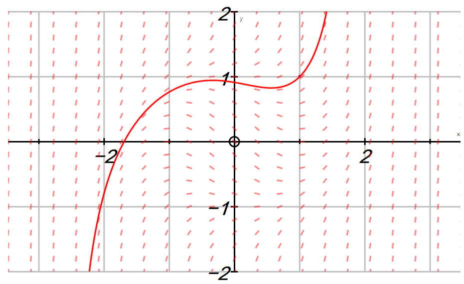
Final Evaluation Solutions (continued)

15a)

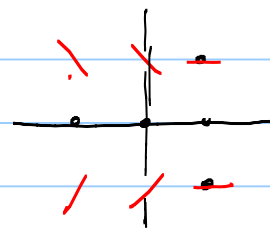


b) $\frac{dy}{dx}=0$ when $x^2+y^2-1=0$

$x^2+y^2=1$

(this would be for all points on the circle with center (0,0) and radius of 1)

16a) $\frac{dy}{dx}=\frac{x-1}{y}$



b) $\left.\frac{dy}{dx}\right|_{\substack{x=3\\y=4}}=\frac{3-1}{4}=\frac{1}{2}$

$\frac{1}{2}=\frac{y-4}{x-3}$

$\frac{1}{2}(x-3)+4=y$

c) $\frac{1}{2}(2.9-3)+4=y$

$\frac{1}{2}(-0.1)+4=y$

$-0.05+4=y$

$y=3.95$

$\therefore f(2.9)\doteq 3.95$

$\frac{d^2y}{dx^2}=\frac{(1)y+(x-1)\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2}=\frac{y+(x-1)\left(\frac{x-1}{y}\right)}{y^2}$

$\left.\frac{d^2y}{dx^2}\right|_{\substack{x=3\\y=4}}=\frac{4+(2)\left(\frac{2}{4}\right)}{4^2}=\frac{4+1}{16}$

Since $y''>0$ $f(x)$ is concave up at $x=3$ and $\therefore f(2.9)>3.95$

d) $\int y\,dy=\int(x-1)\,dx$

$\frac{1}{2}y^2=\frac{1}{2}x^2-x+c$

(3,4): $\frac{1}{2}(4)^2=\frac{1}{2}(3)^2-3+c$

$8=4.5-3+c$

$8=1.5+c$

$6.5=c$

$\frac{1}{2}y^2=\frac{1}{2}x^2-x+6.5$

$y^2=x^2-2x+13$

$y=\sqrt{x^2-2x+13}$

$\therefore f(x)=\sqrt{x^2-2x+13}$

e) $f(2.9)=\sqrt{(2.9)^2-2(2.9)+13}$

$f(2.9)=\sqrt{15.61}$

$f(2.9)\doteq 3.95095$

17 a) $\int \sec^2 x \sqrt{\tan^5 x}\, dx$

let $u = \tan x$
$du = \sec^2 x\, dx$

$\int u^{5/2}\, du = \frac{2}{7} u^{7/2} + C$

$= \frac{2}{7}(\tan x)^{7/2} + C$

b) $\int x \sin(x^2+5)\, dx$

let $u = x^2 + 5$
$du = 2x\, dx$
$\frac{1}{2} du = x\, dx$

$\int \frac{1}{2} \sin u\, du$

$= \frac{1}{2}(-\cos u) + C$

$= -\frac{1}{2}\cos(x^2+5) + C$

18 a) $\left.\frac{dw}{dt}\right|_{\substack{t=0 \\ y=1400}} = \frac{1}{25}(1400-300) = 44$

$44 = \frac{y-1400}{t-0}$

$\therefore$ When $t = 1/4$ $\quad y = 44\left(\frac{1}{4}\right) + 1400$
$= 1411$ tons

$\boxed{44t + 1400 = y}$

$\therefore W\left(\frac{1}{4}\right) \doteq 1411$ tons

b) $\frac{d^2w}{dt^2} = \frac{1}{25}\left(\frac{dw}{dt}\right) = \frac{1}{25}\left(\frac{1}{25}(W-300)\right) = \frac{1}{625}(W-300)$

For $t=0$ $W=1400$ $\frac{d^2w}{dt^2} > 0$ $\therefore$ $W\left(\frac{1}{4}\right) > 1411$ since

W is concave up and thus the tangent line provides an under-estimate.

c) $\frac{dW}{dt} = \frac{1}{25}(W-300)$

$\int \frac{1}{W-300}\, dW = \int \frac{1}{25}\, dt$

$\ln|W-300| = \frac{1}{25}t + c$

$\ln|1100| = c$

$\ln|W-300| = \frac{1}{25}t + \ln 1100$

$|W-300| = e^{\frac{1}{25}t + \ln 1100}$

$|W-300| = 1100e^{\frac{1}{25}t}$

$W-300 = 1100e^{\frac{1}{25}t}$

$W = 1100e^{\frac{1}{25}t} + 300$

19a) $\int_0^1 \frac{6x}{\sqrt{10-6x^2}}\, dx$

let $u = 10-6x^2$
$du = -12x\, dx$
$-\frac{1}{2} du = 6x\, dx$

$= \int_{10}^{4} -\frac{1}{2}(u)^{-1/2}\, du$

$= -\frac{1}{2}\left(2u^{1/2}\right)\Big|_{10}^{4} = -u^{1/2}\Big|_{10}^{4} = -\sqrt{4} + \sqrt{10} = \boxed{\sqrt{10} - 2}$

b) $\int 2e^{4x}\, dx = 2\left(\frac{1}{4}e^{4x}\right) + C = \boxed{\frac{1}{2}e^{4x} + C}$

c) $\int \sin^3 x \cos x\, dx$

let $u = \sin x$
$du = \cos x\, dx$

$\int u^3\, du = \frac{1}{4}u^4 + C$

$= \boxed{\frac{1}{4}(\sin x)^4 + C}$

d) $\int x^3 e^{2x^4}\, dx = \int \frac{1}{8}e^u\, du$

let $u = 2x^4$
$du = 8x^3\, du$
$\frac{1}{8} du = x^3\, du$

$= \frac{1}{8}e^u + C = \boxed{\frac{1}{8}e^{2x^4} + C}$

20) $y = \int_0^x \frac{1}{8t-2t^2}\,dt$

$y' = \frac{1}{8x-2x^2} = (8x-2x^2)^{-1}$

$y'' = -1(8x-2x^2)^{-2}(8-4x) = \frac{-(8-4x)}{(8x-2x^2)^2}$

$y''<0$ when $-(8-4x)<0$

$-8+4x<0$

$4x<8$

$\boxed{x<2}$