## The Basics of the <br> 17.2 Special Theory of Relativity

Einstein's special theory of relativity changed our fundamental understanding of distance, time, and mass. He used his famous thought experiments to illustrate these new concepts. This section contains several thought experiments similar to the ones Einstein used.

## Thought Experiment 1: Simultaneity

Imagine that you are sitting high on a hill on Canada Day and you can see two different celebrations going on in the distance. You are startled when two sets of fireworks ignite at exactly the same time - one off to your left and the other far to your right. About 100 m behind you, a car is travelling along a highway at $95 \mathrm{~km} / \mathrm{h}$. Do the passengers in the car see the fireworks igniting simultaneously or do they think that one set ignited before the other? Your immediate reaction is probably, "Of course they saw the fireworks igniting simultaneously — they were simultaneous!"

According to Einstein's special theory of relativity, however, the answer is not quite so simple. To restate the question more precisely, are two events that are simultaneous for an observer in one inertial reference frame simultaneous for observers in all inertial reference frames? The answer is no. The constancy of the speed of light creates problems with the simultaneity of events, as the situation in Figure 17.9 illustrates.

In Figure 17.9 (A), observers A and B are seated equidistant from a light source (S). The light source flashes. Since the light must travel an equal distance to both observers, they would say that they received the flash at exactly the same time, that the arrival of the flash was simultaneous for both of them.

Now imagine that these two observers are actually sitting on a railway flatcar that is moving to the right with velocity $\vec{V}$ relative to the ground and to observer C in Figure 17.9 (B). Observer C makes two observations.

1. $B$ is moving away from the point from which the light was emitted.
2. A is moving toward the point from which the light was emitted.


Figure 17.9 Do events that appear to be simultaneous to observers $A$ and $B$ also appear to be simultaneous to observers C and D ?

Observer C concludes that it takes longer for light to reach B than it does to reach A . Thus, according to observer C , observer A received the flash first and B received it second. The arrivals are not simultaneous in C's frame of reference, and yet it is an inertial reference just as much as is the frame of reference of the flatcar.

In the frame of reference for observer D, who is moving to the right with a velocity of $2 \vec{v}$, the flatcar is moving toward the left with a velocity of $\vec{v}$. Now, it is A who is moving away from the point from which the flash was emitted and B is moving toward that emission point. The light would take longer to reach A , so the light would arrive at observer B first.

As you can see from this example, the whole concept of simultaneity, of past, present, and future, is fuzzy in relativity. What is a future event in one frame of reference becomes a past event in another. This is due entirely to the fact that the speed of light is the same in all inertial frames of reference, regardless of their relative velocities.

## Thought Experiment 2: Time Dilation

Imagine yourself back on the hilltop, watching fireworks. You look at your watch at the moment that the fireworks ignite and it says 11:23 P.m. What do the watches of the passengers in the car read? If they saw the fireworks ignite at different times, their watches cannot possibly agree with yours.


Figure 17.10 If the speed of light is the same to all observers, then light takes longer to travel from $A$ to $B^{\prime}$ than it does to travel from $A$ to $B$.

The constancy of the speed of light creates problems with time intervals. The term time dilation applies to situations in which time intervals appear different to observers in different inertial frames of reference. To understand the implications of this constant speed of light for time measurement, assume that an experimenter has devised a light clock. In it, a pulse of light reflects back and forth between two mirrors, A and B. The time that it takes for the pulse to travel between the mirrors is the basic tick of this clock. Figure 17.10 (A) shows such a "tick."

Now, picture this clock in a spacecraft that is speeding past Earth. An observer in the spacecraft sees the light as reflecting back and forth as it was before, so the basic tick of the clock has not changed. However, an observer on Earth would see that the mirrors moved
while the light pulse travelled from A to B, as shown in Figure 17.10 (B). Since the pulse actually has to travel from A to B', it must take longer, as indicated in Figure 17.10 (C). The tick of the clock therefore takes longer to occur in the Earth frame of reference than in the spacecraft observer's frame of reference. In fact, if the spacecraft observer was wearing a watch, the Earth observer would say that the watch was counting out the seconds too slowly. The spacecraft observer, however, would say that the watch and the light clock were working properly.

