18.2 Light Particles and Matter Waves

SECTION OUTCOMES

- Explain how photon momentum revolutionized thinking in the scientific community.
- Explain the Compton effect and the de Broglie wavelength.

KEY TERMS

- Compton effect
- de Broglie wavelength
- wave-particle duality

When Millikan's experimental results verified Einstein's interpretation of the photoelectric effect, the scientific community began to accept the particle nature of light. Physicists started to ask more questions about the extent to which particles of light, or photons, resembled particles of matter. U.S. physicist Arthur Compton (1892–1962) decided to study elastic collisions between photons and electrons. Would the law of conservation of momentum apply to such collisions? How could physicists determine the momentum (mv) of a particle that has no mass?

The Compton Effect

The ideal way to study collisions between particles is to start with free particles. Preferably, the only force acting on either particle at the moment of the collision is the impact of the other particle. However, electrons rarely exist free of atoms. So, Compton reasoned that if the photon's energy was significantly greater than the work function of the metal, the energy required to free an electron from the metal would be negligible when compared to the energy of the interaction. He needed a source of highly energetic photons.

About 30 years prior to Compton's work, Wilhelm Conrad Röntgen (1845–1923) discovered X rays and demonstrated that they are high-frequency electromagnetic waves. Thus, X-ray photons would have the amount of energy that Compton needed for his studies. In 1923, Compton carried out some very sophisticated experiments on collisions between X-ray photons and electrons. The phenomenon that he discovered is now known as the **Compton effect** and is illustrated in Figure 18.10. When a very high-energy X-ray photon collides with a "free" electron, it gives some of its energy to the electron and a lower-energy photon scatters off the electron.

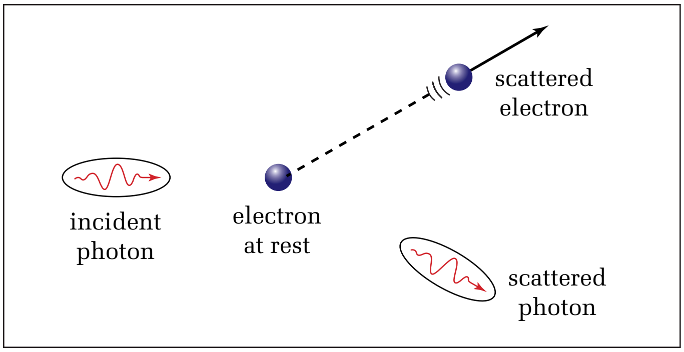

Figure 18.10 When a high-energy photon collides with a "free" electron, both energy and momentum are conserved.

You can describe mathematically the conservation of energy in a photon-electron collision as follows, where hf is the energy of the photon before the collision, hf' is the energy of the photon after the collision, and $\frac{1}{2}m_e v'^2$ is the kinetic energy of the electron after the collision.

$$hf = hf' + \frac{1}{2}m_e v'^2$$

Since the scattered photon has a lower energy, it must have a lower frequency and a longer wavelength than the original photon. Compton's measurements showed that the scattered photon had a lower frequency, and that kinetic energy gained by an electron in a collision with a photon was equal to the energy lost by the photon.

The more difficult task for Compton was finding a way to determine whether momentum had been conserved in the collision. The familiar expression for momentum, $p = mv$, contains the object's mass, but photons have no mass. So Compton turned to Einstein's now famous equation, $E = mc^2$, to find the mass equivalence of a photon. The following steps show how Compton used Einstein's relationship to derive an expression for the momentum of a photon. Since the goal is to find the magnitude of the momentum, vector notations are omitted.

- Write Einstein's equation that describes the energy equivalent of mass. $E = mc^2$
- Divide both sides of the equation by c^2 to solve for mass. $m = \frac{E}{c^2}$
- Write the equation for momentum. $p = mv$
- Substitute the energy equivalent of mass into the equation for momentum. $p = \frac{E}{c^2}v$
- Since the velocity of a photon is c, substitute c for v and simplify. $P = \frac{E}{c^{\cancel{2}}}\cancel{c} = \frac{E}{c}$
- Substitute the expression for the energy of a photon (hf) for E in the equation for momentum. $p = \frac{hf}{c}$
- The momentum of a photon is usually expressed in terms of wavelength, rather than frequency. Use the equation for the velocity of a wave to find the expression for f in terms of v. Note that the velocity of a light wave is c.
  $f\lambda = v$
  $f\lambda = c$
  $f = \frac{c}{\lambda}$
- Substitute the expression for frequency into the momentum equation and simplify.
  $p = \frac{h\frac{\cancel{c}}{\lambda}}{\cancel{c}}$
  $p = \frac{h}{\lambda}$

When Compton calculated the momentum of a photon using $p = \frac{h}{\lambda}$, he was able to show that momentum is conserved in collisions between photons and electrons. These collisions obey all of the laws for collisions between two masses. The line between matter and energy was becoming more and more faint.

