Related Rate Problems

ex) An oil spill spreads out in a circular pattern, whose radius increases at a constant rate of 0.8 m/s. How fast is the area of the spill increasing when the radius is 20 m?

$A = \pi r^2$ Want $\frac{dA}{dt}$

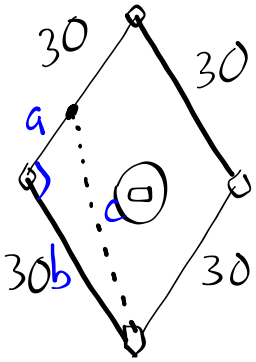
$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$ Have $\frac{dr}{dt} = 0.8 \text{ m/s}$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi(20)(0.8)$$

$$= 32\pi = 100.5 \text{ m}^2/\text{sec.}$$

ex)



The runner runs at a constant speed of 10m/s. When he is 6m from 3rd, how fast is his distance from home changing?

$c^2 = a^2 + b^2$

$\frac{d}{dt}(c^2) = \frac{d}{dt}(a^2 + 30^2)$

$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 0$

Want: $\frac{dc}{dt}$ Have: $\frac{da}{dt} = -10 m/s$

$a = 6m$

Find c when a=6...

$c^2 = a^2 + b^2$

$c^2 = 6^2 + 30^2$

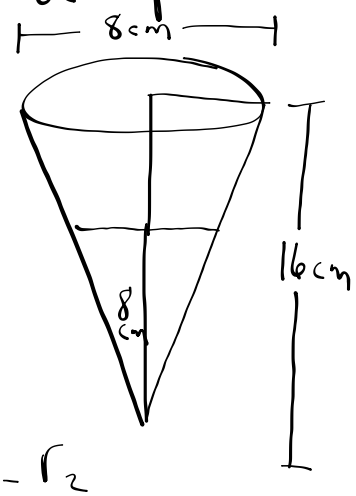
$c^2 = 936$

$c = 30.6 m$

$\frac{2(\sqrt{936})\frac{dc}{dt}}{2\sqrt{936}} = \frac{2(6)(-10)}{2\sqrt{936}}$

$\frac{dc}{dt} = -1.96 m/s$

ex) Liquid is poured into a cone at a rate of $2cm^3/min$. At what rate is the depth changing when it is 8cm deep?



$V = \frac{1}{3}\pi r^2 h$

Want: $\frac{dh}{dt}$ Have: $\frac{dV}{dt} = 2$

$h = 8cm$

Can we write r in terms of

$\frac{r_1}{h_1} = \frac{r_2}{h_2}$

$\frac{4}{16} = \frac{r}{h}$

$r = \frac{1}{4}h$

$V = \frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h$

$V = \frac{1}{48}\pi h^3$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{1}{48}\pi h^3\right)$

$\frac{dV}{dt} = (3)\left(\frac{1}{48}\pi\right)(h^2)\frac{dh}{dt}$

$2 = \frac{1}{16}\pi(8)^2\frac{dh}{dt}$

$\frac{dh}{dt} = 0.16\ cm/min$

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