The relationship between times as measured in the spacecraft and on Earth can be deduced from Figure 17.10 (D). Assume that

- $c$ is the speed of light, which is the same for all observers
- $\Delta t$ is the time that the Earth observer says it takes for the pulse to travel between the mirrors
- $\Delta t_{0}$ is the time that the spacecraft observer says it takes for the pulse to travel between the mirrors

The distance from A to B would be $c \Delta t_{0}$. The distance travelled by the spacecraft would be $v \Delta t$, since this involves a distance, speed, and time observed by the Earth observer.

The Earth observer claims that the light pulse actually travelled a distance of $c \Delta t$. These distances represent the lengths of the sides of a right-angled triangle, as seen in Figure 17.10 (D). Notice how similar this result is to the arrival-time equation in the boat X-boat Y scenario on pages 743 and 744.

- Apply the Pythagorean theorem and expand.
- Solve for $c^{2} \Delta t_{0}^{2}$.
- Factor out a $\Delta t^{2}$.
- Divide by $c^{2}$.
- Simplify, then take the square root of both sides of the equation.
- Solve for $\Delta t$.

$$
\begin{aligned}
& (c \Delta t)^{2}=\left(c \Delta t_{\mathrm{o}}\right)^{2}+(v \Delta t)^{2} \\
& c^{2} \Delta t^{2}=c^{2} \Delta t_{\mathrm{o}}^{2}+v^{2} \Delta t^{2} \\
& c^{2} \Delta t_{\mathrm{o}}^{2}=c^{2} \Delta t^{2}-v^{2} \Delta t^{2} \\
& c^{2} \Delta t_{\mathrm{o}}^{2}=\Delta t^{2}\left(c^{2}-v^{2}\right) \\
& \Delta t_{\mathrm{o}}^{2}=\frac{\Delta t^{2}\left(c^{2}-v^{2}\right)}{c^{2}} \\
& \Delta t_{\mathrm{o}}^{2}=\Delta t^{2}\left(1-\frac{v^{2}}{c^{2}}\right) \\
& \Delta t_{\mathrm{o}}=\Delta t \sqrt{1-\frac{v^{2}}{c^{2}}} \\
& \Delta t=\frac{\Delta t_{\mathrm{o}}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}
\end{aligned}
$$

In any question involving relativistic times, it is important to carefully identify the times.

- $\Delta t_{0}$ is the time as measured by a person at rest relative to the object or the event. It is called the proper time. You could think of it as the "rest time," although this term is not generally used. Another way to picture it is as the "one-point" time, the time for an observer who sees the clock as staying at only one point.


## Relativity

Experiment with near-light speeds and time dilation by using your Electronic Learning Partner.

## (1) Math Link

Note that the negative square root has no meaning in this situation. Both times will be seen as positive. In addition, $v$ must be less than $c$. If it was greater than $c$, the denominator would become the square root of a negative number. Although such a square root can be expressed using complex numbers, it is not expected that a time measurement would involve anything other than the set of real numbers.

- $\Delta t$ is the expanded or dilated time. Since the denominator $\sqrt{1-\frac{V^{2}}{c^{2}}}$ is less than one, $\Delta t$ is always greater than $\Delta t_{0}$. It can also be thought of as the "two-point" time, the time as measured by an observer who sees the clock as moving between two points.


## DILATED TIME

The dilated time is the quotient of the proper time and the expression: square root of one minus the velocity of the moving reference frame squared divided by the speed of light squared.

$$
\Delta t=\frac{\Delta t_{0}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}
$$

Quantity
dilated time
proper time
velocity of the moving reference frame speed of light

Symbol
$\Delta t \quad \mathrm{~s}$ (seconds)
$\Delta t_{0} \quad \mathrm{~s}$ (seconds)
$v \quad \frac{\mathrm{~m}}{\mathrm{~s}}$ (metres per second)
$c \quad \frac{\mathrm{~m}}{\mathrm{~s}}$ (metres per second)

## Unit Analysis



## MODEL PROBLEM

## Relative Times

A rocket speeds past an asteroid at 0.800 c . If an observer in the rocket sees 10.0 s pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

## Frame the Problem

- Proper time, $\Delta t_{0}$, and dilated time, $\Delta t$, are not the same. Time intervals appear to be shorter to the observer who is moving at a velocity close to the speed of light.
- Proper time, $\Delta t_{0}$, and dilated time, $\Delta t$, are related by the speed of light, c.


