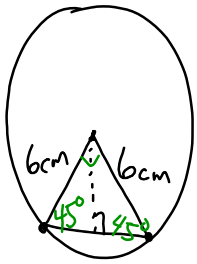
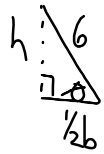
Optimization cont.



Find the area of the largest possible triangle.



$A = \frac{1}{2}bh$ $\qquad \tan\theta = \frac{h}{\frac{1}{2}b}$

$\sin\theta = \frac{h}{6} \rightarrow 6\sin\theta = h$

$\cos\theta = \frac{\frac{1}{2}b}{6} \rightarrow 12\cos\theta = b$

$A = \frac{1}{2}(12\cos\theta)(6\sin\theta)$

$A = 36\cos\theta\sin\theta$

$A = 18(2\cos\theta\sin\theta)$

$A = 18(\sin 2\theta)$

$\frac{dA}{d\theta} = 18\cos 2\theta \cdot 2$

$0 = 36\cos 2\theta$

$0 = \cos 2\theta$

$\frac{2\theta}{2} = \begin{cases} \pi/2 \div 2 \\ \frac{3\pi}{2} \div 2 \end{cases}$

$\theta = \begin{cases} \pi/4 \\ \cancel{3\pi/4} \end{cases}$

$A = 36\cos\theta\sin\theta$

$= 36\cos\frac{\pi}{4}\sin\frac{\pi}{4}$

$= 36\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= 36\left(\frac{2}{4}\right) = 18\text{cm}^2$

ex) Design the most economical open-top barrel $\bar{w}$ a volume of 20L (20000cm^3)

The cost of material for the bottom is 3 times the cost of material for the sides.

$$V = \pi r^2 h$$

$$20000 = \pi r^2 h$$

$$\frac{20000}{\pi r^2} = h$$

<u>cost</u>

$$\text{cost} = 3\pi r^2 + 2\pi r h \rightarrow$$

$$SA = \pi r^2 + 2\pi r h$$

$$SA = \pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{20000}{\cancel{\pi} r^{\cancel{2}}}\right)$$

$$SA = \pi r^2 + 40000 r^{-1}$$

$$\frac{dSA}{dr} = 6\pi r - 40000 r^{-2}$$

$$0 = 6\pi r - \frac{40000}{r^2} \quad (r^2)$$

$$0 = 6\pi r^3 - 40000$$

$$\sqrt[3]{\frac{40000}{6\pi}} = r = 12.85\text{cm}$$

$$h = \frac{20000}{\pi(12.85)^2}$$

$$= 38.55\text{cm}$$

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