Functions and Implicit Differentiation (ANS)

Note Title 21/03/2012

1) $y^2 = 2x^2 - x^3$

$2y\frac{dy}{dx} = 4x\frac{dx}{dx} - 3x^2\frac{dx}{dx}$

$2y\frac{dy}{dx} = 4x - 3x^2$

$\frac{dy}{dx} = \frac{4x-3x^2}{2y}$

$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=1}} = \frac{4(1)-3(1)^2}{2(1)}$

$= \frac{4-3}{2} = \frac{1}{2}$

$\therefore \frac{1}{2} = \frac{y-1}{x-1}$

$\frac{1}{2}(x-1) + 1 = y$

2) $2x\frac{dx}{dx}y^2 + x^2\left(2y\frac{dy}{dx}\right) = 2(y+1)^1\frac{dy}{dx}(4-y^2) + (y+1)^2\left(-2y\frac{dy}{dx}\right)$

$2xy^2 + 2x^2y\frac{dy}{dx} = 2(y+1)(4-y^2)\frac{dy}{dx} - 2y(y+1)^2\frac{dy}{dx}$

At $\left.\begin{matrix}x=0\\y=-2\end{matrix}\right\}$

$2(0)(-2)^2 + 2(0)^2(-2)\frac{dy}{dx} = 2(-2+1)(4-4)\frac{dy}{dx} - 2(-2)(-2+1)^2\frac{dy}{dx}$

$0 + 0 = 0 + 4\frac{dy}{dx}$

$4\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$

$0 = \frac{y+2}{x-0}$

$0(x-0) - 2 = y$

$y = -2$

3 a) $2y\frac{dy}{dx} = 3x^2\frac{dx}{dx} + 6x\frac{dx}{dx}$

$2y\frac{dy}{dx} = 3x^2 + 6x$

$\frac{dy}{dx} = \frac{3x^2+6x}{2y}$

$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=-2}} = \frac{3(1)^2+6(1)}{2(-2)} = \frac{3+6}{-4} = \frac{9}{-4}$

$-\frac{9}{4} = \frac{y+2}{x-1}$

$-\frac{9}{4}(x-1) - 2 = y$

b) $\frac{dy}{dx} = \frac{3x^2+6x}{2y}$

$\frac{dy}{dx} = 0$ when $3x^2+6x=0$

$3x(x+2)=0$

$x=0 \quad x=-2$

$y^2 = x^3+3x^2$

$y^2 = (0)^3+3(0)^2$

$y^2=0$

$y=0$

Point: $(0,0)$

But at $y=0$
$\frac{dy}{dx}$ DNE so this is
<u>NOT</u> a solution

$y^2 = x^3+3x^2$

$y^2 = (-2)^3+3(-2)^2$

$y^2 = -8+12$

$y^2=4$

$y = \pm 2$

Points: $(-2,2)$ and $(-2,-2)$

4) $x^2y^2+xy=2$

$2x\frac{dx}{dx}y^2 + x^2(2y\frac{dy}{dx}) + \frac{dx}{dx}y + x\frac{dy}{dx} = 0$

$2xy^2 + 2x^2y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x^2y+x) = -2xy^2-y$

$\frac{dy}{dx} = \frac{-2xy^2-y}{2x^2y+x}$

$\frac{dy}{dx} = -1$ when $\frac{-2xy^2-y}{2x^2y+x} = -1$

$-2xy^2-y = -2x^2y-x$

$-y(2xy+1) = -x(2xy+1)$

$-y=-x$

$y=x$

But $x^2y^2+xy=2$
for all values

<u>So:</u> substitution
of $y=x$ yields:

$x^2(x)^2+x(x)=2$

$x^4+x^2=2$

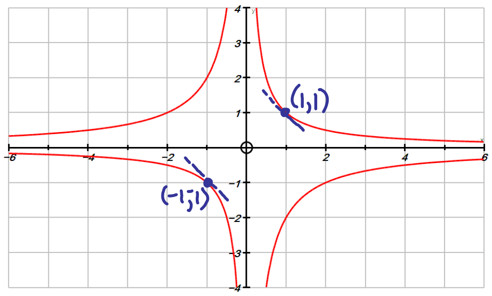
$x^4+x^2-2=0$

$(x^2+2)(x^2-1)=0$

$\cancel{x^2=-2} \quad x^2=1$

$\boxed{x=\pm 1}$

$\therefore$ Slope of -1 at $(1,1)$ and $(-1,-1)$


(1,1)
(-1,-1)
4
3
2
1
-1
-2
-3
-4
-6
-4
-2
2
4
6