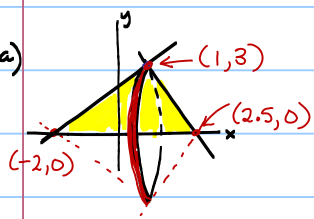
Calculus Examination Review Solutions

Note Title 13/06/2007

1 a) 

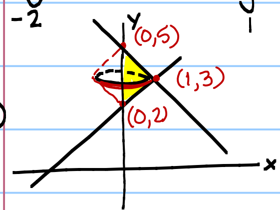
Intersection:

$x+2 = 5-2x$

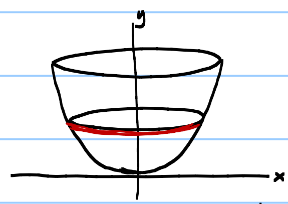
$3x = 3$

$x = 1$

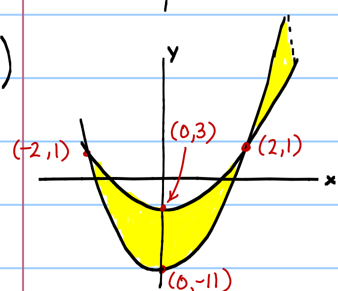
$$\pi\int_{-2}^{1}(x+2)^2\,dx + \pi\int_{1}^{2.5}(5-2x)^2\,dx = 9\pi + 4.5\pi = 13.5\pi$$

b) 

$$\pi\int_{2}^{3}(y-2)^2\,dy + \pi\int_{3}^{5}\left(2.5-\tfrac{1}{2}y\right)^2 dy = \tfrac{1}{3}\pi + \tfrac{2}{3}\pi = \pi$$

2) 

$$\pi\int_{0}^{9}(\sqrt{y})^2\,dy = \pi\int_{0}^{9} y\,dy = 40.5\pi$$

3) 

Intersection: $x^2-3 = 3x^2-11$

$2x^2 = 8$

$x = \pm 2$

$$\int_{-1}^{2}(x^2-3-3x^2+11)\,dx + \int_{2}^{3}(3x^2-11-x^2+3)\,dx$$

$$= 18 + 4\tfrac{2}{3} = 22\tfrac{2}{3}$$

4 a) $y' = 4(3x-5)^{1/2}$

$y = 4\cdot\frac{2}{3}(3x-5)^{3/2}\cdot\frac{1}{3} + c$

$y = \frac{8}{9}(3x-5)^{3/2} + c$

At (2,50): $50 = \frac{8}{9}(6-5)^{3/2} + c$

$50 = 8/9 + c$

$49\tfrac{1}{9} = c$

$y = \frac{8}{9}(3x-5)^{3/2} + 49\tfrac{1}{9}$

c) $y' = e^{3x-4}$

$y = \frac{1}{3}e^{3x-4} + c$

d) $y' = \frac{5}{2x-4}$

$y = \frac{5}{2}\ln|2x-4| + c$

5) Intersection:

$x^2+3 = 11-4x^2$

$5x^2 = 8$

$x^2 = 8/5$

$x = \pm\frac{2\sqrt{2}}{\sqrt{5}} = \pm\frac{2\sqrt{10}}{5}$

$$\int_{-\frac{2\sqrt{10}}{5}}^{\frac{2\sqrt{10}}{5}} (11-4x^2-x^2-3)\,dx = \int_{-\frac{2\sqrt{10}}{5}}^{\frac{2\sqrt{10}}{5}} (8-5x^2)\,dx \doteq 13.5$$

6) $a = -2t$

$v = -t^2+27$

$d = -\frac{1}{3}t^3 + 27t$

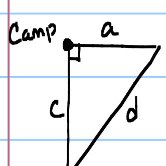
$v = 0$ when $-t^2+27 = 0$

$t^2 = 27$

$t = 3\sqrt{3}$ sec

$d(3\sqrt{3}) = \frac{1}{3}(3\sqrt{3})^3 + 27(3\sqrt{3}) = -27\sqrt{3} + 81\sqrt{3} = 54\sqrt{3}\text{ m} \doteq 93.53\text{m}$

$\therefore$ It stops in time and doesn't hit the barrier!

