Tangents & Normals

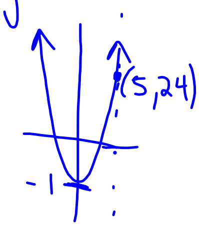
If you are finding the derivative (slope of a tangent) of a polynomial, the power rule can be used.

$$\boxed{\text{If } y = ax^n \text{ then } y' = anx^{n-1}}$$

ex) Find the slope (and then the equation) of the tangent to $y = x^2 - 1$ at $(5, 24)$.

$y = x^2 - 1$

$y' = 2x$



slope at $(5, 24)$

$2(5) = 10$.

Method 1

$y = mx + b$

$24 = 10(5) + b$

$24 = 50 + b$

$-50 \quad -50$

$-26 = b$

$y = 10x - 26$

Method 2

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$(x_2 - x_1)m = y_2 - y_1$

Let $(5, 24)$ be (x_1, y_1)

$\boxed{(x - 5)10 = y - 24}$

$10x - 50 = y - 24$

$10x - 26 = y$

$y_1 = x^2 - 1$

$y_2 = 10x - 26$

GRAPH & TABLE

x	y_1	y_2
5	24	24

ex) Find the equation of the tangent line

to $y = x^2 - 5x - 6$ at $x = 2$. $(2, -12)$

$y' = 2x - 5$

slope @ $x = 2$ $\quad 2(2) - 5 = -1$

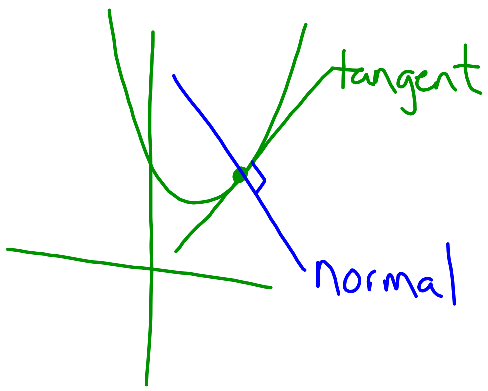
$m(x - x_1) = y - y_1$

$-1(x - 2) = y + 12$

$-x + 2 = y + 12$

$-x - 10 = y$

Normal lines are perpendicular to tangent lines at the point of intersection.



Find the equation of the normal line in the previous example.

$m_{tan} = -1 \quad \therefore m_{norm} = 1$

(neg. reciprocals)

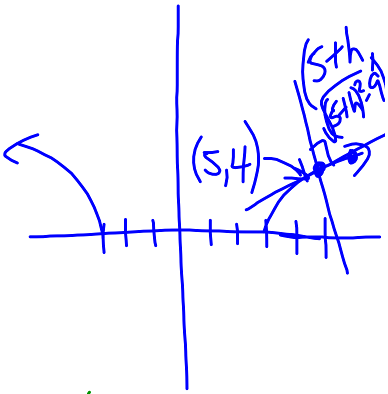
$m(x - x_1) = y - y_1$

$1(x - 2) = y + 12$

$x - 2 = y + 12$

$x - 14 = y$

ex) Find the equation of the normal line to $y=\sqrt{x^2-9}$ at x=5.



$$m_{tan} = \lim_{h\to 0} \frac{\sqrt{(5+h)^2-9}-4}{5+h-5}$$

$$= \lim_{h\to 0} \frac{\sqrt{h^2+10h+16}-4}{h} \cdot \frac{\sqrt{h^2+10h+16}+4}{\sqrt{h^2+10h+16}+4}$$

$$= \lim_{h\to 0} \frac{h^{\cancel{2}}+10\cancel{h}+\cancel{16}-\cancel{16}}{\cancel{h}\left(\sqrt{h^2+10h+16}+4\right)}$$

$$m_{norm} = -4/5$$

$$= \lim_{h\to 0} \frac{h+10}{\sqrt{h^2+10h+16}+4}$$

$$= \frac{10}{\sqrt{16}+4}$$

$$= 10/8 = 5/4$$

$$m(x-x_1)=y-y_1$$

$$-\frac{4}{5}(x-5)=y-4$$

$$-\frac{4}{5}x+4=y-4$$

$$-\frac{4}{5}x+8=y$$

(5,4) intersect.

- pg. 88 #10bc, #11bc
- Worksheet
- pg. 101 #1-6