Applying the Derivative

1. The memory recall of unrelated facts without review was tested over time. The following equation modelled the *average* marks of the students:

$$A(t) = 75 - 5\ln(t+1); t \geq 0$$

(t is the number of weeks since the facts were memorized, and A is the average mark, in %.)

a) What is the average mark when a student writes the test immediately?

$A(0) = 75 - 5\ln(0+1) = 75 - \cancel{5\ln(1)} = 75$

b) Find $A'(t)$ and explain what it means.

$A'(t) = 0 - 5\left(\frac{1}{\cancel{\ln e}\cdot t+1}\right) = \frac{-5}{t+1}$ $\quad \ln_e(t+1) \to \frac{1}{\ln e \cdot t+1}$

c) Determine how quickly the average mark is changing 4 weeks after the facts were memorized.

$A'(4) = \frac{-5}{4+1} = -1\%/\text{week}$ $\quad A'(1) = \frac{-5}{1+1} = -2.5$

d) When would the average mark reach 50%?

$50 = 75 - 5\ln(t+1)$

$-25 = -5\ln(t+1) \rightarrow 5 = \ln(t+1)$

$e^5 = t+1 \quad t = e^5 - 1$

$t = 147.4$ weeks

e) When is the memory loss **changing** at a rate of less than 0.5%?

$+0.5 = \frac{+5}{t+1}$ $\quad 0.5(t+1) = 5$

$t+1 = 10 \quad t = 9$ After 9 weeks.

2. The spread of a virus is modelled by $P(t) = \frac{800}{1+e^{6-t}}$ where t is the number of days since the first case(s) were diagnosed, and P is the number of people that have been infected.

a) How many people were infected initially? (t=0)

$$P(0) = \frac{800}{1+e^{6-0}} \doteq 1.978\ldots \rightarrow 2 \text{ people}$$

b) About how many people will ultimately be infected? (Find the limit as $t \to \infty$)

$$\lim_{t\to\infty} \frac{800}{1+e^{6-\infty}} = \frac{800}{1+e^{-\infty}} = \frac{800}{1+\frac{1}{e^{\infty}}} \rightarrow 0 = \frac{800}{1}$$

c) How fast is the virus spreading after 4 days?

$$P(t) = 800(1+e^{6-t})^{-1}$$

$$P'(t) = 800(-1(1+e^{6-t})^{-2}(e^{6-t}\cdot \cancel{\ln e} \cdot -1)$$

$$P'(4) = 800(1+e^2)^{-2}(e^2)$$

$$= \frac{800e^2}{(1+e^2)^2}$$

$$\doteq 84 \text{ ppl/day}$$

d) When will the virus be spreading to more than 20 people per day?

[Consider substituting $k = e^{6-t}$ and solving the quadratic, then replace k and solve the resulting two equations using natural logs.]

$$20 = \frac{800(e^{6-t})}{(1+e^{6-t})^2} \qquad 20 = \frac{800k}{(1+k)^2}$$

$$20(1+k)^2 = 800k$$

$$\frac{20(1+2k+k^2)}{20} = \frac{800k}{20}$$

$$k^2+2k+1 = 40k$$

$$k^2-38k+1=0$$

Factor? X

CTS? X

Graph?

Quad Form?

$$k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$k = 0.026\ldots$$

$$k = 37.97\ldots$$

$$k = e^{6-t} = 0.026\ldots$$

$$6-t \cancel{\ln e} = \ln 0.026\ldots$$

$$-t = \ln 0.026\ldots - 6$$

$$t = 9.64 \text{ days}$$

$$e^{6-t} = 37.97\ldots$$

$$6-t = \ln 37.97\ldots$$

$$t = 2.36 \text{ days}$$

More than 20...

$$20 < \frac{800e^{6-t}}{(1+e^{6-t})^2}$$

0	2.36	3	9.64	10
	−	+	−	

$$t \in (2.36, 9.64) \text{ days}$$

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+ Sheet