7) 

$\frac{da}{dt} = 8$ km/h

$\frac{dc}{dt} = 6$ km/hr

find $\frac{dd}{dt}$

$a = 12$ km

$c = 18$ km

$d = \sqrt{468} = 2\sqrt{117}$

$d^2 = a^2 + c^2$

$2d\frac{dd}{dt} = 2a\frac{da}{dt} + 2c\frac{dc}{dt}$

$2(2\sqrt{117})\frac{dd}{dt} = 2(12)(8) + 2(18)(6)$

$4\sqrt{117}\frac{dd}{dt} = 408$

$\therefore \frac{dd}{dt} = \frac{102}{\sqrt{117}}$ or $\frac{102\sqrt{117}}{117} \doteq 9.43$ km/h

8) Let $n=$ # of additional trees planted

$Y=(25+n)(500-10n)$

$Y=12500-250n+500n-10n^2$

$Y=12500+250n-10n^2$

$\frac{dY}{dn}=250-20n$

$\frac{dY}{dn}=0$ when $250=20n$; $n=12.5$

$\therefore$ Plant 37 trees [12 more than the standard 25 trees]

9a) $\int_{-1}^{2}\sqrt{4x+6}\,dx=$

let $u=4x+6$

$du=4\,dx$

$\frac{1}{4}du=\ dx$

$=\int_{2}^{14}\frac{1}{4}u^{1/2}\,du=\frac{1}{4}\cdot\frac{2}{3}u^{3/2}\Big|_{2}^{14}$

$=\frac{1}{6}\left(14^{3/2}-2^{3/2}\right)=\frac{14\sqrt{14}}{6}-\frac{2\sqrt{2}}{6}=\frac{7\sqrt{14}-\sqrt{2}}{3}$ or $\frac{1}{3}\left(7\sqrt{14}-\sqrt{2}\right)$

b) $\int_{-4}^{-2}\frac{5x}{4+5x^2}\,dx$

let $u=4+5x^2$

$du=10x\,dx$

$\frac{1}{2}du=5x\,dx$

$=\int_{84}^{24}\frac{1}{2}\frac{1}{u}\,du=\frac{1}{2}\ln|u|\Big|_{84}^{24}$

$=\frac{1}{2}\left[\ln(24)-\ln(84)\right]=\frac{1}{2}\ln\left(\frac{24}{84}\right)=\frac{1}{2}\ln\left(\frac{2}{7}\right)$

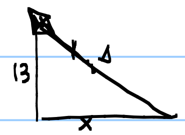
c) $\int_{\pi}^{2.5\pi}\cos\left(\frac{3x-\pi}{2}\right)dx=\int_{\pi}^{2.5\pi}\cos\left(\frac{3}{2}x-\frac{\pi}{2}\right)dx=\frac{2}{3}\sin\left(\frac{3}{2}x-\pi/2\right)\Big|_{\pi}^{2.5\pi}$

$=\frac{2}{3}\left[\sin(3.25\pi)-\sin(\pi)\right]=\frac{2}{3}\left[-\frac{\sqrt{2}}{2}-0\right]=-\sqrt{2}/3$

d) $\int_{0}^{1}2^{4-7x}\,dx=\frac{1}{\ln 2}\cdot\frac{-1}{7}\cdot 2^{4-7x}\Big|_{0}^{1}=\frac{1}{\ln 2}\cdot\frac{-1}{7}\left[2^{-3}-2^{4}\right]$

$=\frac{-1}{7\ln 2}\left[\frac{1}{8}-16\right]=\frac{-1}{7\ln 2}\left(\frac{-127}{8}\right)=\frac{127}{56\ln 2}$

10) Handout

① 

$\frac{ds}{dt} = 10\ m/min$

find $\frac{dx}{dt}$ when $\begin{cases} s = 50m \\ x = 48.28m \end{cases}$

$s^2 = 13^2 + x^2$

$2s\frac{ds}{dt} = 2x\frac{dx}{dt}$

$2(50)(10) = 2(48.25)\frac{dx}{dt}$

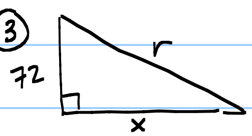
$\therefore \frac{dx}{dt} = 10.36\ m/min$

