

Differentiating Logarithmic Functions

Find $\frac{dy}{dx}$ for $y = \log_b x$

Write $y = \log_b x$ in exp. form: $b^y = x$

Now differentiate (w.r.t. x): $\frac{b^y \cdot \ln b \cdot \frac{dy}{dx}}{b^y \cdot \ln b} = \frac{1}{b^y \cdot \ln b}$

Rule:

$$y = \log_b x$$

$$y' = \frac{1}{\ln b \cdot x} = \frac{1}{x \cdot \ln b}$$

$$\frac{dy}{dx} = \frac{1}{b^y \cdot \ln b}$$

since $x = b^y$ $\rightarrow \frac{1}{x \cdot \ln b}$

Differentiate:

$$a) y = \log_{10}(3x-4)$$

$$\begin{aligned} \frac{dy}{dx} &= \left(\frac{1}{\ln 10 \cdot (3x-4)} \right) (3) \\ &= \frac{3}{\ln 10 \cdot (3x-4)} \end{aligned}$$

$$b) y = \log_3(\sin x)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\ln 3 \cdot \sin x} \cdot \cos x \\ &= \frac{\cos x}{\ln 3 \cdot \sin x} = \frac{\cot x}{\ln 3} \\ &= \frac{1}{\ln 3 \cdot \tan x} \end{aligned}$$

$$c) y = \ln(5x^2 + 4) \rightarrow \log_e(5x^2 + 4)$$

$$y' = \frac{10x}{\ln e \cdot 5x^2 + 4} = \frac{10x}{5x^2 + 4}$$

$$d) y = x^2 \cdot \ln(2x)$$

$$y' = 2x \cdot \ln(2x) + x^2 \cdot \left(\frac{2}{\ln e \cdot 2x} \right) \quad \frac{1}{\ln e \cdot 2x} \quad 2$$

$$y' = 2x \ln(2x) + x$$

Find max/mins
of $y \dots$

$$0 = x(2 \ln(2x) + 1)$$

$$x=0 \quad 2 \ln(2x) + 1 = 0$$

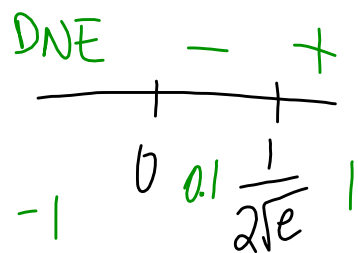
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$$\ln(2x) = -\frac{1}{2}$$

$$e^{-\frac{1}{2}} = 2x$$

$$\frac{1}{2\sqrt{e}} = x$$

min



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