The Fundamental Theorem(s) of Calculus

The first fundamental theorem of calculus states that, if f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

This result, while taught early in elementary calculus courses, is actually a very deep result connecting the purely algebraic indefinite integral and the purely analytic (or geometric) definite integral.

The second fundamental theorem of calculus holds for f a continuous function on an open interval I and a any point in I, and states that if F is defined by

$$F(x) = \int_a^x f(t)\,dt,$$

then

$$F'(x) = f(x)$$

at each point in I.

Source:mathworld.wolfram.com/FundamentalTheoremsofCalculus.html ex) Find the area b/w $y=\sqrt{x}$ and the x-axis, from $x=1$ to $x=4$.

$$A=\int_1^4 \sqrt{x}\,dx \rightarrow \int_1^4 x^{1/2}\,dx$$

$$=\frac{2}{3}x^{3/2}\Big|_1^4$$

$$=\frac{2}{3}(4)^{3/2}-\frac{2}{3}(1)^{3/2}$$

$$=\frac{2}{3}\left(\sqrt{4}^3-\sqrt{1}^3\right)$$

$$=\frac{2}{3}(8-1) \quad \text{- fnInt}$$

$$=\frac{2}{3}(7) \quad \text{- Graph}$$

$$=14/3$$

ex) Evaluate $\int_1^3 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_1^3$

$$= \frac{1}{2}e^{2(3)} - \frac{1}{2}e^{2(1)}$$

$$= \frac{1}{2}\left(e^{6} - e^{2}\right)$$

ex) Evaluate $\int_0^{\pi} 2\sin x\,dx = -2\cos x \Big|_0^{\pi}$

$= -2\cos\pi + 2\cos 0$

$= -2(-1) + 2(1)$

$= 4$

ex) $\int_1^4 \frac{1}{x}\,dx = \ln x \Big|_1^4$

$= \ln 4 - \ln 1$ OR $\ln(4/1)$

$= \ln 4 - 0$ $\qquad \ln 4$

$= \ln 4$

Why is $\ln 1 = 0$

$e^0 = 1$

(1) Read pg. 277, then skim pg. 278

(2) Read pg. 282 (FTC pt. 2) & pg. 283-4 Area (net vs. total)

(3) pg. 286 #15-18

(4) Do the 8 posted questions on the next page...

For each of the following, find the area between the function and the horizontal line, within the indicated interval. Please sketch a picture to assist you.

Function	Horizontal Bound	Lower Bound	Upper Bound
$y = x^2 + 1$	$y = 0$	$x = -1$	$x = 4$
$y = 2^x$	$y = 0$	$x = -2$	$x = 3$
$y = 2\sqrt{x}$	$y = 0$	$x = 1$	$x = 9$
$y = 6\sin x$	$y = 0$	$x = 0$	$x = \pi$
$y = 3e^{-x}$	$y = 5$	$x = 0$	$x = 4$
$y = \lvert x^3 + 2x^2 - 3x \rvert$	$y = 0$	$x = -2$	$x = 2$

Find the **total** area between $y = \sin(x)$ and the x-axis from $x = 0$ to $x = 2\pi$.

Find the **net** area between $y = \sin(x)$ and the x-axis from $x = 0$ to $x = 2\pi$.