March 06, 2013

Chain Rule

Used to find the derivative of composite functions.

ex) $f(x) = \sqrt{x-3} \rightarrow f(x) = g(h(x))$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$g(x) = \sqrt{x} = x^{1/2}$

$h(x) = x-3$

$g'(x) = \frac{1}{2}x^{-1/2}$

$h'(x) = 1$

$$f'(x) = \frac{1}{2}(x-3)^{-1/2} \cdot 1$$

$$= \frac{1}{2}(x-3)^{-1/2} = \frac{1}{2\sqrt{x-3}}$$

ex) $f(x) = \frac{2}{x-1}$ Find $f'(x)$.

$f(x) = 2(x-1)^{-1}$

$g(x) = \frac{2}{x} = 2x^{-1}$

$h(x) = x-1$

$f'(x) = -2(x-1)^{-2} \cdot 1$

$= \frac{-2}{(x-1)^2}$

ex) Find y' if $y=(3x+5)^4$

$g(x)= x^4$

$h(x)=3x+5$

$y'=4(3x+5)^3\cdot 3$

$=12(3x+5)^3$

$g'(x)=4x^3$

$h'(x)=3$

ex) Differentiate $y=\sqrt{x^2+5x+6}$.

$y=(x^2+5x+6)^{1/2}$

$y'=\frac{1}{2}(x^2+5x+6)^{-1/2}\cdot 2x+5$

$=\dfrac{2x+5}{2(x^2+5x+6)^{1/2}}$

ex) $y=\dfrac{3}{(4x^2+8x+7)^2}$. Find $\dfrac{dy}{dx}$.

$y=3(4x^2+8x+7)^{-2}$

$y'=-6(4x^2+8x+7)^{-3}(8x+8)$

$y'=\dfrac{-48(x+1)}{(4x^2+8x+7)^3}$

ex) For $y=\sqrt{5-x^2}$ find $\left.\dfrac{dy}{dx}\right|_{x=1}$

$y=(5-x^2)^{1/2}$

$\dfrac{dy}{dx}=\frac{1}{2}(5-x^2)^{-1/2}(-2x)$

$=\dfrac{-\cancel{2}x}{\cancel{2}\sqrt{5-x^2}}$

$\left.\dfrac{dy}{dx}\right|_{x=1}=\dfrac{-1}{\sqrt{5-1^2}}=\dfrac{-1}{2}$

ex) Find the vertices of $f(x)=\frac{3}{2x^3-8x}$

$$f(x)=3(2x^3-8x)^{-1}$$

$$f'(x)=-3(2x^3-8x)^{-2}(6x^2-8)$$

$$f'(x)=\frac{-3(2)(3x^2-4)}{(2x^3-8x)^2}$$

$$(2x^3-8x)^2\,0=\frac{-6(3x^2-4)}{\cancel{(2x^3-8x)^2}}\cancel{(2x^3-8x)^2}$$

$$\frac{0}{-6}=\frac{\cancel{-6}(3x^2-4)}{\cancel{-6}}$$

$$0=3x^2-4$$

$$x=\pm\sqrt{4/3}$$

$$x=\pm\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$x=\pm\frac{2\sqrt{3}}{3}$$

$\left(\frac{2\sqrt{3}}{3},\quad\right)$ & $\left(\frac{-2\sqrt{3}}{3},\quad\right)$

Check with your calculator!

ex) This is a hint as to why the Chain Rule works...

$x = \sqrt{t}$ $\quad y = t^3 + 5$ $\quad$ Find $\left.\frac{dy}{dx}\right|_{t=4}$

$x = t^{1/2}$ $\quad \frac{dy}{dt} = 3t^2$

$\frac{dx}{dt} = \frac{1}{2}t^{-1/2}$

$= \frac{1}{2\sqrt{t}}$

Want $\boxed{\frac{dy}{dx}} = \frac{dy}{\cancel{dt}} \times \frac{\cancel{dt}}{dx}$

$\frac{dy}{dx} = 3t^2 \times \frac{2\sqrt{t}}{1}$

$= 6t^2\sqrt{t}$

$\left.\frac{dy}{dx}\right|_{t=4} = 6(4)^2\sqrt{4}$

$= \underline{\underline{192}}$

- Read pg. 141-143 (up to but not incl. Repeated Use of C.R.)
- Do pg. 146-148 #9, 15, 33, 34, 45, 46
- Continue working on Application Q's given yesterday.