

# Implicit Differentiation

Sometimes we have to differentiate equations that have more than one variable. These are often not functions.

ex) Differentiate  $x^2 + y^2 = 1$ .

$\frac{dy}{dx}$  means "the derivative of  $y$  with respect to  $x$ "

$\frac{d}{dx}$  means "take the derivative of  $\_$  with respect to  $x$ "

ex) Diff.  $x^2 + y^2 = 1$  (Find  $\frac{dy}{dx}$ )

$$\frac{d}{dx}(x^2 + y^2 = 1)$$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1)$$

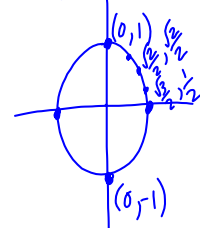
$$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = 0$$

$$\begin{matrix} 2x & + & 2y \frac{dy}{dx} & = & 0 \\ -2x & & & & -2x \end{matrix}$$

$$\frac{2y \frac{dy}{dx}}{2y} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

"unit circle"



Find the slope of some tangent

$$\frac{dy}{dx} = \frac{-x}{y} = \frac{-0}{1} = 0$$

= undefined

ex) Find  $\frac{dy}{dx}$  for  $x^2 + xy + y^2 = 3.75$

$$\frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}(3.75)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = 0$$

$$2x + (1 \cdot y + x \frac{dy}{dx}) + 2y \frac{dy}{dx} = 0$$

$$2x + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} \frac{(x + 2y)}{(x + 2y)} = \frac{-2x - y}{x + 2y}$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

③ Find  $\frac{dy}{dx}$  for  $x + \frac{x}{y} - y^3 = 4\frac{3}{8}$

$xy^{-1}$

$$\frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{x}{y}\right) - \frac{d}{dx}(y^3) = \frac{d}{dx}\left(4\frac{3}{8}\right)$$

$$y^2 \left( 1 + \frac{y^2}{y^2} \left( 1 - y - x \cdot \frac{dy}{dx} \right) - 3y^2 \frac{dy}{dx} \right) = 0 \cdot y^2$$

$$y^2 + y - x \frac{dy}{dx} - 3y^4 \frac{dy}{dx} = 0$$

$$y^2 + y = x \frac{dy}{dx} + 3y^4 \frac{dy}{dx}$$

$$y^2 + y = \frac{dy}{dx} (x + 3y^4)$$

$$\frac{y^2 + y}{x + 3y^4} = \frac{dy}{dx}$$

Read pg. 149-150 incl. ex1 & ex2

pg. 155 #9-12, 37a, 41, 42