Test 3 - Review Solutions

1. a) $y' = 3x^2 - 4 + \frac{1}{2}(2x)^{-1/2}(2)$
$= 3x^2 - 4 + (2x)^{-1/2}$

b) $y' = 2(2x-5)(2)(x+3)^3 + (2x-5)^2 3(x+3)^2$
$= (2x-5)(x+3)^2\left(4(x+3) + 3(2x-5)\right)$
$= (2x-5)(x+3)^2(4x+12+6x-15)$
$= (2x-5)(x+3)^2(10x-3)$

c) $y' = \dfrac{-3(x^2-2) - (5-3x)(2x)}{(x^2-2)^2}$
$= (-3x^2+6+6x^2-10x)/(x^2-2)^2$
$= (3x^2-10x+6)/(x^2-2)^2$

d) $y' = 2\sin(4x)\cos(4x)\cdot 4$
$= 8\sin(4x)\cos(4x) \to 4\sin(8x)$

e) $1\cdot y^2 + x\cdot 2y\frac{dy}{dx} + 6y\frac{dy}{dx} = 1$

$\frac{dy}{dx}(2xy+6y) = 1-y^2$

$\frac{dy}{dx} = \frac{1-y^2}{2xy+6y}$

f) $y' = \sec^2(x^2-3)\cdot 2x = 2x\sec^2(x^2-3)$

g) $2\sin(x)\cos(x) = 3y^2\frac{dy}{dx} + 5$

$\frac{2\sin(x)\cos(x)-5}{3y^2} = \frac{dy}{dx}$

$\frac{\sin(2x)-5}{3y^2} = \frac{dy}{dx}$

h) $y' = \cos(3x) - 3x\sin(3x)$

i) $y' = 2e^{3x} + 2x\cdot 3\cdot e^{3x}$
$= 2e^{3x}(1+3x)$

j) $y' = \frac{1}{3x}\cdot\frac{3(2x-4) - \ln(3x)\cdot 2}{(2x-4)^2}$

$y' = \dfrac{\frac{2x-4}{x} - \frac{2\ln(3x)}{x}\cdot x}{(2x-4)^2}$

$= \dfrac{2x-4-2x\ln(3x)}{x(2x-4)^2}$

k) $y' = \frac{1}{\ln 4 \sin(5x)} \cdot \cos(5x) \cdot 5 = \frac{5\cot(5x)}{\ln 4}$

l) $y' = \frac{\ln 4 \cdot 4^{2x-5} \cdot 2 \cdot \tan(2x) - 4^{2x-5} \cdot \sec^2(2x) \cdot 2}{\tan^2(2x)}$

$= \frac{2 \cdot 4^{2x-5} (\ln 4 \cdot \tan(2x) - \sec^2(2x))}{\tan^2(2x)}$

2. a) $y' = -\sin(3x+5) \cdot 3 = -3\sin(3x+5)$

b) $y' = \frac{3\ln^2(x^2-4x+1) \cdot (2x-4)}{x^2-4x+1} = \frac{6(x-2)\ln^2(x^2-4x+1)}{x^2-4x+1}$

c) $y' = 3\left(\frac{1}{2}(\tan(2x))^{-1/2} \cdot \sec^2(2x) \cdot 2\right) = \frac{3\sec^2(2x)}{\sqrt{\tan(2x)}}$

d) $y' = 2x \cdot e^{2x} + x^2 \cdot e^{2x} \cdot 2 = 2xe^{2x}(1+x)$

e) $\cos(2\theta - 5) \cdot 2 + \frac{dw}{d\theta} = 0 \quad \frac{dw}{d\theta} = -2\cos(2\theta - 5)$

f) $y = 3x^{2x}$

$\ln y = \ln(3x^{2x})$

$\ln y = \ln 3 + 2x \ln x$

$\frac{1}{y}\frac{dy}{dx} = 0 + 2\ln x + 2x\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = 2y(\ln x + 1)$

$= 2 \cdot 3x^{2x}(\ln x + 1)$

$= 6x^{2x}(\ln x + 1)$

3. $y' = 10xe^{2x} + 5x^2e^{2x}\cdot 2$

$0 = 10xe^{2x}(1+x)$

$x=0 \quad e^{2x}=0 \quad 1+x=0$

DNE $\quad x=-1$

Decreasing: $x\in(-1,0)$

4. $y' = \frac{3\cdot 2x}{x^2+4} = \frac{6x}{x^2+4}$

$y'' = \frac{6(x^2+4) - 6x\cdot 2x}{(x^2+4)^2}$

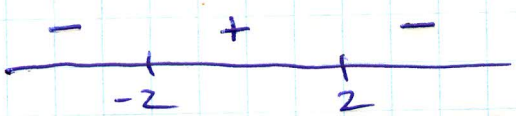
$0 = \frac{6x^2+24-12x^2}{(x^2+4)^2}$

$0 = 24-6x^2$

$6x^2 = 24$

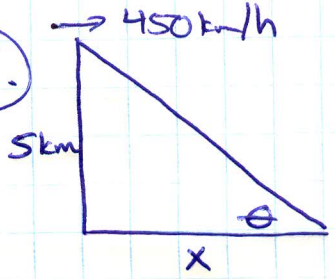
$x^2 = 4$

$x = \pm 2$



Concave up: $x\in(-2,2)$

5.



