# Using Logarithmic Differentiation

variable$^{\text{constant}}$ power rule $y = x^5$ $\frac{dy}{dx} = 5x^4$

constant$^{\text{variable}}$ exponential $y = 5^x$ $\frac{dy}{dx} = 5^x \ln 5$

constant$^{\text{constant}}$ ? $y = 5^5$ $\frac{dy}{dx} = 0$

variable$^{\text{variable}}$ ? $y = x^x$ $\frac{dy}{dx} = ?$

$y = x^x \rightarrow \ln y = x \ln x$ (x)

$\log_x y = x$ $\frac{d}{dx}(\ln y = x \ln x)$

$\frac{1}{\ln e^y} \cdot \frac{dy}{dx} = (1)(\ln x) + x \frac{1}{\ln e \cdot x}$

$\frac{1}{y} \frac{dy}{dx} = \ln x + 1$ $y$

$\frac{dy}{dx} = y(\ln x + 1)$

$\frac{dy}{dx} = x^x(\ln x + 1)$

ex) Find $\frac{dy}{dx}$ of $y = x^{\sin(x)}$

$$\frac{d}{dx}(\ln y = \sin(x)\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \cos(x)\ln x + \sin(x)\frac{1}{x}$$

$$\frac{dy}{dx} = x^{\sin(x)}\left(\cos x \ln x + x^{-1}\sin(x)\right)$$

ex) Find $\frac{dy}{dx}$ for $y = (\sin x)^{x^2}$

$$\frac{d}{dx}\left(\ln y = x^2 \ln(\sin x)\right)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = 2x \ln \sin(x) + x^2 \cdot \frac{1}{\sin x} \cdot \cos x$$

$$= (\sin x)^{x^2}\left[2x \ln \sin(x) + x^2 \cot(x)\right]$$

ex) $y = x^{10}$ $\frac{dy}{dx} = 10x^9$

$\ln y = 10 \ln x$

$\frac{1}{y}\frac{dy}{dx} = 10 \cdot \frac{1}{x}$

$\frac{dy}{dx} = x^{10} \cdot \frac{10}{x}$

$\frac{dy}{dx} = 10x^9$

ex) $y = 2^x$ $\frac{dy}{dx} = \ln 2 \cdot 2^x$

$\ln y = x \ln 2$

$\frac{1}{y}\frac{dy}{dx} = \ln 2$

$\frac{dy}{dx} = \ln 2 \cdot 2^x$

ex) $y = \dfrac{(5x+1)^{10}(2x-5)^3}{(4x+7)^{14}}$ Find $\dfrac{dy}{dx}$

$\ln y = 10\ln(5x+1) + 3\ln(2x-5) - 14\ln(4x+7)$

$\dfrac{1}{y}\dfrac{dy}{dx} = 10\dfrac{1}{5x+1}\cdot 5 + 3\dfrac{1}{2x-5}\cdot 2 - 14\dfrac{1}{4x+7}\cdot 4$

$\dfrac{dy}{dx} = \left(\dfrac{50}{5x+1} + \dfrac{6}{2x-5} - \dfrac{56}{4x+7}\right) y$

ex) Find $\left.\frac{dy}{dx}\right|_{x=2}$ for $y=(5x+1)(2x-5)^4$

Using ln  Using Product & Chain  Using DRAWTAN

$$\frac{dy}{dx}=\left(\frac{5}{5x+1}+\frac{8}{2x-5}\right)y$$

$$\left.\frac{dy}{dx}\right|_{x=2}=-83$$

Practice:
pg. 170 #19, 20, 43-46
pg. 172 #13-16, 21