② $2x - 6y^3 + xy^2 = 12x + 1$

$2 - 18y^2\frac{dy}{dx} + x \cdot 2y\frac{dy}{dx} + y^2 = 12$

$\frac{dy}{dx}(2xy - 18y^2) = 12 - 2 - y^2$

$\frac{dy}{dx} = \frac{10 - y^2}{2xy - 18y^2}$ or $\frac{10 - y^2}{2y(x - 9y)}$

③ 

$\frac{dr}{dt} = -50\ cm/min$

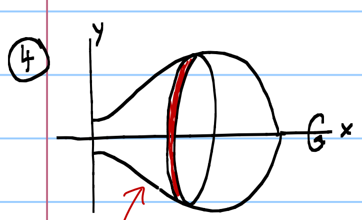
find $\frac{dx}{dt}$ when $\begin{cases} x = 96cm \\ r = 120cm \end{cases}$

$r^2 = 72^2 + x^2$

$2r\frac{dr}{dt} = 2x\frac{dx}{dt}$

$2(120)(-50) = 2(96)\frac{dx}{dt}$

$\therefore \frac{dx}{dt} = -62.5\ cm/min$

④ 

$y = x^3 - 4x^2 - 11x - 6$

$\pi\int_0^6 (x^3 - 4x^2 - 11x - 6)^2\,dx \doteq 7835.66\pi\ m^3$

or $24616.44\ m^3$

⑤ max height where $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = -10t + 100$

$\therefore$ Max height at $t = 10\ sec$

$x(10) = -5(10)^2 + 100(10) + 1500 = 2000m$

Max height

$v(15) = -10(15) + 100$

$v(15) = -50\ m/s$

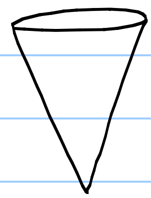
$x = 0$ when $-5t^2 + 100t + 1500 = 0$

$t = 30\ sec$ ~~t = -10 sec~~

$v(30) = -10(30) + 100$

$= -200\ m/s$ when it hits the ground

(6)



$h = 4r \Rightarrow \frac{1}{4}h = r$

$\frac{dV}{dt} = 0.05\,m^3/min$

find $\frac{dh}{dt}$ when $h=2$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{4}h\right)^2 h$

$V = \frac{1}{48}\pi h^3$

$\frac{dV}{dt} = \frac{1}{16}\pi h^2 \frac{dh}{dt}$

$0.05 = \frac{1}{16}\pi (2)^2 \frac{dh}{dt}$

$\frac{1}{5\pi} = \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = \frac{1}{5\pi}$ m/min

$\doteq 0.064$ m/min

(7) $A = \pi r^2$ $\frac{dr}{dt} = 1.5\,m/s$ when $r = 8\,m$ find $\frac{dA}{dt}$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(8)(1.5)$

$\frac{dA}{dt} = 24\pi\, m^3/sec$

(8) $h = 3 + 12t - 5t^2$

$\frac{dh}{dt} = 12 - 10t$

$\frac{dh}{dt} = 0$ when $10t = 12$

$t = 1.2\,sec$

$h(1.2) = 3 + 12(1.2) - 5(1.2)^2$

$= 10.2\,m$

(9) a) $a = 5 + 1.04\,t^{0.4}$ ft/sec^2

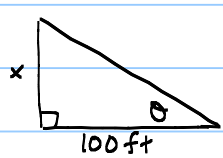
b) $d = \frac{5}{2}t^2 + \frac{1}{2.04}t^{2.04}$ ft

$d(10) = \frac{5}{2}(10)^2 + \frac{1}{2.04}(10)^{2.04}$

$\therefore d(10) \doteq 250 + 53.75$

$\doteq 303.75$ ft.

