Calculus Test 1 Solutions

Note Title 3/6/2009

1) $\lim_{x\to 5^-} f(x) = (5)^2 - 25 = 0$

$\lim_{x\to 5^+} f(x) = 5 - a$

$\therefore 5 - a = 0$

$\boxed{5 = a}$

2) $g(x) = 5x^2 - 3x + 2$

$g'(x) = 10x - 3$

$g'(2) = 20 - 3 = 17$

$g(2) = 5(2)^2 - 3(2) + 2 = 16$

Tangent line: slope $= 17$

Normal line: slope $= \frac{-1}{17}$

$\therefore -\frac{1}{17} = \frac{y-16}{x-2}$

$\boxed{-\frac{1}{17}(x-2) + 16 = y}$

3) see below to the right

4) $y = ax^2 + bx + c$

(x, y)

$(7,0)$ $0 = 49a + 7b + c$

$(5,4)$ $4 = 25a + 5b + c$

$\frac{dy}{dx} = 2ax + b$

(x, y')

$(5,1)$ $\boxed{1 = 10a + b}$

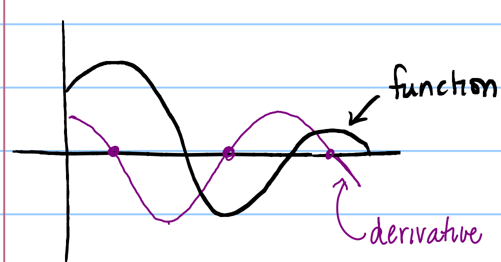
$$\therefore \begin{pmatrix} 49 & 7 & 1 \\ 25 & 5 & 1 \\ 10 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$$

$a = -\frac{3}{2}$ $b = 16$ $c = \frac{-77}{2}$

Equation:

$y = \frac{-3}{2}x^2 + 16x - \frac{77}{2}$

5)



6a) 1 b) -2 c) DNE since $\lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x)$ d) 1

3 a) $\lim_{x\to 4} \frac{x(x-4)}{x-4} = \lim_{x\to 4}(x) = 4$

b) $\lim_{x\to 9} \frac{\sqrt{x} - 3}{(x+9)(x-9)} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$

$= \lim_{x\to 9} \frac{x-9}{(x+9)(x-9)(\sqrt{x}+3)}$

$= \lim_{x\to 9} \frac{1}{(x+9)(\sqrt{x}+3)} = \frac{1}{(18)(6)} = \frac{1}{108}$

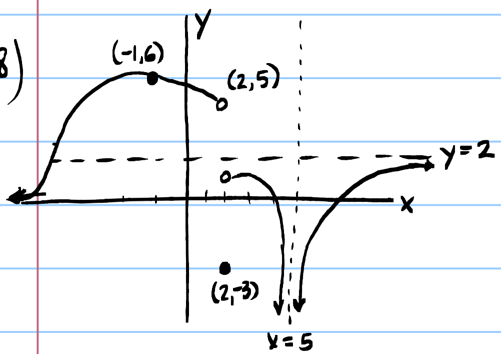
7 a) $\lim_{x\to\infty} \dfrac{\frac{5x^3}{x^2} - \frac{6x}{x^2} - \frac{3}{x^2}}{\frac{4x^2}{x^2} + \frac{9}{x^2}} = \lim_{x\to\infty} \frac{5x}{4} = +\infty$

b) $\lim_{x\to 0} \frac{2\sin(5x)}{3x} = \lim_{x\to 0} \frac{2(5x)}{3x} = 10/3$

7c) $\lim\limits_{x\to-\infty} \frac{|x+5|}{x-2} = \lim\limits_{x\to-\infty} \frac{-(x+5)}{x+2} = \lim\limits_{x\to-\infty} \frac{-\frac{x}{x}-\frac{5}{x}}{\frac{x}{x}+\frac{2}{x}} = -1$

d) $\lim\limits_{x\to 0} \frac{\sin^3(5x)}{8x^3} = \lim\limits_{x\to 0} \frac{[\sin(5x)]^3}{8x^3} = \lim\limits_{x\to 0} \frac{(5x)^3}{8x^3} = \frac{125}{8}$

8)



9a) $\lim\limits_{x\to 2} \frac{(x+2)(x-2)}{(2-x)(4+2x+x^2)} = \lim\limits_{x\to 2} \frac{-(x+2)}{4+2x+x^2} = \frac{-4}{12} = -1/3$

b) $\lim\limits_{x\to 3} \frac{\sqrt{x-1}-\sqrt{2}}{x-3} \cdot \frac{\sqrt{x-1}+\sqrt{2}}{\sqrt{x-1}+\sqrt{2}} = \lim\limits_{x\to 3} \frac{x-1-2}{(x-3)(\sqrt{x-1}+\sqrt{2})}$

