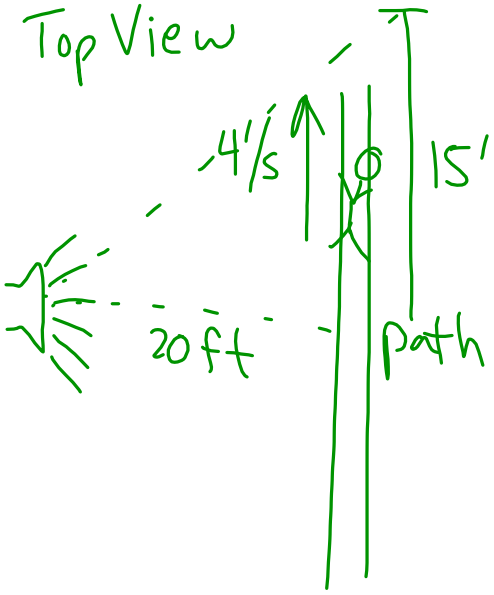
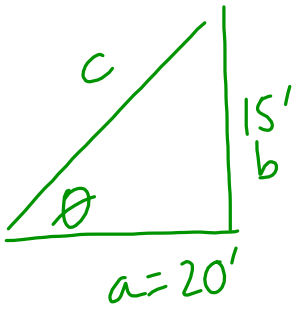
3. A searchlight is located on the ground, 20 feet from a straight path. A man is walking along the path, away from the light, at a speed of 4 feet/second. If the searchlight is to stay focused on the man, what is the rate that it must rotate when the man is 15 feet from the point on the path that is closest to the light?





$\frac{db}{dt} = 4$ ft/s

$b = 15$ ft

$a = 20$

$\frac{da}{dt} = 0$ ft/s

$\frac{d\theta}{dt} = ?$

$c^2 = 15^2 + 20^2$

$c^2 = 625$

$c = 25$

$\tan\theta = \frac{b}{20}$

$\sec\theta = \frac{1}{\cos\theta}$

$\frac{d}{dt}(20\tan\theta = b)$

$\sec\theta = \frac{hyp}{adj}$

$= \frac{25}{20}$

$= \frac{5}{4}$

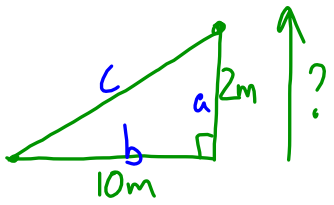
$20\sec^2\theta \frac{d\theta}{dt} = \frac{db}{dt}$

$20\left(\frac{5}{4}\right)^2 \frac{d\theta}{dt} = 4$

$\frac{d\theta}{dt} = \frac{4 \times 16}{20 \times 25}$

$\frac{d\theta}{dt} = \frac{16}{125}$ rads/sec $\xrightarrow{\times \frac{180}{\pi}}$ 7.3°/sec

4. A prison search light, located on the ground, rotates 3 times per minute. The fence is 10 meters away from the light at its closest point. Determine how quickly the light moves along the fence when it is 2 meters from the closest point to the light. Could you outrun the light?



3 rotations per min...?

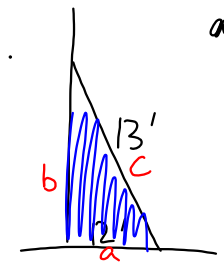
1 rotation per 20 sec.

$\frac{d\theta}{dt} = \boxed{6\pi \text{ rads/min}}$

$1080^\circ/\text{min}$

$\frac{da}{dt} = ?$

13.



$\frac{da}{dt} = 5'/\text{sec}$

a) $\frac{db}{dt} = ?$

$a^2 + b^2 = c^2$

$a^2 + b^2 = 169$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$

$a^2 + b^2 = c^2$

$12^2 + b^2 = 13^2$

$\vdots$

$b = 5$

$2(12)(5) + 2(5)\frac{db}{dt} = 0$

$\frac{db}{dt} = -12'/\text{sec}$

b) $\frac{dA}{dt} = ?$ $2xA = \frac{bh}{2} \times 2$

$2\frac{dA}{dt} = \frac{db}{dt}h + \frac{dh}{dt}b$

$= \frac{(5)(5) + (-12)(12)}{2}$

$= -\frac{119}{2} = -59.5\ ft^2/\text{sec}$

c) $\frac{d\theta}{dt} = ?$

$\sin\theta = \frac{b}{13}$

$13\sin\theta = b$

$13\cos\theta\frac{d\theta}{dt} = \frac{db}{dt}$

$\cos\theta = \frac{adj}{hyp}$

$= \frac{12}{13}$

$\frac{d\theta}{dt} = -\frac{12}{13} \times \frac{13}{12}$

$\frac{d\theta}{dt} = -1\ rads/\text{sec}.$