MOMENTUM OF A PHOTON

The momentum of a photon is the quotient of Planck's constant and the wavelength of the photon.

$$p = \frac{h}{\lambda}$$

Quantity	Symbol	SI unit
momentum	p	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$ (kilogram metres per seconds)
Planck's constant	h	J · s (joule seconds)
wavelength	λ	m (metres)

Unit Analysis

$$\frac{\text{kilogram} \cdot \text{metre}}{\text{second}} = \frac{\text{joule} \cdot \text{second}}{\text{metre}}$$

$$\frac{\text{kg} \cdot \text{m}}{\text{s}} = \frac{\text{J} \cdot \text{s}}{\text{m}} = \frac{\frac{\text{kg} \cdot \text{m}^{\cancel{2}}}{\text{s}^{\cancel{2}}} \cdot \cancel{\text{s}}}{\cancel{\text{m}}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

The following problem will help you to develop a feeling for the amount of momentum that is carried by photons.

MODEL PROBLEM

Momentum of a Photon

Calculate the momentum of a photon of light that has a frequency of 5.09×10^{14} Hz.

Frame the Problem

- The *momentum* of a *photon* is related to its *wavelength*.
- A photon's *wavelength* is related to its *frequency* and the speed of light.

Identify the Goal

The momentum, p, of the photon

Variables and Constants

Known	Implied	Unknown
$f = 5.09 \times 10^{14}$ Hz	$c = 3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}$	λ
		p

Strategy

Find the wavelength by using the equation for the speed of waves and the value for the speed of light.

Calculations

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}}{5.09 \times 10^{14}\ \text{s}^{-1}}$$

$$\lambda = 5.8939 \times 10^{-7}\ \text{m}$$

Use the equation that relates the momentum of a photon to its wavelength.

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{5.8939 \times 10^{-7}\ \text{m}}$$

$$p = 1.1249 \times 10^{-27}\ \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

$$p \cong 1.12 \times 10^{-27}\ \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

The momentum of a photon with a frequency of 5.09×10^{14} Hz is $1.12 \times 10^{-27}\ \frac{\text{kg} \cdot \text{m}}{\text{s}}$.

Validate

You would expect the momentum of a photon to be exceedingly small, and it is. Check to see if the units cancel to give $\frac{\text{kg} \cdot \text{m}}{\text{s}}$.

$$\frac{\text{J} \cdot \text{s}}{\text{m}} = \frac{\frac{\text{kg} \cdot \text{m}^{\cancel{2}}}{\text{s}^{\cancel{2}}} \cdot \cancel{\text{s}}}{\cancel{\text{m}}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

PRACTICE PROBLEMS

1. Find the momentum of a photon with a wavelength of 1.55 m (radio wave).
2. Find the momentum of a gamma ray photon with a frequency of 4.27×10^{20} Hz.
3. What would be the wavelength of a photon that had the same momentum as a neutron travelling at 8.26×10^{7} m/s?
4. How many photons with a wavelength of 5.89×10^{-7} m would it take to equal the momentum of a 5.00 g Ping-Pong™ ball moving at 8.25 m/s?
5. What would be the frequency of a photon with a momentum of 2.45×10^{-32} kg · m/s? In what part of the electromagnetic spectrum would this photon be?

Matter Waves

By the 1920s, physicists had accepted the quantum theory of light and continued to refine the concepts. Once again, however, the scientific community was startled by the revolutionary theory proposed by a young French graduate student, who was studying at the Sorbonne. As part of his doctoral dissertation, Louis de Broglie (1892–1987) proposed that not only do light waves behave as particles, but also that particulate matter has wave properties.

De Broglie's professors at the Sorbonne thought that the concept was rather bizarre, so they sent the manuscript to Einstein and asked for his response to the proposal. Einstein read the dissertation with excitement and strongly supported de Broglie's proposal. De Broglie was promptly granted his Ph.D., and six years later, he was honoured with the Nobel Award in Physics for his theory of matter waves. The following steps lead to what is now called the **de Broglie wavelength** of matter waves.

