

Applying the Derivative

1. The memory recall of unrelated facts without review was tested over time. The following equation modelled the *average* marks of the students:

$$A(t) = 75 - 5 \ln(t + 1); t \geq 0$$

(t is the number of weeks since the facts were memorized, and A is the average mark, in %.)

a) What is the average mark when a student writes the test immediately?

$$A(0) = 75 - 5 \ln(0+1) = 75 - 5 \ln(1) = 75$$

b) Find $A'(t)$ and explain what it means.

$$A'(t) = 0 - 5 \left(\frac{1}{\ln e \cdot t+1} \right) = -\frac{5}{t+1} \quad \ln_e(t+1) \rightarrow \frac{1}{\ln e \cdot t+1}$$

c) Determine how quickly the average mark is changing 4 weeks after the facts were memorized.

$$A'(4) = -\frac{5}{4+1} = -1\%/\text{week} \quad A'(1) = -\frac{5}{1+1} = -2.5$$

d) When would the average mark reach 50%?

$$\begin{aligned} 50 &= 75 - 5 \ln(t+1) & 5 &= \ln(t+1) \\ -25 &= -5 \ln(t+1) & e^5 &= t+1 & t &= e^5 - 1 \\ & & & & t &= 147.4 \text{ weeks} \end{aligned}$$

e) When is the memory loss changing at a rate of less than 0.5%?

$$\begin{aligned} +0.5 &= +\frac{5}{t+1} & 0.5(t+1) &= 5 & t+1 &= 10 & t &= 9 & \text{After } 9 \text{ weeks.} \end{aligned}$$

2. The spread of a virus is modelled by $P(t) = \frac{800}{1+e^{6-t}}$ where t is the number of days since the first case(s) were diagnosed, and P is the number of people that have been infected.

a) How many people were infected initially? ($t=0$)

$$P(0) = \frac{800}{1+e^{6-0}} = 1.978... \rightarrow 2 \text{ people}$$

b) About how many people will ultimately be infected? (Find the limit as $t \rightarrow \infty$)

$$\lim_{t \rightarrow \infty} \frac{800}{1+e^{6-t}} = \frac{800}{1+e^{-\infty}} = \frac{800}{1+\frac{1}{e^{\infty}}} \rightarrow 0 = \frac{800}{1}$$

c) How fast is the virus spreading after 4 days?

$$P(t) = 800(1+e^{6-t})^{-1} \quad P'(4) = 800(1+e^2)^{-2}(e^2)$$

$$P'(t) = 800(-1)(1+e^{6-t})^{-2}(e^{6-t} \cdot -1) \quad \downarrow$$

$$= \frac{800e^2}{(1+e^2)^2}$$

d) When will the virus be spreading to more than 20 people per day?

[Consider substituting $k = e^{6-t}$ and solving the quadratic, then replace k and solve the resulting two equations using natural logs.]

$$20 = \frac{800(e^{6-t})}{(1+e^{6-t})^2} \quad 20 = \frac{800k}{(1+k)^2} \quad = \underline{84 \text{ ppl/day}}$$

$$20(1+k)^2 = 800k$$

$$\frac{20(1+2k+k^2)}{20} = \frac{800k}{20}$$

$$k^2 + 2k + 1 = 40k$$

$$k^2 - 38k + 1 = 0$$

$$e^{6-t} = 37.97...$$

$$6-t = \ln 37.97...$$

$$t = 2.36 \text{ days}$$

Factor? \times $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

CTS? \times

Graph? $k = 0.026...$

Quad Form? $k = 37.97...$

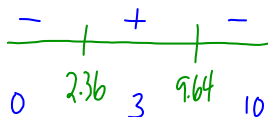
$$k = e^{6-t} = 0.026...$$

$$6-t = \ln 0.026...$$

$$-t = \ln 0.026... - 6$$

$$t = 9.64 \text{ days}$$

More than 20...



$$20 < \frac{800e^{6-t}}{(1+e^{6-t})^2}$$

$$t \in (2.36, 9.64) \text{ days}$$

pg. 173-175 #61, 71-73, 78
+ Sheet