(10)



$\frac{dx}{dt} = 5\,ft/sec$

find $\frac{d\theta}{dt}$ when $\begin{cases} x = 100\,ft \\ \theta = \pi/4 \end{cases}$

$\tan\theta = \frac{x}{100}$

$100\tan\theta = x$

$100\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$

$100\sec^2(\pi/4)\frac{d\theta}{dt} = 5$

$100(2)\frac{d\theta}{dt} = 5$

$\therefore \frac{d\theta}{dt} = \frac{1}{40}$ radians/sec

$\doteq 1.43°/sec.$

11) Multiple Choice

1) C	5) A	9) C	13) A	17) D	21) A
2) B	6) D	10) D	14) E	18) C	22) D
3) D	7) B	11) D	15) C	19) C	23) C
4) A	8) E	12) E	16) A	20) E	

12) p243 #34

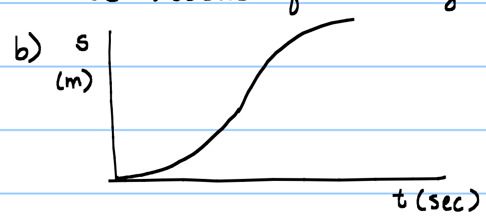
Changes concavity at approximately 23 days.

13) p298 #11

a) represents $(1\,\text{sec})(\frac{1}{2}\,\text{m/s}) = \frac{1}{2}\,\text{m}$

53 blocks of this size $\therefore 53(\frac{1}{2}) = 26.5\,\text{m}$

b)



14a) $\int \frac{3}{6x-4}\,dx = \frac{1}{2}\ln|6x-4| + C$

d) $\int \frac{3}{x^2}\,dx = \int 3x^{-2}\,dx$

$= -3x^{-1} + C$

or $= \frac{-3}{x} + C$

b) $\int \sec^2(4x)\,dx = \frac{1}{4}\tan(4x) + C$

c) $\int 2e^{-3x}\,dx = -\frac{2}{3}e^{-3x} + C$

e) $\int x^2(x^3-4)^2\,dx =$

let $u = x^3 - 4$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3}\int u^2\,du = \frac{1}{3}\cdot\frac{1}{3}u^3 + C$

$= \frac{1}{9}(x^3-4)^3 + C$

f) $\int \frac{2x-5}{x}\,dx = \int(2 - \frac{5}{x})\,dx = 2x - 5\ln|x| + C$

15a) $y = 5\sqrt{4x^2-6x}$

$y = 5(4x^2-6x)^{1/2}$

$y' = \frac{5}{2}(4x^2-6x)^{-1/2} \cdot (8x-6)$

$y' = \dfrac{5(4x-3)}{\sqrt{4x^2-6x}}$

b) $y = \dfrac{3x^2-4}{x+2x^3}$

$y' = \dfrac{(6x)(x+2x^3)-(3x^2-4)(1+6x^2)}{(x+2x^3)^2}$

$= \dfrac{6x^2+12x^4-[3x^2+18x^4-4-24x^2]}{(x+2x^3)^2}$

$= \dfrac{12x^4-18x^4+3x^2+24x^2+4}{(x+2x^3)^2}$

$= \dfrac{-6x^4+27x^2+4}{(x+2x^3)^2}$

c) $xy^3+2x-3y^4=8$

$x \cdot 3y^2 \cdot y' + y^3 + 2 - 12y^3y' = 0$

$y'(3xy^2-12y^3) = -2-y^3$

$y' = \dfrac{-2-y^3}{3xy^2-12y^3}$ or $\dfrac{y^3+2}{12y^3-3xy^2}$ or $\dfrac{y^3+2}{3y^2(4y-x)}$

