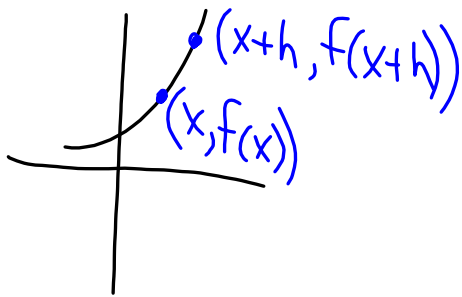


# Differentiating Exponential Functions

Let  $f(x) = 2^x$ . Find  $f'(x)$  by 1<sup>st</sup> principles.  
(Use the definition of the derivative)



$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2^{x+h} - 2^x}{(x+h) - x}\end{aligned}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2^x(2^h - 1)}{h}$$

$$f'(x) = 2^x \lim_{h \rightarrow 0} \frac{(2^h - 1)}{h} \quad \text{let } h = 0.000001$$

$$f'(x) = 2^x (0.6931...)$$

$$f'(x) = 2^x \ln 2$$

Now find  $f'(x)$  for  $f(x) = 3^x$

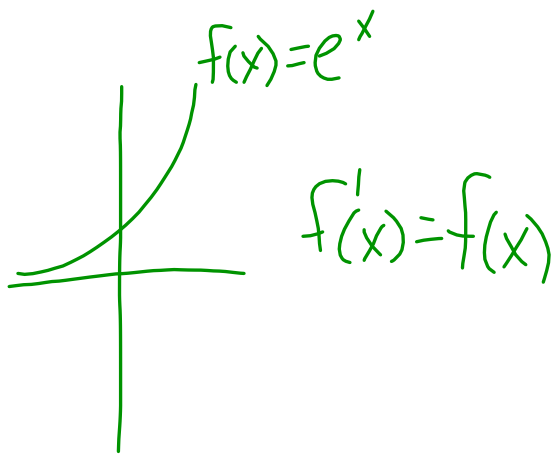
$$f'(x) = 3^x \left( \frac{3^h - 1}{h} \right) \underline{\underline{h \rightarrow 0}}$$

$$= 3^x (1.0986\dots)$$

$$= 3^x \ln 3$$

$$\text{Rule: } f(x) = a^x \rightarrow f'(x) = a^x \ln a$$

ex) Find  $f'(x)$  for  $f(x) = e^x$



$$\begin{aligned} f'(x) &= e^x \cdot \ln e \\ &= e^x (1) \\ &= e^x \end{aligned}$$

find  $\frac{dy}{dx}$  for the following:

a)  $y = 3^{\sin x}$  Think of  $3^{\square}$

$$y' = 3^{\sin x} \cdot \ln 3 \cdot \cos x$$

$$= \ln 3 \cdot 3^{\sin x} \cdot \cos x$$

b)  $y = 2^{5-x^2}$

$$y' = 2^{5-x^2} \cdot \ln 2 \cdot (-2x)$$

c)  $y = 50(e^{0.15x})$

$$y' = 50(e^{0.15x}) \cdot \ln e \cdot 0.15$$

$$y' = 7.5 e^{0.15x}$$

d)  $y = (15e)^{0.2x}$

$$y' = (15e)^{0.2x} \cdot \ln(15e) \cdot 0.2$$

$$= 0.74 \cdot (15e)^{0.2x}$$

e)  $y = \frac{6x+4}{e^{2x}}$

$$y' = \frac{6e^{2x} - (6x+4)e^{2x} \cdot \ln e \cdot 2}{(e^{2x})^2}$$

$$= \frac{2(e^{2x})(3 - (6x+4))}{e^{4x}}$$

$$= \frac{2(-6x-1)}{e^{2x}}$$

$$= e^{2x-4x}$$

$$= e^{-2x}$$

$$= \frac{1}{e^{2x}}$$

$$f) y = x^2 e^{2x}$$

$$y' = x^2 \cdot e^{2x} \cdot \cancel{\ln e \cdot 2} + 2xe^{2x}$$

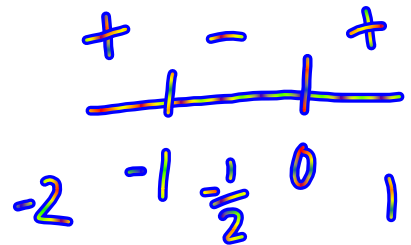
$$= 2x^2 e^{2x} + 2xe^{2x}$$

$$= 2xe^{2x}(x+1)$$

Where does  $y = x^2 e^{2x}$  have max/min?

Where  $y' = 0$ !

$$0 = 2xe^{2x}(x+1)$$



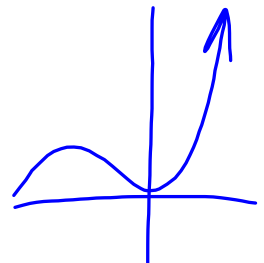
$$2x=0 \quad e^{2x}=0 \quad x+1=0$$

$$x=0 \quad \text{Never!}$$

$$x=-1$$

min

max



Ag. 170 #1-18 ☺