Quiz #4 Solutions for Practice Questions

Note Title 25/05/2008

1) $$\int_0^{2.5} \frac{1}{2}\sin\left(\frac{2\pi}{5}t\right)\partial t = -\frac{1}{2}\cdot\frac{5}{2\pi}\cos\left(\frac{2\pi}{5}t\right)\Big|_0^{2.5}$$

since half of 5 second period would be for inhalation.

$$= -\frac{5}{4\pi}\cos\left(\frac{2\pi}{5}t\right)\Big|_0^{2.5}$$

$$= -\left[\frac{5}{4\pi}\cos(\pi) - \frac{5}{4\pi}\cos(0)\right]$$

$$= -\frac{5}{4\pi}(-1) + \frac{5}{4\pi}(1)$$

$$= \frac{10}{4\pi} \doteq 0.8 \text{ litres}$$

2) $$1200 + \int_0^8 R(t)\,\partial t$$

3a) $$\int_1^3 (x^2-1)\,\partial x = \frac{1}{3}x^3 - x\Big|_1^3 = \left(\frac{27}{3}-3\right) - \left(\frac{1}{3}-1\right) = (6) - \left(-\frac{2}{3}\right) = 6\tfrac{2}{3}$$

b) $$\int_{-1}^4 (6x-2)\,\partial x = 3x^2 - 2x\Big|_{-1}^4 = (48-8) - (3+2) = 40 - 5 = 35$$

c) $$\int_{-\pi}^{3\pi} 2\sin x\,\partial x = -2\cos x\Big|_{-\pi}^{3\pi} = -2\cos(3\pi) + 2\cos(-\pi) = 2-2 = 0$$

d) $$\int_0^2 e^{3x}\,\partial x = \frac{1}{3}e^{3x}\Big|_0^2 = \frac{1}{3}e^6 - \frac{1}{3}e^0 = \frac{1}{3}(e^6 - 1)$$

e) $$\int_0^{\pi/3} \sec^2\left(\tfrac{1}{2}x\right)\partial x = 2\tan\left(\tfrac{1}{2}x\right)\Big|_0^{\pi/3} = 2\tan\left(\frac{\pi}{6}\right) - 2\tan(0) = \frac{2\sqrt{3}}{3}$$

f) $$\int_1^3 \frac{2}{4p-1}\,\partial p = \frac{1}{2}\ln(4p-1)\Big|_1^3 = \frac{1}{2}\ln(11) - \frac{1}{2}\ln(3) = \frac{1}{2}\ln\left(\frac{11}{3}\right)$$

g) $$\int_1^3 6(2^{3x})\,\partial x = \frac{2}{\ln 2}2^{3x}\Big|_1^3 = \frac{2}{\ln 2}(2^9 - 2^3) = \frac{2^4}{\ln 2}(2^6 - 1)$$

3h) $\int_{-\pi/6}^{\pi/2} 2\cos(4x)\,dx = \frac{1}{2}\sin(4x)\Big|_{-\pi/6}^{\pi/2} = \frac{1}{2}\sin(2\pi) - \frac{1}{2}\sin\left(\frac{-2}{3}\pi\right)$

$= \frac{1}{2}(0) - \frac{1}{2}\left(\frac{-\sqrt{3}}{2}\right) = \sqrt{3}/4$

i) $\int_4^9 \left(2\sqrt{r} - \frac{6}{r}\right)dr = \left[\frac{4}{3}r^{3/2} - 6\ln(r)\right]\Big|_4^9$

$= \left[\frac{4}{3}(9)^{3/2} - 6\ln(9)\right] - \left[\frac{4}{3}(4)^{3/2} - 6\ln(4)\right]$

$= 36 - 6\ln 9 - \frac{32}{3} + 6\ln(4)$

$= \frac{76}{3} - 6\ln\left(\frac{9}{4}\right)$

4) $\int_0^{12} (53t + 2.4t^2 - t^3)\,dt = \left(\frac{53}{2}t^2 + 0.8t^3 - \frac{1}{4}t^4\right)\Big|_0^{12} = 14.4\,°F$

This would mean that there was a 14.4° rise in air temperature during this 12 hour period.