16) $f(x) = -3x^2-18x+10$ at $x=-2$

$f(-2) = -3(-2)^2-18(-2)+10$

$f(-2) = 34$

$f'(x) = -6x-18$

$f'(-2) = -6(-2)-18 = -6$

$\therefore -6 = \dfrac{y-34}{x+2}$

$-6(x+2)+34 = y$

$-6x-12+34 = y$

$\therefore \boxed{y=-6x+22}$

17) $y = \int_2^t \sqrt{3-4x+x^2}\, dx$

$y' = \sqrt{3-4t+t^2}$

$\therefore$ Critical values occur where $y'=0$ or y' is undefined

$\therefore 3-4t+t^2=0$

$t^2-4t+3=0$

$(t-3)(t-1)=0$

$\therefore$ Critical values occur when $t=3$ and $t=1$

18) $y = \int_2^x 6t\sqrt{3+12t^2}\,dt$

Concave up when $y'' > 0$

$y' = \sqrt{3+12x^2} \cdot 6x$

$y' = 6x\sqrt{3+12x^2}$

$y'' = 6\sqrt{3+12x^2} + 6x \cdot \frac{1}{2}(3+12x^2)^{-1/2} \cdot 24x$

$= 6(3+12x^2)^{1/2} + 72x^2(3+12x^2)^{-1/2}$

$= 6(3+12x^2)^{-1/2}\left[(3+12x^2)+72x^2\right]$

$= \dfrac{6\left[3+84x^2\right]}{\sqrt{3+12x^2}}$

y'' is always positive since $3+84x^2 > 0$ for all $t \in \mathbb{R}$

and $3+12x^2 > 0$ for all $t \in \mathbb{R}$

$\therefore$ This function is always concave up!

19) $f(x) = \int_x^2 \sin^2(6t)\,dt$

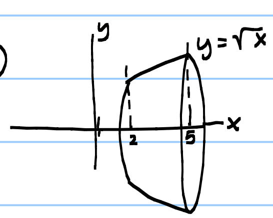
$f(x) = -\int_2^x \sin^2(6t)\,dt$

$f'(x) = -\sin^2(6x)$

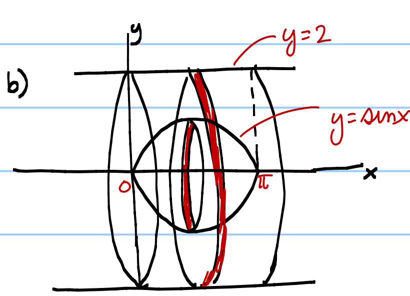
$f'(\frac{\pi}{12}) = -\sin^2(\frac{\pi}{2}) = -(1)^2 = -1$

$\therefore -1 = \dfrac{y-2}{x-\pi/12}$ $\therefore$ Equation is: $\boxed{y = -(x-\pi/12)+2}$

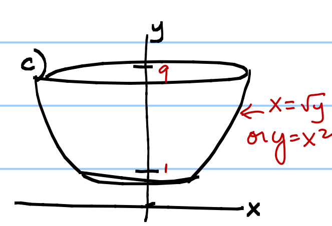
20a)



b)



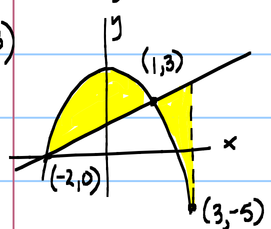
c)



21) The differential equations describe the slope fields directly above them

22) It would contain both x and y since there are no horizontal or vertical patterns.

23)



$$\int_{-2}^{1} (4-x^2-x-2)\,dx + \int_{1}^{3} (x+2-4+x^2)\,dx$$

$$= 4.5 + \frac{26}{3} = \frac{79}{6} \text{ units}^2$$

24) $v(t) = 12 - 2t^{0.6}$

$$\int_{1}^{14} (12-2t^{0.6})\,dt + 72 = 144 \text{ thousand}$$

OR $c'(t) = 12 - 2t^{0.6}$

$$c(t) = 12t - \frac{2}{1.6}t^{1.6} + c$$

$$c(1) = 12(1) + \frac{2}{1.6}(1)^{1.6} + c$$

$$72 = 12 - 1.25 + c$$

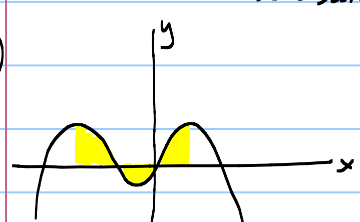
$$61.25 = c$$

$$\therefore c(t) = 12t - \frac{2}{1.6}t^{1.6} + 61.25$$

$$c(14) = 12(14) + \frac{2}{1.6}(14)^{1.6} + 61.25$$

$$= 144 \text{ thousand}$$

25)



$\int_{-3}^{1} g(x)\,dx$ should be positive since there is more area above than below the curve.

Note: $a(t)\,dt$ should not be $f(x)\,dx$ ... Sorry!

26a) # apples consumed in 4 weeks

b) 35 apples were consumed during weeks 4–10

c) 105 apples were consumed during the first 10 weeks

$$\int_{0}^{10} a(t)\,dt = 105$$