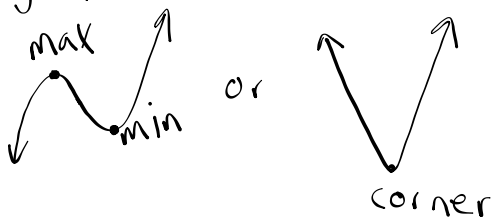
Critical Points

Consider the graph of a continuous function such as



critical points are where the function has either $\frac{dy}{dx} = 0$ or $\frac{dy}{dx} = DNE$

ex) $y = (2x+1)^{10}(5x+6)^8$ find all critical values

$y' = 10(2x+1)^9(2)(5x+6)^8 + (2x+1)^{10}8(5x+6)^7(5)$

$0 = 20(2x+1)^9(5x+6)^8 + 40(2x+1)^{10}(5x+6)^7$

$0 = 20(2x+1)^9(5x+6)^7\left[(5x+6) + 2(2x+1)\right]$

$4x+2$

$0 = 20(2x+1)^9(5x+6)^7(9x+8)$

$0 = (2x+1)^9 \quad 0 = (5x+6)^7 \quad 0 = 9x+8$

$x = -1/2 \qquad x = -6/5 \qquad x = -8/9$

These tell us the location of the critical pts.

To get the value of each C.P., sub each x into the original function.

$y = (2x+1)^{10}(5x+6)^8$

$(-1/2, 0) \quad (-6/5, 0) \quad (-8/9, 2.7774\ldots)$

ex) $y = (3x+1)(x+1)^{1/3}$ Find critical points

$$y' = 3(x+1)^{1/3} + (3x+1)\frac{1}{3}(x+1)^{-2/3}(1)$$

$$0 = (x+1)^{-2/3}\left[3(x+1)^{1} + \frac{1}{3}(3x+1)\right]$$

$3x+3 \quad +x+1/3$

$$0 = (x+1)^{-2/3}(4x+10/3)$$

$$0 = \frac{4x+10/3}{(x+1)^{2/3}} \qquad 0 = 4x+10/3 \qquad \text{DNE } (x+1)^{2/3}=0$$

$$x = -5/6 \qquad x = -1$$

What happens when we put these x-values into the original function?

$(-5/6, -0.8333\ldots)$ $(-1, 0)$

ex) $y = (3+x)(x^{2/3})$

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$$\frac{dy}{dx} = 1(x^{2/3}) + (3+x)(\tfrac{2}{3}x^{-1/3})$$

$$= x^{-1/3}\left(1x + (3+x)(\tfrac{2}{3})\right)$$

$2 + \frac{2}{3}x$

$$0 = \frac{\frac{5}{3}x+2}{x^{1/3}}$$

$\frac{5}{3}x + 2 = 0$

$x = -6/5$ (max or min)

$x^{1/3} = 0$ corner cusp inf. tan

$x = 0$

$(-6/5, 2.0\ldots)$ $(0, 0)$