Differentiating Logarithmic Functions

Find $\frac{dy}{dx}$ for $y = \log_b x$

Write $y = \log_b x$ in exp. form: $b^y = x$

Now differentiate (w.r.t. x): $\frac{b^y \cdot \ln b \cdot \frac{dy}{dx}}{b^y \cdot \ln b} = \frac{1}{b^y \ln b}$

$$\frac{dy}{dx} = \frac{1}{b^y \cdot \ln b}$$

since $x = b^y$

$$= \frac{1}{x \cdot \ln b}$$

Rule:

$y = \log_b x$

$y' = \frac{1}{\ln b \cdot x} = \frac{1}{x \cdot \ln b}$

Differentiate:

a) $y = \log_{10}(3x-4)$

$$\frac{dy}{dx} = \left(\frac{1}{\ln 10 \cdot (3x-4)}\right)(3)$$

$$= \frac{3}{\ln 10 (3x-4)}$$

b) $y = \log_3(\sin x)$

$$\frac{dy}{dx} = \frac{1}{\ln 3 \cdot \sin x} \cdot \cos x$$

$$= \frac{\cos x}{\ln 3 \cdot \sin x} = \frac{\cot x}{\ln 3}$$

$$= \frac{1}{\ln 3 \cdot \tan x}$$

c) $y=\ln(5x^2+4) \rightarrow \log_e(5x^2+4)$

$$y' = \frac{10x}{\cancel{\ln e}\cdot 5x^2+4} = \frac{10x}{5x^2+4}$$

d) $y = x^2 \cdot \ln(2x)$

$$\frac{1}{\ln e \cdot 2x} \quad 2$$

$$y' = 2x\cdot\ln(2x) + x^2\cdot\left(\frac{\cancel{2}}{\cancel{\ln e}\cdot\cancel{2x}}\right)$$

$$y' = 2x\ln(2x) + x$$

Find max/mins of y...

$$0 = x(2\ln(2x)+1)$$

$x=0$ $2\ln(2x)+1=0$

?

$$\ln(2x) = -1/2$$

$$e^{-1/2} = 2x$$

$$\frac{1}{2\sqrt{e}} = x$$

min

DNE $-$ $+$

-1 0 0.1 $\frac{1}{2\sqrt{e}}$ 1

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