QUIZ FRIDAY

Even/Odd Function

Limits
- substitution
- "numerical" approach (sub in close values on either side)
- looking at a graph → LH & RH lims.
- factoring
- conjugate
- let $\sqrt{x} = p$
- common denominator

Limits Involving ∞

If the limit becomes:

$\frac{0}{\#} \rightarrow$ the limit is 0.

$\frac{\#}{0} \rightarrow$ the limit DNE, or ∞, $-\infty$

$\frac{\infty}{\infty}$ OR $\frac{\infty}{-\infty}$ OR $\frac{-\infty}{\infty}$ OR $\frac{-\infty}{-\infty}$

means there are lots of possibilities, and we must investigate further.

ex) $\lim_{x\to 2} \frac{\sqrt{x-2}}{x^2-4} \cdot \frac{\sqrt{x-2}}{\sqrt{x-2}} = \lim_{x\to 2} \frac{\cancel{x-2}\ 1}{\cancel{(x-2)}(x+2)\sqrt{x-2}}$

$\lim_{x\to 2} \frac{1}{(x+2)\sqrt{x-2}} = \frac{1}{4(0)} = \frac{1}{0}$ Answer is either $\pm\infty$ OR DNE.

$\lim_{x\to 2^-} \frac{1}{(x+2)\sqrt{x-2}}$ DNE since $\sqrt{x-2}$ is undefined.

$\lim_{x\to 2^+} \frac{1}{(x+2)\sqrt{x-2}}$ ← small + # $= \frac{1}{\text{small pos\#}} = \infty$

$\lim_{x\to 2} \frac{1}{(x+2)\sqrt{x-2}} \rightarrow$ DNE since LH $\neq$ RH ex) $\lim\limits_{x \to 1} \dfrac{x^3 - x^2 - x + 1}{x^3 - 1}$

$= \lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(x^2 - 1)}{\cancel{(x-1)}(x^2 + x + 1)}$

$= \lim\limits_{x \to 1} \dfrac{(x^2 - 1)}{x^2 + x + 1} = \dfrac{1^2 - 1}{1+1+1} = \dfrac{0}{3} = 0$

$$
\begin{array}{r|l}
 & x^2 \quad\quad -1 \\
\hline
x-1 & x^3 - x^2 - x + 1 \\
 & -(x^3 - x^2) \\
\hline
 & 0x^3 + 0x^2 - x + 1 \\
 & -x + 1 \\
\hline
 & 0
\end{array}
$$

ex) $\lim_{x \to 2} \frac{x}{(x-2)^2} = \frac{2}{0}$ so ∞, $-\infty$, or DNE

$$\lim_{x \to 2^-} \frac{x}{(x-2)^2} = \frac{2}{\text{very small pos \#}} = +\infty$$

$$\lim_{x \to 2^+} \frac{x}{(x-2)^2} = \frac{2}{\text{very small pos \#}} = +\infty$$

ex) $$\lim_{x\to\infty} \frac{5x+3}{2x-1} \left(\frac{\frac{1}{x}}{\frac{1}{x}}\right)$$

$$\lim_{x\to\infty} \frac{\frac{5\cancel{x}}{\cancel{x}} + \frac{3}{x} \to 0}{\frac{2\cancel{x}}{\cancel{x}} - \frac{1}{x} \to 0}$$

$$= 5/2$$

ex) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+4}} = \lim_{x\to\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2+4}}{x}} = \lim_{x\to\infty} \frac{1}{\frac{\sqrt{x^2+4}}{\sqrt{x^2}}}$

$x = \sqrt{x^2}$
if $x > 0$
Since $x \to \infty$, it is positive

$= \lim_{x\to\infty} \frac{1}{\sqrt{\frac{x^2}{x^2}+\frac{4}{x^2}}}$ $\frac{4}{x^2} \to 0$

$= \frac{1}{\sqrt{1+0}} = \frac{1}{\sqrt{1}} = \frac{1}{1} = 1$

ex) $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+4}} = \lim_{x\to-\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2+4}}{-\sqrt{x^2}}} = \lim_{x\to-\infty} \frac{1}{-\sqrt{\frac{x^2}{x^2}+\frac{4}{x^2}}}$ $\frac{4}{x^2} \to 0$

$x = -\sqrt{x^2}$
$x < 0$
since $x \to -\infty$

$= \frac{1}{-\sqrt{1+0}}$
$= \frac{1}{-\sqrt{1}} = -1$

ex) $\displaystyle\lim_{x\to-\infty} \frac{|x-3|}{x-3} = \frac{\text{really big \#}}{\text{same as numerator but neg.}} = -1$

$$\downarrow$$

$$\lim_{x\to-\infty} \frac{-1\cancel{(x-3)}}{\cancel{x-3}} = -1$$

ex) $\lim_{x \to 3} \frac{|x-3|}{x-3} = \text{DNE}$

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$\lim_{x \to 3^-} \frac{|x-3|}{x-3} \rightarrow \lim_{x \to 3^-} \frac{-1\cancel{(x-3)}}{\cancel{x-3}} = -1$

$\lim_{x \to 3^+} \frac{|x-3|}{x-3} \rightarrow \lim_{x \to 3^+} \frac{x-3}{x-3} = 1$