Note Title 29/05/2011

Calculus Evaluation 7 (Solutions for Practice)

1 a) $f'(x)$ = \$/ounce ; $\frac{dC}{dx}$

b) $f'(800) = 17$ means that when 800 oz of gold is being produced the cost for producing one more oz will be approx \$17.

c) In the short term $f'(x)$ will decrease (supply is likely plentiful). In the long term $f'(x)$ will likely increase.

2 a) LRAM: $\frac{1}{12}[2.5+2.4+2.3+2.4+2.4+2.5+2.6+2.5+2.4+2.3+2.4+2.4]$

$\frac{1}{12}(29.1) = 2.425$ gal

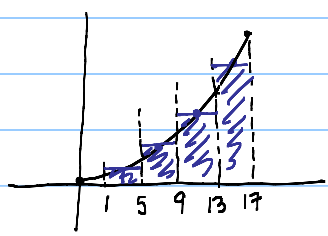
RRAM: $\frac{1}{12}[2.4+2.3+2.4+2.4+2.5+2.6+2.5+2.4+2.3+2.4+2.4+2.3]$

$\frac{1}{12}(28.9) = 2.408\bar{3}$ gal

b) $\frac{60}{2.425} = 24.74$ mi/gal ; $\frac{60}{2.408\bar{3}} = 24.91$ mi/gal

or using average : $\frac{60}{2.4167} = 24.83$ mi/gal

3)



$4[v(3)+v(7)+v(11)+v(15)]$

$4[207+567+1023+1575]$

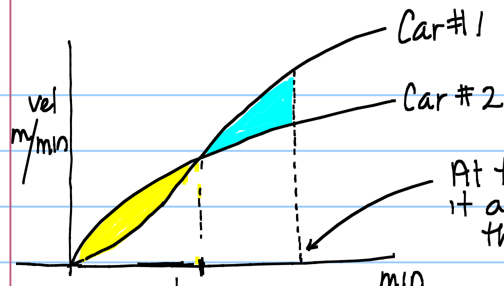
$4(3167.7) = 12670.8$ units.

4) See Answers in Solution Guide

5) $\int_{w=100}^{w=120} F(w)\,dw$ represents the change in items produced when the number of workers employed by this factory is increased from 100 to 120.

6) $\int_{t=3}^{t=6} f(t)\,dt$ represents the number of houses sold by this realtor in the months of April, May and June

7)



time when velocities are equal

At this time Car #2 has travelled further than Car #1 ...

At this time it appears that the extra distance Car #1 has gained since beginning to go faster than Car #2 has made up the extra distance that Car #2 had achieved prior to that point.

My estimate: 1.4 min

8a) $\int_0^4 f(x)\,dx = 10 \quad \therefore \quad \int_0^2 f(2x)\,dx = \frac{10}{2} = 5$

width of all rectangles half as much [height not changed]

$\therefore$ Area = ½

$$\int_0^2 5f(2x)\,dx = 5\int_0^2 f(2x)\,dx = 5(5) = 25$$

b) $\int_0^9 (x + 2g(x))\,dx = \int_0^9 x\,dx + 2\int_0^9 g(x)\,dx$

$= \frac{1}{2}x^2\Big|_0^9 + 2(4)$

$= \frac{1}{2}(9)^2 - \frac{1}{2}(0)^2 + 8$

$= \frac{81}{2} + 8 = 48.5$

9. Critical values occur where $C'(t) = 0$ and where $C'(x)$ does not exist

i) $C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt$

$C'(x) = \cos\left(\frac{\pi}{2}x^2\right)$

$\therefore x = \pm\sqrt{1+2k}$, $k \in \mathbb{Z}$

$C'(x) = 0$ when $\cos\left(\frac{\pi}{2}x^2\right) = 0$

$\frac{\pi}{2}x^2 = \frac{\pi}{2} + \pi k$

$x^2 = 1 + 2k$

iii) Inflection Points occur where $C''(t)$ changes sign.

$$C''(x) = -\sin\left(\frac{\pi}{2}x^2\right) \cdot 2\frac{\pi}{2}x = -\pi x \sin\left(\frac{\pi}{2}x^2\right)$$

