1. $y = 2x^3 + 6x^2 - 48x - 4$

$\frac{dy}{dx} = 6x^2 + 12x - 48$

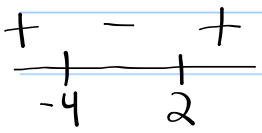
$\frac{dy}{dx} = 0$ when $6x^2 + 12x - 48 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$ $x = 2$

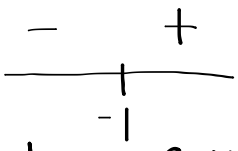
max min



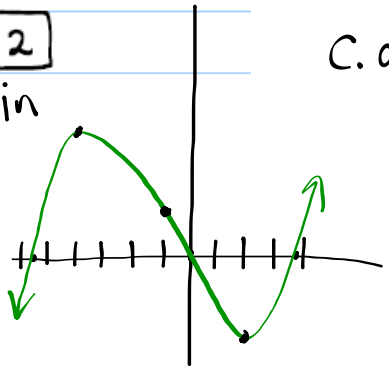
$\frac{d^2y}{dx^2} = 12x + 12$

$0 = 12x + 12$

$x = -1$



C. down C. up



2. a) $g'(x) = \frac{4}{3}x^{1/3}(3x+6) + x^{4/3}(3)$

$= x^{1/3}[4/3(3x+6) + 3(x)']$

$= x^{1/3}[4x + 8 + 3x]$

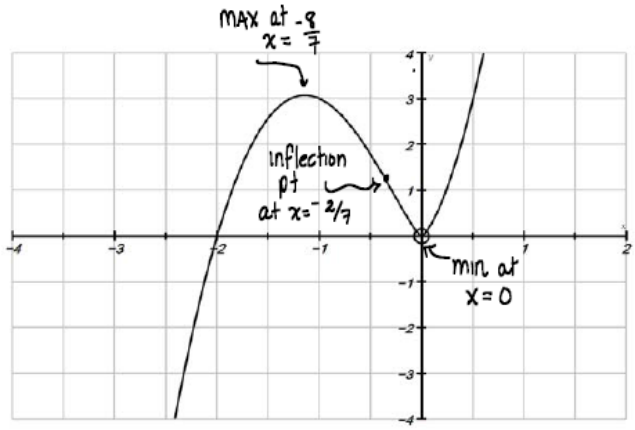
$= x^{1/3}[7x + 8]$

$g'(x) = 0$ when $x = 0$ and $x = -8/7$

b) $g''(x) = (\frac{1}{3})x^{-2/3}(7x+8) + x^{1/3}(7)$ $\quad \therefore g''(x) = 0$ when $x = -2/7$

$= x^{-2/3}[1/3(7x+8) + 7x']$

$= \frac{7/3x + 8/3 + 7x}{x^{2/3}} = \frac{28/3x + 8/3}{x^{2/3}}$



a) Increasing on intervals $(-\infty, -8/7) \cup (0, +\infty)$

b) Concave up on interval $(-2/7, +\infty)$

3. $\frac{dV}{dt} = 5\ cm^3/min$

find $\frac{dr}{dt}$ when $r = 10\ cm$

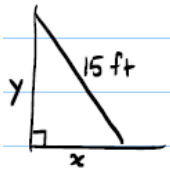
$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$5 = 4\pi (10)^2 \frac{dr}{dt}$

$\frac{5}{400\pi} = \frac{1}{80\pi}\ cm/min = \frac{dr}{dt}$

4.



$x^2 + y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dx}{dt} = -\frac{1}{4}$ ft/s

$2(7)\left(-\frac{1}{4}\right) + 2(\sqrt{176})\frac{dy}{dt} = 0$

$-3.5 + 2\sqrt{176}\frac{dy}{dt} = 0$

find $\frac{dy}{dt}$ when $\begin{cases} t = 12 \text{ sec.} \\ x = 10 - \frac{1}{4}(12) = 7 \text{ ft} \end{cases}$

$\frac{dy}{dt} = \frac{3.5}{2\sqrt{176}} \approx 0.132$ ft/sec

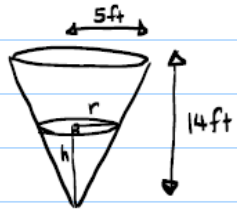
$x^2 + y^2 = 15^2$

$7^2 + y^2 = 15^2$

$y^2 = 176$

$y = \sqrt{176} \approx 13.3$

5.



$\frac{dV}{dt} = -2 \text{ ft}^3/\text{hr}$

Find $\frac{dh}{dt}$ when $h = 6\text{ft}$

$\frac{14}{5} = \frac{h}{r}$

$14r = 5h$

$r = \frac{5}{14}h$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{5}{14}h\right)^2 h$

$= \frac{1}{3}\pi\left(\frac{25}{196}h^2\right)h$

$V = \frac{25}{588}\pi h^3$

$\frac{dV}{dt} = \frac{25}{196}\pi h^2 \frac{dh}{dt}$

$-2 = \frac{25}{196}\pi(6)^2 \frac{dh}{dt}$

$\frac{-98}{225\pi}$ ft/hr $= \frac{dh}{dt}$

6.

$\frac{dr}{dt} = -3$ cm/min

find $\frac{dV}{dt}$ when $r = 20$ cm

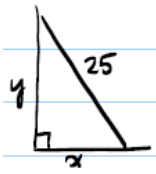
$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi (20)^2 (-3)$

$\frac{dV}{dt} = -4800\pi$ cm^3/min

7.



$x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(20)(3) + 2(15)\frac{dy}{dt} = 0$

$30\frac{dy}{dt} = 120$

$\frac{dy}{dt} = 4$ ft/sec

$\frac{x}{t} = 3$ ft/sec

find $\frac{dy}{dt}$ when $\begin{cases} x = 20 \\ y = 15 \end{cases}$

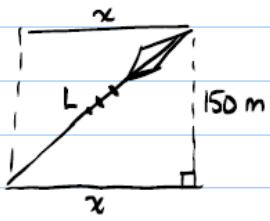
$x^2 + y^2 = 25^2$

$20^2 + y^2 = 25^2$

$y^2 = 225$

$y = 15$

8.



$x^2 + 150^2 = L^2$

$2x\frac{dx}{dt} = 2L\frac{dL}{dt}$

$2(200)(5) = 2(250)\frac{dL}{dt}$

$\frac{dx}{dt} = 5\ m/s$

$\frac{2000}{500} = \frac{dL}{dt}$

find $\frac{dL}{dt}$ when $\begin{cases} L = 250m \\ x = 200m \end{cases}$

$4\ m/s = \frac{dL}{dt}$

9.a)

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$= \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = 7\ ft^3/min$

$r = \frac{1}{2}h$

find $\frac{dh}{dt}$ when $h = 2ft$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$7 = \frac{1}{4}\pi (2)^2 \frac{dh}{dt}$

$\frac{7}{\pi}\ ft/min = \frac{dh}{dt}$

b) $\frac{dV}{dt} = 7\ ft/sec - 2\ ft/sec$

$= 5\ ft/sec$

$r = \frac{1}{2}h$

find $\frac{dh}{dt}$ when $h = 2$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$5 = \frac{1}{4}\pi (2)^2 \frac{dh}{dt}$

$\frac{5}{\pi}\ ft/min = \frac{dh}{dt}$

$r = \frac{1}{2}h$

$\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$

$\therefore \frac{dr}{dt} = \frac{5}{2\pi}\ ft/min$