Calculus **Derivative Quiz** **/25** **Name:**

All work must be shown to earn full value.

1. Find the slope of the tangent line to $y = \frac{2}{x-1}$ at $x = 5$ using the difference quotient. (4 marks)

$\left(5, \frac{1}{2}\right)$ & $\left(5+h, \frac{2}{4+h}\right)$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{2}{4+h} - \frac{1}{2}}{5+h-5} = \frac{\frac{4-4-h}{2(4+h)}}{h}$$

$$= \frac{-\cancel{h}}{2(4+h)} \cdot \frac{1}{\cancel{h}} = \frac{-1}{2(4+h)}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{-1}{2(4+h)} = \frac{-1}{8}$$

$\left(5, \frac{1}{2}\right)$ & $\left(a, \frac{2}{a-1}\right)$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{2}{a-1} - \frac{1}{2}}{a-5} = \frac{\frac{4-a+1}{2(a-1)}}{a-5}$$

$$= \frac{-a+5}{2(a-1)} \cdot \frac{1}{a-5} = \frac{-\cancel{(a-5)}}{2(a-1)\cancel{(a-5)}}$$

$$\frac{dy}{dx} = \lim_{a \to 5} \frac{-1}{2(a-1)} = -\frac{1}{8}$$

2. Do either A or B from below. (3 marks)

A) Use the Sandwich Theorem to determine the limit of $f(x) = x^2 \cdot cos\left(\frac{1}{x^2}\right)$ as $x \to 0$.

B) Use the Intermediate Value Theorem to prove that there is at least one real number such that the cube of the number and the square of the number differ by exactly one.

$x^3 - x^2 = 1$ ← This is continuous

If $x=0$ $f(x) = x^3 - x^2$ $f(0) = 0^3 - 0^2 = 0$

If $x=10$ $f(10) = 10^3 - 10^2 = 900$

$$-1 \leq \cos\left(\frac{1}{x^2}\right) \leq 1$$

$$-x^2 \leq x^2\cos\left(\frac{1}{x^2}\right) \leq x^2$$

$$\lim_{x\to 0} -x^2 = 0 \qquad \lim_{x\to 0} x^2 = 0$$

$\therefore$ The $\lim_{x\to 0} x^2\cos\left(\frac{1}{x^2}\right)$ must be 0 since it is between 0 & 0.

3. Consider the function $f(x) = -x^3 + x^2 - 2x$. Find $f(1)$, $f'(1)$ and $f''(1)$ and explain what they tell you about the curve at $x = 1$. (6 marks)

$f(1) = -2$ The point $(1,-2)$ is on the curve.

$f'(x) = -3x^2 + 2x - 2$ The slope of the tangent at $x=1$ is -3

$f'(1) = -3 + 2 - 2 = -3$

$f''(x) = -6x + 2$ The slope is decreasing or getting steeper.

$f''(1) = -6(1) + 2 = -4$ Concave down

4. Find the equations of the tangent and the normal line to $y = \sqrt{x-3}$ at $x = 4$. (6 marks)

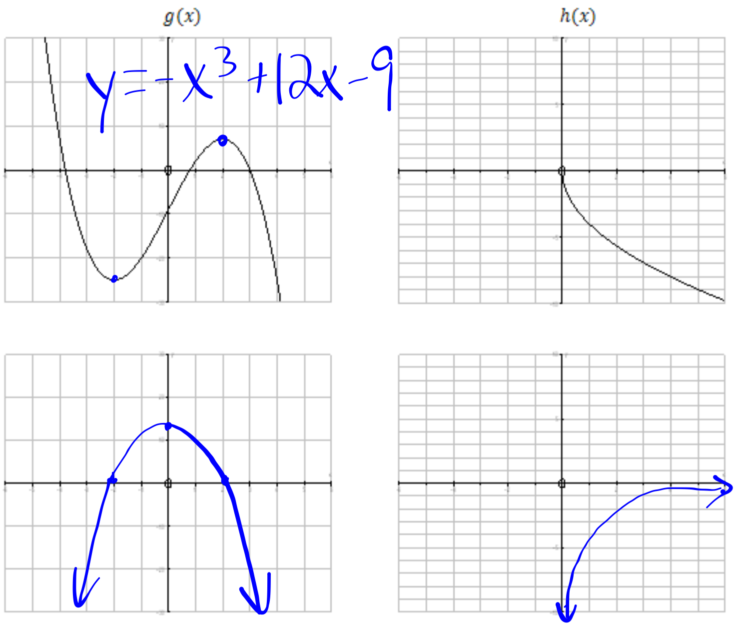
$y = (x-3)^{1/2}$ $\quad (4,1) \quad (a, \sqrt{a-3})$

$\frac{dy}{dx} = \frac{\sqrt{a-3}-1}{a-4} \cdot \frac{\sqrt{a-3}+1}{(a-4)(\sqrt{a-3}+1)} = \frac{a-3-1}{\longrightarrow} = \frac{\cancel{a-4}}{\cancel{(a-4)}(\quad)}$

$\frac{dy}{dx} = \lim_{a \to 4} \frac{1}{\sqrt{a-3}+1} = \frac{1}{2}$ $\quad y = \frac{1}{2}x - 1$

$y = -2x + 9$

5. Using the graphs of $g(x)$ and $h(x)$ shown below, sketch $g'(x)$ and $h'(x)$. (6 marks)



APPLYING & UNDERSTANDING THE DERIVATIVE

ex) An arrow is shot, so that its height (m) above the ground is given by:

$h(t) = 1.5 + 32t - 5t^2$ where t is time (s) since the arrow was shot

a) Determine the max height the arrow reache

$h'(t) = 32 - 10t$

$0 = 32 - 10t$

$3.2 = t$

b) Find velocity when the arrow hits the ground.

*When does it hit ground? (6.45)

$v(t) = 32 - 10(6.45) = -32.5 m/s$

c) Acceleration? $h''(t) = -10$

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#3, 5, 12, 1
" 23, 24