- Write Compton's equation for the momentum of a photon. $$p = \frac{h}{\lambda}$$
- Solve the equation for wavelength. $$\lambda = \frac{h}{p}$$
- Substitute the value for the momentum of a particle for p. $$\lambda = \frac{h}{mv}$$

DE BROGLIE WAVELENGTH OF MATTER WAVES

The de Broglie wavelength of matter waves is the quotient of Planck's constant and the momentum of the mass.

$$\lambda = \frac{h}{mv}$$

Quantity	Symbol	SI unit
wavelength (of a matter wave)	λ	m (metres)
Planck's constant	h	J · s (joule seconds)
mass	m	kg (kilograms)
velocity	v	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\frac{\text{joule} \cdot \text{second}}{\text{kilogram}\frac{\text{metre}}{\text{second}}} = \frac{\text{J} \cdot \text{s}}{\text{kg}\frac{\text{m}}{\text{s}}} = \frac{\text{J} \cdot \text{s}}{\text{kg}} \cdot \frac{\text{s}}{\text{m}} = \frac{\frac{\cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\cancel{\text{s}^2}}{\cancel{\text{kg}} \cdot \cancel{\text{m}}} = \text{m}$$

Note: Since wavelength is a scalar quantity, vector notations are not used for velocity.

MODEL PROBLEM

Matter Waves

Calculate the wavelength of an electron moving with a velocity of 6.39×10^6 m/s.

Frame the Problem

- *Moving particles* have *wave* properties.
- The *wavelength* of particle waves depends on *Planck's constant* and the *momentum* of the particle.

Identify the Goal

The wavelength, λ, of the electron

Variables and Constants

Known	Implied	Unknown
$v = 6.39 \times 10^6 \; \frac{\text{m}}{\text{s}}$	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$ $m_\text{e} = 9.11 \times 10^{-31}\ \text{kg}$	λ

Strategy

Use the equation for the de Broglie wavelength.

Calculations

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})(6.39 \times 10^6\ \frac{\text{m}}{\text{s}})}$$

$$\lambda = 1.1389 \times 10^{-10}\ \text{m}$$

$$\lambda \cong 1.14 \times 10^{-10}\ \text{m}$$

The de Broglie wavelength of an electron travelling at 6.39×10^6 m/s is 1.14×10^{-10} m.

Validate

Since Planck's constant is in the numerator, you would expect that the value would be very small. Check the units to ensure that the final answer has the unit of metres.

$$\frac{\text{J}\cdot\text{s}}{\text{kg}\frac{\text{m}}{\text{s}}} = \frac{\frac{\cancel{\text{kg}}\cdot\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\cdot\cancel{\text{s}}}{\cancel{\text{kg}}}\cdot\frac{\cancel{\text{s}}}{\cancel{\text{m}}} = \text{m}$$

PRACTICE PROBLEMS

6. Calculate the wavelength of a proton that is moving at 3.79×10^6 m/s.
7. Calculate the wavelength of an alpha particle that is moving at 1.28×10^7 m/s.
8. What is the wavelength of a 5.00 g Ping-Pong™ ball moving at 12.7 m/s?
9. Find the wavelength of a jet airplane with a mass of 1.12×10^5 kg that is cruising at 891 km/h.
10. Calculate the wavelength of a beta particle (electron) that has an energy of 4.35×10^4 eV.
11. What is the speed of an electron that has a wavelength of 3.32×10^{-10} m?

To verify de Broglie's hypothesis that particles have wavelike properties, an experimenter would need to show that electrons exhibit interference. A technique such as Young's double-slit experiment would be ideal. This technique is not feasible for particles such as electrons, however, because the electrons have wavelengths in the range of 10^{-10} m. It simply is not possible to mechanically cut slits this small and close together. Fortunately, a new technique for observing interference of waves with very small wavelengths had recently been devised.

PHYSICS FILE

As you know, in 1897, J.J. Thomson provided solid evidence for the existence of the electron, a subatomic particle that is contained in all atoms. Ironically, just 30 years later, his son George P. Thomson demonstrated that electrons behave like waves.

During the 10 years prior to de Broglie's proposal, physicists Max von Laue (1879–1960) and Sir Lawrence Bragg (1890–1971) were developing the theory and technique for diffraction of X rays by crystals. The spacing between atoms in crystals is in the same order of magnitude as both the wavelength of X rays and electrons, about 10^{-10} m. As illustrated in Figure 18.11, when X rays scatter from the atoms in a crystal, they form diffraction patterns in much the same way that light forms diffraction patterns when it passes through a double slit or a diffraction grating. If electrons have wave properties, then the same crystals that diffract X rays should diffract electrons and create a pattern.

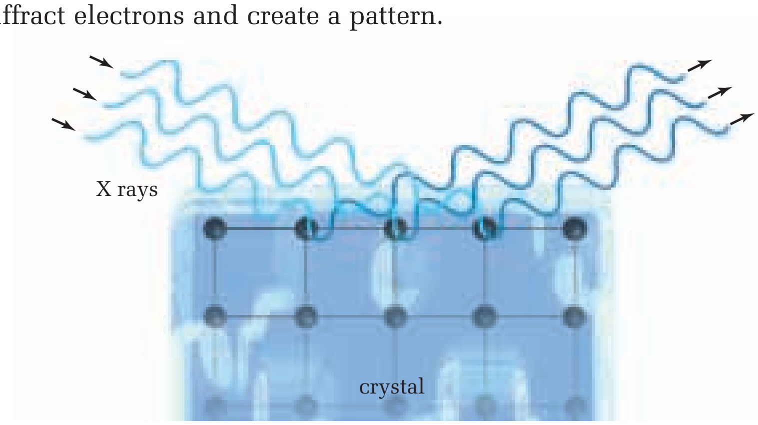

Figure 18.11 X rays scattered from regularly spaced atoms in a crystal will remain in phase only at certain scattering angles.

