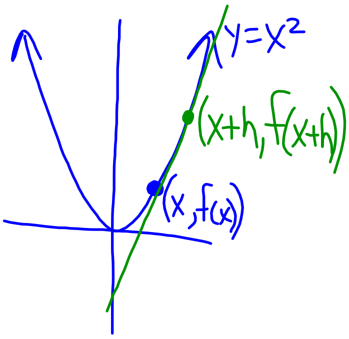
Definition of the Derivative (Difference Quotient)

ex) Find the derivative of $y=x^2$ at (x,y).



$f(x)=x^2 \qquad (x, f(x))$

$$m_{secant}=\frac{y_2-y_1}{x_2-x_1}=\frac{f(x+h)-f(x)}{x+h-x}$$

$$m_{tan}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{x+h-x}$$

$$=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$=2x+0$$

$$=2x$$

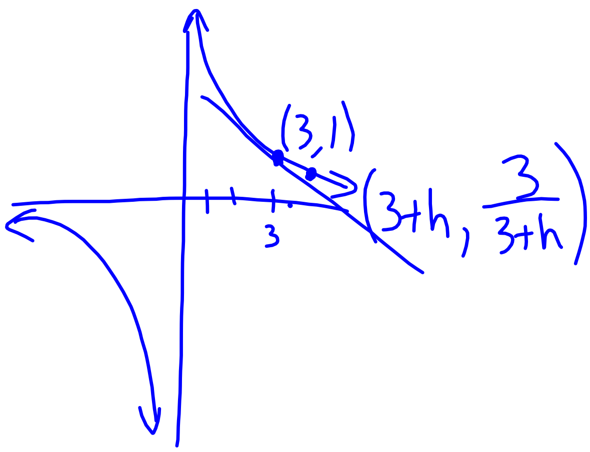
Use the derivative to evaluate the slope at:

a) $x=5 \quad m_{tan}=10$

b) $x=-2 \quad m_{tan}=-4$

c) $x=0 \quad m_{tan}=0$

ex) Find the slope of $y = \frac{3}{x}$ at $(3,1)$



$$m_{tan} = \lim_{h \to 0} \frac{\frac{3}{3+h} - 1}{3+h-3}$$

$$= \lim_{h \to 0} \frac{\frac{3-(3+h)}{3+h}}{h}$$

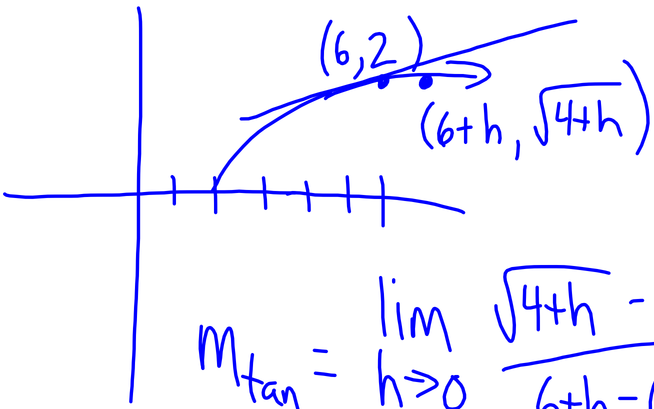
$$= \lim_{h \to 0} \frac{-\cancel{h}/3+h}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{3+h}$$

$$= \frac{-1}{3}$$

ex) Find the slope of the tangent to $y=\sqrt{x-2}$ at $x=6$.

$$y_1=\sqrt{6-2}=\sqrt{4}=2$$



$$y_2=\sqrt{6+h-2}=\sqrt{4+h}$$

$$m_{tan}=\lim_{h\to 0}\frac{\sqrt{4+h}-2}{6+h-6}$$

$$=\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\cdot\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

$$=\lim_{h\to 0}\frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$=\lim_{h\to 0}\frac{\cancel{h}\,1}{\cancel{h}(\sqrt{4+h}+2)}$$

$$=\frac{1}{\sqrt{4}+2}=1/4$$

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#1, 2, 3, 10a, 11a
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