Implicit Differentiation

Sometimes we have to differentiate equations that have more than one variable. These are often not functions.

ex) Differentiate $x^2+y^2=1$.

$\frac{dy}{dx}$ means "the derivative of y with respect to x"

$\frac{d}{dx}$ means "take the derivative of _ with respect to x"

ex) Diff. $x^2+y^2=1$ (Find $\frac{dy}{dx}$)

$\frac{d}{dx}(x^2+y^2=1)$

$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(1)$

$\frac{d}{dx}x^2+\frac{d}{dx}y^2=0$

$2x+2y\frac{dy}{dx}=0$

$-2x \qquad -2x$

$\frac{2y\frac{dy}{dx}}{2y}=\frac{-2x}{2y}$

$\frac{dy}{dx}=-\frac{x}{y}$

Find the slope of some tangent

$\frac{dy}{dx}=-\frac{x}{y}=-\frac{0}{1}=0$

= undefined

ex) Find $\frac{dy}{dx}$ for $x^2 + xy + y^2 = 3.75$

$$\frac{d}{dx}\left(x^2 + xy + y^2\right) = \frac{d}{dx}(3.75)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = 0$$

$$2x + \left(1 \cdot y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$\frac{\frac{dy}{dx}\left(x + 2y\right)}{(x + 2y)} = \frac{-2x - y}{x + 2y}$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

③ Find $\frac{dy}{dx}$ for $x + \frac{x}{y} - y^3 = 4\frac{3}{8}$

xy^{-1}

$$\frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{x}{y}\right) - \frac{d}{dx}(y^3) = \frac{d}{dx}(4\tfrac{3}{8})$$

$$1 + \frac{(1\cdot y - x\cdot\frac{dy}{dx})}{y^2} - 3y^2\frac{dy}{dx} = 0 \quad (\times y^2)$$

$$y^2 + y - x\frac{dy}{dx} - 3y^4\frac{dy}{dx} = 0$$

$$y^2 + y = x\frac{dy}{dx} + 3y^4\frac{dy}{dx}$$

$$y^2 + y = \frac{dy}{dx}(x + 3y^4)$$

$$\frac{y^2+y}{x+3y^4} = \frac{dy}{dx}$$

Read pg. 149-150 incl. ex1 & ex2

pg. 155 #9-12, 37a, 41, 42