Coke Can Optimization

Have you ever considered whether that can of Coke that so many students handle on a daily basis is the “right” size? In other words, could a 355 ml aluminum can be made which would use less material?

This is a study involving volume and surface area, where we want to **minimize** the surface area for a given volume — in this case, 355 ml.

(a) Complete the following table by determining the radius for each height, then calculating the surface area of the can. Remember to include both the “side” and the top/bottom of the can, and that volume = 355 cm^3.

height	radius	total surface area
6 cm		
8 cm		
10 cm		
12 cm		

(b) Which of the 4 sizes above yields the smallest (most efficient) surface area? Make an educated guess as to what height might yield the minimum surface area.

(c) What is the formula for the total surface area of the can? Express this using r and h.

(d) We know that for a cylinder, $V = \pi r^2 h$. In order to isolate the variable **h** in the next step equation, solve the volume equation for **r**.

(e) Now, replace **r** in your surface area equation from part (c) using your answer to part (d). You should now have the equation for the surface area of a 355 ml can in terms of its height, **h**.

(f) Use your knowledge of calculus to find the value of **h** that makes the surface area a **minimum**. Be careful! Your derivative could get messy! It's OK to graph the derivative if you find it helpful, but this problem **can** (pardon the pun!) be solved algebraically.

(g) How tall **is** a Coke can, anyway? Did they choose an optimum size for the millions of containers that are manufactured every day?