Test #4 – Outline and Practice

You should be able to:

1. Understand the LRAM, RRAM and MRAM given a function or data points.
2. Write Riemann sums as an integral.
3. Evaluate an integral using geometric formulas.
4. Determine an indefinite integral using anti-differentiation.
5. Determine the area of a region using integrals.
6. Understand the difference between net area and total area.
7. Interpret integrals.
8. Evaluate definite integrals.
9. Use integration by substitution.
10. Use the rules of integration to evaluate integrals.
11. Work with integral applications including motion questions.
12. Use and understand the Fundamental Theorem of Calculus.

Practice Questions:

1. The cost of producing x ounces of gold from a new gold mine is $C = f(x)$ dollars.
   (a) What is the meaning of $f'(x)$? What are its units?
   (b) What does the statement $f'(800) = 17$ mean?
   (c) Do you think the values of $f'(x)$ will increase or decrease in the short term? What about in the long term? Explain.

2. An automobile computer gives a digital readout of fuel consumption in gallons per hour. During a one hour trip you record the fuel consumption every five minutes.
   Estimate the approximate total fuel consumption during the hour trip using:
   (a) LRAM (b) RRAM

time (min.)	consumption (gal/hour)	time (min.)	consumption (gal/hour)
0	2.5	35	2.5
5	2.4	40	2.4
10	2.3	45	2.3
15	2.4	50	2.4
20	2.4	55	2.4
25	2.5	60	2.3
30	2.6		

   (c) If the automobile covered 60 miles in the hour, what was its fuel efficiency (in miles per gallon) for the trip?

3. Given that the velocity of a rocket can be described by the function: $v(t) = 60t + 3t^2$ determine the distance it has traveled during the time interval $t \in [1,17]$ using four rectangles and MRAM. *Include a diagram.*

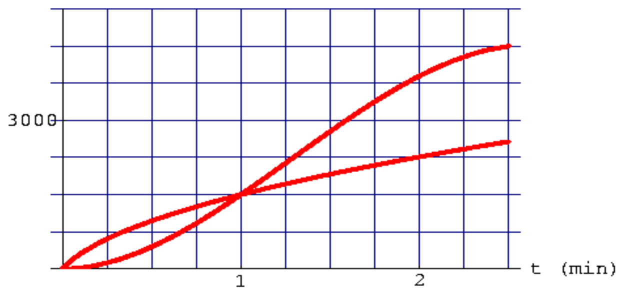
4. page 298 # 9, 11, 15-24
   page 299 # 25 – 33, 43
   page 300 # 47, 50

5. Given that F(w) represents the rate of productivity of a factory in items produced per workers and w represents the number of workers, what is the meaning of the integral

$$\int_{100}^{120} F(w)\,dw$$

6. If f(t) represents the rate at which a realtor is selling houses (houses per month) where $\int_{3}^{6} f(t)dt$, $t \in [0,1)$ would represent January. Explain the meaning of the integral.

7. If the graph below describes the velocity of two cars for a 2.4 minute time period, determine when the two cars have traveled the same distance. Explain how you determined this answer.



8. (a) If $f(x)$ is continuous and $\int_{0}^{4} f(x)\,dx = 10$, find $\int_{0}^{2} 5f(2x)\,dx$

(b) If $g(x)$ is continuous and $\int_{0}^{9} g(x)\,dx = 4$, find $\int_{0}^{9} (x + 2g(x))\,dx$

9. Fresnel also used the function $C(x) = \int_{0}^{x} \cos(\frac{\pi}{2}t^2)\,dt$ to describe the diffraction of light waves.

i) What are the critical values of C?
ii) What are the x coordinates for the inflection points of C?

10. If $\frac{dy}{dx} = \frac{2x}{y}$ and $y(2) = -1$, determine an equation for y and use it to determine $y(5)$.

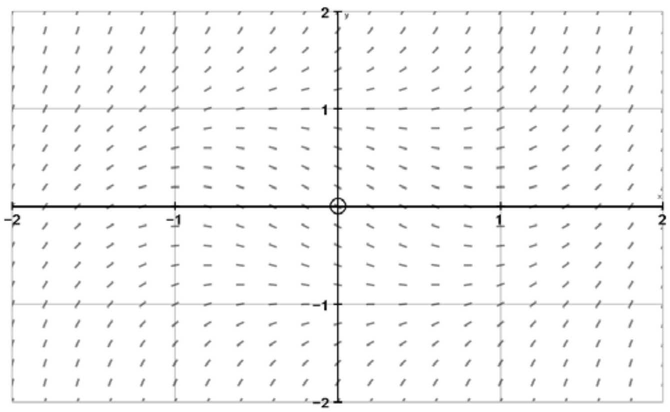
11. If $\frac{dy}{dx} = 6x^2(y-2)$ and $y(3) = 5$, determine an equation for y and use it to determine $y(7)$.

12. About how accurately should we measure the radius or a sphere to calculate the surface area ($S = 4\pi r^2$) to within 1% of its true value?

13. Find a linear approximation to estimate the value of $y = f(0.1)$; given that : $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} - \sin x$ when x = 0 and $f(0) = 2$. Then explain how you could tell if this was an under-estimate or an over-estimate of $f(0.1)$.

14. A car is decelerating at a rate of If $2\sqrt{t}$ m/s^2 where t is time in seconds since it began to decelerate. If the car was travelling 23m/s when it first began to decelerate, determine the distance it travels prior to stopping.

15. The following slope field can be drawn for the differential equation: $\frac{dy}{dx} = x^2 + y^2 - 1$.

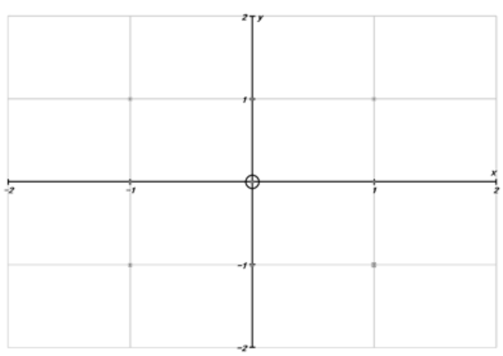


a) Sketch, on the field, a solution with the initial condition (1, 1)

b) For what value(s) of x and y would the slope be zero?

16 The slope of a function at any point (x,y) is $\frac{dy}{dx} = \frac{x-1}{y}$.

a) Sketch a slope field for this differential equation on the grid shown below.



b) Given that the point (3, 4) is on the graph of $f(x)$ write an equation of the tangent line to the graph of $f(x)$ at $x = 3$

c) Use the tangent line in part (a) to approximate $f(2.9)$. Determmine if this is an overestimate or an underestimate.

d) Solve the differential equation: $\frac{dy}{dx} = \frac{x-1}{y}$ with the initial condition $f(3) = 4$

e) Use the solution in part (c) to find $f(2.9)$

17. Evaluate the integrals:

a) $\int \sec^2 x\sqrt{\tan^5 x}\ dx$

b) $\int x\sin(x^2+5)\,dx$

18. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W, models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation: $\frac{dW}{dt} = \frac{1}{25}(W-300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time = ¼)

b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time *t=1/4*.

c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W-300)$ with the initial condition $W(0)=1400$.

19. Evaluate:

a) *by hand* $\int_0^1 \frac{6x}{\sqrt{10-6x^2}}\,dx$

b) $\int 2e^{4x}\ dx$

c) $\int \left(\sin^3 x\cdot\cos x\right)dx$

d) $\int x^3e^{2x^4}\,dx$

20. Find the interval on which the curve: $y = \int_0^x \frac{1}{8t-2t^2}\,dt$ is concave downwards.