$C''(x)=0$ when $-\pi x = 0$, $x=0$

$\sin\left(\frac{\pi}{2}x^2\right)=0$

$\pi/_2 x^2 = 0 + \pi k$

$x^2 = 0 + 2k$

$x = \pm\sqrt{2k}, \; k \in \mathbb{Z}$

10) $\frac{dy}{dx} = \frac{2x}{y}$

$\int y\,dy = \int 2x\,dx$

$(2,-1)$ $\frac{1}{2}y^2 = x^2 + C$

$\frac{1}{2}(-1)^2 = (2)^2 + C$

$\frac{1}{2} = 4 + C$

$-3.5 = C$

$\frac{1}{2}y^2 = x^2 - 3.5$

$y^2 = 2x^2 - 7$

$y = \underbrace{-}_{\text{since point } (2,-1) \text{ had a negative } y}\sqrt{2x^2-7}$

$y(5) = -\sqrt{2(5)^2 - 7}$

$y(5) = -\sqrt{43}$

11) $\frac{dy}{dx} = 6x^2(y-2)$

$\int \frac{1}{y-2}\,dy = \int 6x^2\,dx$

$(3,5)$ $\ln|y-2| = 2x^3 + C$

$\ln|5-2| = 2(3)^3 + C$

$\ln(3) = 54 + C$

$\ln(3) - 54 = C$

$\ln|y-2| = 2x^3 + \ln(3) - 54$

$|y-2| = e^{2x^3 + \ln(3) - 54}$

Since $y=5$ $|y-2|>0$

$\therefore y-2 = e^{2x^3+\ln(3)-54}$

$y = e^{2x^3+\ln(3)-54} + 2$

or

$y = e^{2x^3 - 52.9} + 2$

$y(7) = e^{2(7)^3 - 52.9} + 2$

$y(7) \doteq e^{633.1} + 2$

12) $S = 4\pi r^2$ find dr if $dS = 0.01S$

$dS = 8\pi r\,dr$

$0.01S = 8\pi r\,dr$

$0.01(4\pi r^2) = 8\pi r\,dr$

$\dfrac{0.04\pi r^2}{8\pi r} = dr$

$0.005r = dr$ $\therefore$ Measure accurate to 0.5%

13) $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x+1}} - \sin x$

$\left.\dfrac{dy}{dx}\right|_{x=0} = \dfrac{1}{2\sqrt{1}} - \sin 0 = \dfrac{1}{2}$

$\dfrac{1}{2} = \dfrac{y-2}{x-0}$

$\boxed{\tfrac{1}{2}x + 2 = y}$

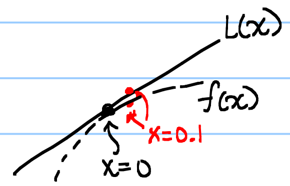
Linearlization

$f(0.1) \doteq \tfrac{1}{2}(0.1) + 2$

$= \dfrac{1}{20} + 2$

$= 2\tfrac{1}{20} = 2.05$

$\dfrac{dy}{dx} = \tfrac{1}{2}(x+1)^{-1/2} - \sin x$



$\dfrac{d^2y}{dx^2} = -\dfrac{1}{4}(x+1)^{-3/2}(1) - \cos x$

$\left.\dfrac{d^2y}{dx^2}\right|_{x=0} = -\tfrac{1}{4}(1)^{-3/2} - \cos 0 = -\tfrac{1}{4} - 1 < 0$

f(x) is concave down at $x=0$

2.05 is an overestimate

14) $\boxed{a = -2t^{1/2}}$

$v = -2 \cdot \dfrac{2}{3} t^{3/2} + C_1$

$t=0$, $v=23$

$v = -\tfrac{4}{3}t^{3/2} + C_1$

$23 = -\tfrac{4}{3}(0)^{3/2} + C_1$

$23 = C_1$

$\boxed{v = -\tfrac{4}{3}t^{3/2} + 23}$

$d = -\tfrac{4}{3} \cdot \tfrac{2}{5} t^{5/2} + 23t + C_2$

$t=0$, $d=0$

$d = -\dfrac{8}{15}t^{5/2} + 23t + C_2$

$0 = C_2$

$\boxed{d = -\tfrac{8}{15}t^{5/2} + 23t}$

$v=0$ when $\tfrac{4}{3}t^{3/2} = 23$

$t^{3/2} = 17.25$

$t = 6.676$ sec

$d(6.676) = -\tfrac{8}{15}(6.676)^{5/2} + 23(6.676)$

$d = 92.13$ m

ANSWER