5a) $4 + \int_2^{3.5} (\underbrace{1 + 0.1t + 0.2t^2}_{})\,dt = 4 + 4.24 = \$\,8.24$

note: gallons per year.

b) $\int_0^4 (1 + 0.1t + 0.2t^2)\,dt = \$\,9.07$

6) $2500 + \int_{30}^{120} 20e^{0.02t}\,dt = 2500 + 1000e^{0.02t}\Big|_{30}^{120}$

$= 2500 + 1000e^{2.4} - 1000e^{0.6}$

$= 11\,701$ gallons

7) $31000 + \int_0^{11} 480(1.024)^t\,dt = 31000 + \frac{480}{\ln 1.024}(1.024)^t\Big|_0^{11}$

$= 31000 + 26271.79 - 20239.05$

$= \$\,37\,032.74$

8) $\int_0^2 (2+6x)\,\partial x = \left(2x+3x^2\right)\Big|_0^2 = (4+12)-(0+0) = 16\text{ g}$

9) $\int_0^{20} 600+300\sin\sqrt{16x+0.6}\,\partial x = \boxed{11\ 570 \text{ cars}}$

(Make sure your calculator is on radian measure!)

10 a) $20-\int_0^{10} 0.15t^{1/3}\,\partial t = 20-0.1125t^{4/3}\Big|_0^{10}$

$= 20-2.42 = 17.6$ cm thick.

b) $20-\int_0^{x} 0.15t^{1/3}\,\partial t = 20-0.1125t^{4/3}\Big|_0^{x}$

$\therefore\quad 20-0.1125x^{4/3} = 13$

$-0.1125x^{4/3} = -7$

$x^{4/3} = 62.\overline{2}$

$x \doteq 22.15$

$\therefore$ The lake should be safe until March 24th.

11 a) $\int_{-2}^{1} 0.25(x^3-2x^2-5x+6)\,\partial x - \int_{1}^{3} 0.25(x^3-2x^2-5x+6)\,\partial x$

$= 3.9375 + 1.\overline{3} \doteq 5.27$

b) $\int_{-1}^{3}\left[4-(2x^2-4x-2)\right]\partial x = \int_{-1}^{3}(6-2x^2+4x)\,\partial x = 21\tfrac{1}{3}$

12a) $-5 + \int_0^{36} 0.2\sqrt{t}\, \partial t = -5 + \frac{2}{15} t^{3/2} \Big|_0^{36} \quad -5 + 28.8 = 23.8 \text{ m}$

$\therefore$ It is 23.8 m right of the supervisor's doorway after 36 seconds.

b) $\underbrace{\int_0^{36} |0.2\sqrt{t}|\, \partial t \quad \text{or} \quad \int_0^{36} 0.2\sqrt{t}\, \partial t}_{\text{these are the same since } v(t) \geq 0 \text{ for } t \in [0,36]} = 28.8 \text{ m.}$

c) Yes! $\int_0^{x} 0.2\sqrt{t}\, \partial t = 5$

$\frac{2}{15} t^{3/2} = 5$

$t^{3/2} = 37.5$

$\boxed{t = 11.2 \text{ seconds}}$

$\therefore$ The chicken passes in front of the supervisor's door at the 11.2 second mark.

(i)

13a) 13 m/s

b) negative

c) $\int_0^5 f(x)\, \partial x = 48.75$ m to the right of where it was at $t=0$.

d) At $t = 7.2$: $\int_0^{7.2} f(x)\, \partial x = 67.58$

(most positive area at this time)

e) At 4.4 sec

f) $\int_0^{10} (-0.12x^3 + 0.3x^2 + 4x + 1)\, \partial x$

$+ \int_{10}^{15} (20x + 249)\, \partial x = 15 \text{ m}$

$\therefore$ 15 m to the right of its initial ($t=0$) position.

(ii)

a) -20 m/s

b) Neither. It is zero m/s^2

c) $\int_0^6 20\sin(\frac{1}{4}\pi x)\, \partial x \doteq 25.465$ $\boxed{\text{OK}}$

$-\frac{80}{\pi} \cos\left(\frac{1}{4}\pi x\right) \Big|_0^6 = \frac{80}{\pi}$

d) $t = 15$ sec

e) $t = 2$ sec, $t = 6$ sec

perhaps $t = 10$ sec.

f) $\int_0^{10} 20\sin\left(\frac{1}{4}\pi x\right) \partial x$

$+ \int_{10}^{15} (-0.5x + 25)\, \partial x = \frac{80}{\pi} + \frac{375}{4}$

$\doteq 119.21$ m to the right of initial spot