$\tan\left(\frac{\pi}{3}\right) = 5/x$

$x = 5/\sqrt{3}$ or $\frac{5\sqrt{3}}{3}$

$\tan\theta = \frac{5}{x}$ or $5x^{-1}$

$\sec^2\theta\frac{d\theta}{dt} = -5x^{-2}\frac{dx}{dt}$

$\sec^2\left(\frac{\pi}{3}\right)\frac{d\theta}{dt} = -5\left(\frac{5}{\sqrt{3}}\right)^{-2}(-450)$

$4\frac{d\theta}{dt} = 2250\left(\frac{3}{25}\right)$

$\frac{d\theta}{dt} = 67.5$ rads/hr

$= 1.125$ rads/min

$= 64.5°$/min

$= 1.07°$/sec

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\sec\frac{\pi}{3} = 2$

$\sec^2\frac{\pi}{3} = 4$

6. $V = l\times w\times h \qquad l = 4w$

$200 = 4w\times w\times h$

$h = \frac{50}{w^2} = 50w^{-2}$

$C = 2\cdot 2(\text{top/bottom}) + 1\cdot 2(\text{sides}) + 1\cdot 2(\text{ends})$

$= 2\cdot 2(4w\cdot w) + 2\cdot 4wh + 2\cdot wh$

$= 16w^2 + 10wh$

$= 16w^2 + 10w\left(\frac{50}{w^2}\right)$

$= 16w^2 + 500w^{-1}$

$\frac{dC}{dw} = 32w - 500w^{-2}$

$0 = 32w^3 - 500$

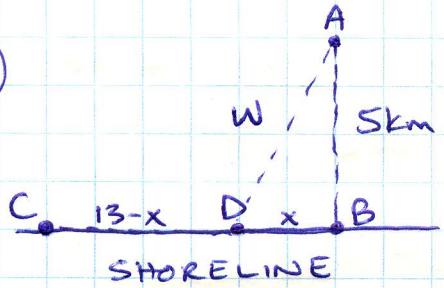
$w = 2.5$

$w = 2.5$ cm

$l = 10$ cm

$h = 8$ cm

7.



$CB = 13$ km

$CD = 13 - x$

$\sqrt{x^2+25} = w$

$x^2 + 5^2 = w^2$

$\text{Energy} = 13 - x + 1.3\sqrt{x^2+25}$

$\frac{dE}{dx} = -1 + 1.3\left(\frac{1}{2}\right)(x^2+25)^{-1/2} \cdot 2x$

$0 = -1 + \frac{1.3x}{\sqrt{x^2+25}}$

$\left(\sqrt{x^2+25}\right)^2 = (1.3x)^2$

$x^2 + 25 = 1.69x^2$

$25 = 0.69x^2$

$36.2\ldots = x^2$

$6\text{km} \doteq x$

$CD \doteq 7$ km

The birds will fly approx. 7 km along the shore, then 7.8 km over the water.

8.

$y' = 2xe^{2x} + x^2e^{2x} \cdot 2$

$= 2xe^{2x}(1+x)$

$x = 0 \quad e^{2x} = 0 \quad 1+x = 0$

DNE $\quad x = -1$

$y'' = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} + 4xe^{2x} + 2x^2 \cdot 2 \cdot e^{2x}$

$= 2e^{2x}(1 + 2x + 2x + 2x^2)$

$0 = 2e^{2x}(2x^2 + 4x + 1)$

$e^{2x} = 0$

DNE

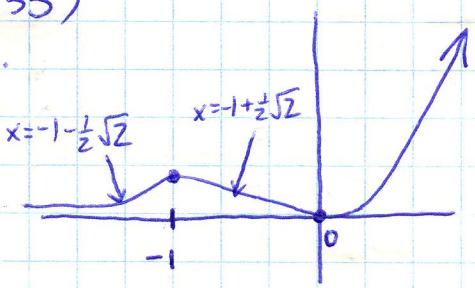
$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(1)}}{2(2)}$

$= \frac{-4 \pm \sqrt{8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4}$

$= -1 \pm \frac{1}{2}\sqrt{2}$

$(0,0)$ min

$(-1, 0.135)$ l.max.



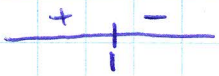
9.

$y' = e^{-x} + x \cdot e^{-x} \cdot (-1)$

$= e^{-x}(1 - x)$

$e^{-x} = 0 \quad 1 - x = 0 \quad (1, 0.368)$

DNE $\quad x = 1 \quad$ max



10.

i) Positive, since temp of yam is increasing.

ii) °F / min

iii) At 20 min, the temp is increasing 2°F / min.