Within three years after de Broglie published his theory of matter waves, Clinton J. Davisson (1881–1958) and Lester H. Germer (1896–1971) of the United States and, working separately, George P. Thomson (1892–1975) of England carried out electron diffraction experiments. Both teams obtained patterns very similar to those formed by X rays. The wave nature of electrons was confirmed. In the years since, physicists have produced diffraction patterns with neutrons and other subatomic particles. Figure 18.12 shows diffraction patterns from aluminum foil formed by a beam of (A) X rays and (B) electrons.

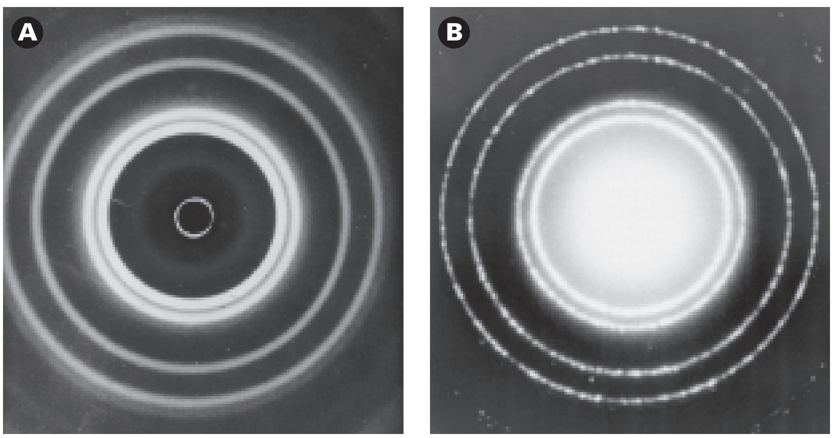

Figure 18.12 These patterns were created by diffraction of **(A)** X rays and **(B)** electrons by aluminum foil. Diffraction occurs as a result of the interference of waves. The similarity of these patterns verifies that electrons behave like waves.

The Wave-Particle Duality

Within 30 years after Planck presented his revolutionary theory to the German Physical Society, physicists had come to accept the particle nature of light and the wave nature of subatomic particles. They did not, however, forsake Maxwellian electromagnetism or Newtonian mechanics. Newton's concepts have made it possible for astronauts to travel to the Moon and back and to put satellites into orbit. Maxwell's electromagnetism permits engineers to develop the technology to send microwaves to and from these satellites. Physicists accept the dual nature of radiant energy that propagates through space as waves and interacts with matter as particles or discrete packets of energy.

Matter also has a dual nature, but only the subatomic particles have a small enough mass, and thus a large enough wavelength, to exhibit their wave nature. In 1924, Albert Einstein wrote, "There are therefore now two theories of light, both indispensable, and — as one must admit today despite twenty years of tremendous effort on the part of theoretical physicists — without any logical connection." Some physicists hope that in the future we will have a clearer picture of matter waves and quanta of energy. For now, we accept the **wave-particle duality**: Both matter and electromagnetic energy exhibit some properties of waves and some properties of particles.

18.2 Section Review

1. **C** Explain how Compton determined the momentum of a photon — a particle that has no mass.
2. **C** Describe the Compton effect.
3. **K/U** What was the most important result of Compton's experiments with the collisions between photons and electrons?
4. **K/U** Compton was able to ignore the work function of the metal in which the electrons were embedded in his momentum calculations. How was he able to justify this?
5. **C** Describe the reasoning that de Broglie used to come up with the idea that matter might have wave properties.
6. **C** When you walk through a doorway, you represent a particle having momentum and, therefore, having a wavelength. Why is it improbable that you will be "diffracted" as you pass through the doorway?
7. **K/U** Attempts to demonstrate the existence of de Broglie matter waves by using a beam of electrons incident on Young's double-slit apparatus proved unsuccessful. Give one possible explanation.
8. **C** Explain the technique that Davisson, Germer, and George P. Thomson used to verify the wave nature of electrons.
9. **MC** Research the production of X rays and prepare a display poster. In your display, include a diagram of the general structure of the X-ray tube, an explanation of how electrons cause the production of X rays, and an indication of the societal importance of the technology.