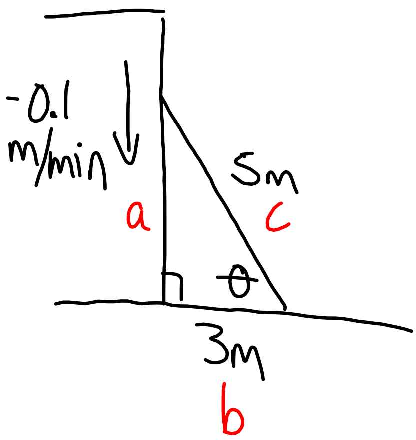
$1m = 100cm$

Related Rates Involving Trigonometry

1. A ladder that is 5 m long is sliding down a wall at a rate of 10cm/min. How quickly is the angle between the ground and the ladder changing when the base is 3 meters from the wall?



$$\frac{d\theta}{dt} = ?$$

$$a^2 + b^2 = c^2$$

$$a^2 + 3^2 = 5^2$$

$$a^2 = 25 - 9$$

$$\underline{a = +4}$$

$$\sin\theta = \frac{a}{c}$$

$$\sin\theta = \frac{a}{5}$$

$$\cos\theta = \frac{3}{5}$$

$\theta =$ not necessary

$$5\sin\theta = a$$

$$5\cos\theta \frac{d\theta}{dt} = \frac{da}{dt}$$

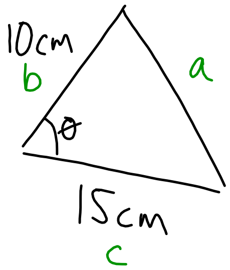
$$\cancel{5}\left(\frac{3}{\cancel{5}}\right)\frac{d\theta}{dt} = -0.1$$

$$\frac{d\theta}{dt} = \frac{-0.1}{3} = -\frac{1}{30} \text{ radians per min.}$$

To change to degrees, $\times \frac{180}{\pi}$

$$= \frac{-6^\circ}{\pi} / \text{min}$$

2. A triangle has two sides of constant length, 10 cm and 15 cm. The angle between these two sides is increasing at a rate of 9 degrees/minute. Find the rate at which the third side is growing when the angle is 60°. (Be sure to work in radians)



$\theta = 60° \to \pi/3$

$\frac{d\theta}{dt} = 9°/\text{min} \to \pi/20$

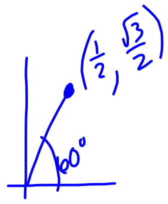
$\boxed{\frac{da}{dt} = ?}$

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

$$a^2 = 100 + 225 - 2(10)(15)\cos\theta$$

$$a^2 = 325 - 300\cos\theta$$

$$2a\frac{da}{dt} = 0 - 300(-\sin\theta)\frac{d\theta}{dt}$$



$$\frac{da}{dt} = \frac{300\sin\theta \, d\theta/dt}{2a}$$

$$= \frac{300(\sin\pi/3)(\pi/20)}{2(\sqrt{175})}$$

$a^2 = b^2 + c^2 - 2bc\cos\theta$

$a^2 = 325 - 300(\cos\pi/3)$

$a^2 = 325 - 300(1/2)$

$a^2 = 325 - 150$

$a = +\sqrt{175}$

$$= \frac{300(\sqrt{3}/2)(\pi/20)}{2(5\sqrt{7})}$$

$$= \frac{3\pi\sqrt{3}}{4\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{21}\pi}{28} \text{ cm/min}$$

$$\doteq 1.54 \text{ cm/min}$$