$= \lim\limits_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x-1}+\sqrt{2})} = \lim\limits_{x\to 3} \frac{1}{\sqrt{x-1}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

c) $\lim\limits_{x\to 8} \frac{x+1}{\sqrt[3]{x}+1} = \frac{9}{3} = 3$

d) $\lim\limits_{x\to 5} \frac{(x-5)(x^2+1)}{x(x-5)(2x+1)} = \lim\limits_{x\to 5} \frac{(x^2+1)}{x(2x+1)} = \frac{26}{55}$

10) $\left.\begin{array}{l} \lim\limits_{x\to 1^+} (3x+5) = 8 \\ \lim\limits_{x\to 1^-} (kx^2-2x+4) = k-2+4 \end{array}\right\}$ To be continuous $8 = k-2+4$

$6 = k$

11a) $17 = \frac{y-16}{x-2}$ b) Already done (#2)

$17(x-2)+16 = y$

12 a) $d = 31.8t^2$ cm

$v = \frac{dd}{dt} = 63.6t$ cm/sec

$v(4) = d'(4) = 254.4$ cm/s

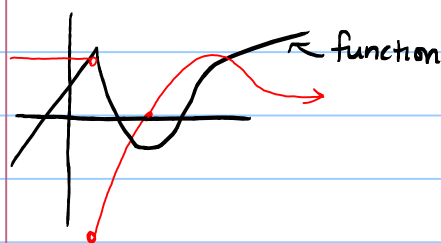
b) $d(4) = 508.8$ cm below the cliff $\therefore$ height is 11491.2 cm above the moon's surface

13) $x=-1$ Corner

$x=1$ Jump discontinuity

$x=3$ Infinite discontinuity

14)



16) $f(-1) = \sqrt{-1+2} = 1$

$f(5) = \sqrt{5+2} = \sqrt{7}$

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{7}-1}{5-(-1)} = \frac{\sqrt{7}-1}{6}$$

15) $x=-2$ Removeable

$x=1$ Jump

$x=3$ Infinite

17) Points: $(x, 3x^2-5x+1)$

$(x+h, 3(x+h)^2-5(x+h)+1)$

$$\frac{\Delta y}{\Delta x} = \frac{[3(x+h)^2-5(x+h)+1]-[3x^2-5x+1]}{[x+h]-[x]}$$

$$= \frac{3(x^2+2xh+h^2)-5x-5h\cancel{+1}-3x^2+5x\cancel{-1}}{h}$$

$$= \frac{\cancel{3x^2}+6xh+3h^2-\cancel{5x}-5h-\cancel{3x^2}+\cancel{5x}}{h}$$

$$= \frac{6xh+3h^2-5h}{h} = 6x+3h-5$$

$$\frac{dy}{dx} = \lim_{h\to 0}(6x+3h-5) = 6x-5$$

b) Pts $\left(x, \frac{5}{2x+3}\right)$

$\left(x+h, \frac{5}{2(x+h)+3}\right)$

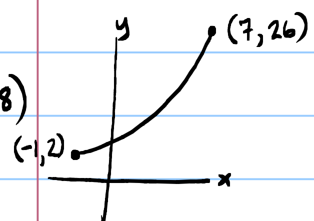
$$\frac{\Delta y}{\Delta x} = \frac{\frac{5}{2x+2h+3} - \frac{5}{2x+3}}{[x+h]-[x]}$$

$$= \frac{\frac{5(2x+3) - 5(2x+2h+3)}{(2x+2h+3)(2x+3)}}{h} = \frac{\frac{10\cancel{x} + 1\cancel{5} - 10\cancel{x} - 10h - \cancel{15}}{(2x+2h+3)(2x+3)}}{h}$$

$$= \frac{-10\cancel{h}}{(2x+2h+3)(2x+3)} \cdot \frac{1}{\cancel{h}} = \frac{-10}{(2x+2h+3)(2x+3)}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{-10}{(2x+2h+3)(2x+3)} = \frac{-10}{(2x+3)^2}$$

18)

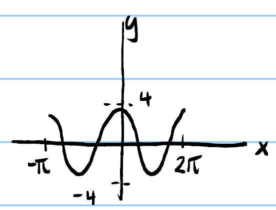


Domain: $x \in [-1, 7]$

Range: $y \in [2, 26]$

$\frac{x+3}{2} = t$ $\therefore$ $y = \left(\frac{x+3}{2}\right)^2 + 1$

19)



$x \in [-2\pi, 2\pi]$

$y \in \underbrace{[-4, 4]}$ since $-1 \leq \cos\theta \leq 1$