 36 seconds after it was placed in the floor?
    b) How far has the chicken traveled in that 36 second period of time?
    c) Did the robotic chicken ever go past the supervisor's door during this time period? Is so when?

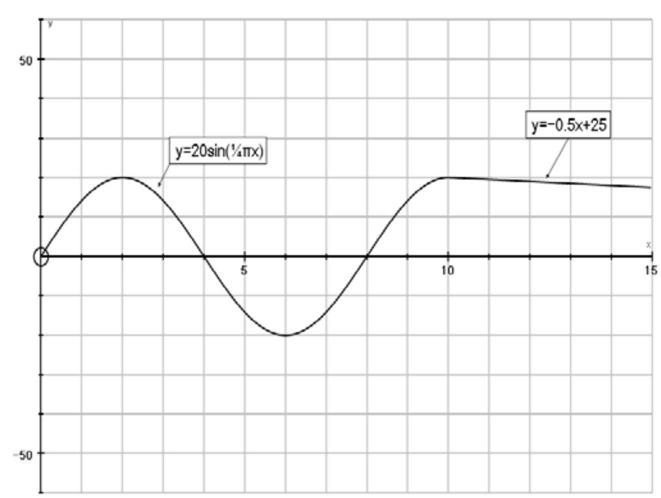
13. The function shown below is a differentiable function. The position at time t seconds of a particle moving along a coordinate axis is $s(t) = \int_0^t f(x)\,dx$ meters. Use the graph of $f(x)$ shown below to answer the questions posed. Give reasons for your answers.

Situation I



a) What is the particle's velocity after 5 seconds?
b) Is the acceleration of the particle after 5 seconds positive or negative?
c) What is the particle's position after 5 seconds?
d) At what time during this 15 second period of time is the particle farthest to the right of its initial position?
e) When is the particle's acceleration zero?
f) Determine the position of the particle after 15 seconds.

Situation II



a) What is the particle's velocity after 6 seconds?
b) Is the acceleration of the particle after 6 seconds positive or negative?
c) What is the particle's position after 6 seconds?
d) At what time during this 15 second period of time is the particle farthest to the right of its initial position?
e) When is the particle's acceleration zero?
f) Determine the position of the particle after 15 seconds.