## PROBLEM TIP

Since $\frac{v^{2}}{c^{2}}$ is a ratio, the speeds can have any units as long as they are the same for both the numerator and the denominator. It is often useful to express $v$ in terms of $c$.

## Identify the Goal

The amount of time, $\Delta t$, that passes for the observer on the asteroid while 10.0 s passes for the observer on the rocket

## Variables and Constants

## Known

$V_{\text {rocket }}=0.800 \mathrm{C}$
$\Delta t_{\mathrm{o}}=10.0 \mathrm{~s}$

## Strategy

Select the equation that relates dilated time to proper time.

Substitute into the equation.

Solve.

$$
\begin{aligned}
& \text { Implied } \\
& c=3.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

## Unknown

$\Delta t$

## Calculations

$$
\begin{aligned}
& \Delta t=\frac{\Delta t_{0}}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& \Delta t=\frac{10.0 \mathrm{~s}}{\sqrt{1-\frac{(0.800 c)^{2}}{c^{2}}}}
\end{aligned}
$$

$$
\Delta t=\frac{10.0 \mathrm{~s}}{0.600}
$$

$$
\Delta t=16.67 \mathrm{~s}
$$

$$
\Delta t \cong 16.7 \mathrm{~s}
$$

The time as seen by an observer on the asteroid would be 16.7 s .

## Validate

The dilated time is expected to be longer than the proper time, and it is.

## PRACTICE PROBLEMS

1. A tau ( $\tau$ ) particle has a lifetime measured at rest in the laboratory of $1.5 \times 10^{-13} \mathrm{~s}$. If it is accelerated to $0.950 c$, what will be its lifetime as measured in (a) the laboratory frame of reference, and (b) the $\tau$ particle's frame of reference?
2. A rocket passes by Earth at a speed of 0.300 c . If a person on the rocket takes

245 s to drink a cup of coffee, according to his watch, how long would that same event take according to an observer on Earth?
3. A kaon particle ( $\kappa$ ) has a lifetime at rest in a laboratory of $1.2 \times 10^{-8} \mathrm{~s}$. At what speed must it travel to have its lifetime measured as $3.6 \times 10^{-8} \mathrm{~s}$ ?

## Thought Experiment 3: Length Contraction

Imagine the following situation. Captain Quick is a comic book hero who can run at nearly the speed of light. In her hand, she is carrying a flare with a lit fuse set to explode in $1.50 \mu \mathrm{~s}$ $\left(1.50 \times 10^{-6} \mathrm{~s}\right)$. The flare must be placed into its bracket before this happens. The distance ( $L$ ) between the flare and the bracket is 402 m .


Figure 17.11 A race against time

- Captain Quick runs at $\frac{2}{3} c$ ( $2.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$ ) and arrives at the bracket in time. According to classical mechanics, this would not be possible because it should take $2.01 \mu \mathrm{~s}$ as shown on the right.
- However, to an observer in the stationary frame of reference, the time for the fuse to burn will be dilated in relation to his own frame of reference. It will take $2.01 \mu$ s for the fuse to burn and therefore, Captain Quick will reach the bracket in time.
- Since Captain Quick and the fuse are in the same frame of reference, however, Captain Quick should observe the fuse burning in $1.50 \mu \mathrm{~s}$. How did she make it in time? Then she realized that the only way she could have arrived in time was if the distance to the bracket in her moving frame of reference was less than the 402 m in the stationary frame. The distance must have been multiplied by the same factor by which the time was divided in the observer's frame of reference. obse

$$
\begin{aligned}
& \Delta t=\frac{L}{V} \\
& \Delta t=\frac{402 \mathrm{~m}}{2.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}} \\
& \Delta t=2.01 \times 10^{-6} \mathrm{~s} \text { or } 2.01 \mu \mathrm{~s}
\end{aligned}
$$

$$
\Delta t=\frac{\Delta t_{0}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}
$$

$$
\Delta t=\frac{1.50 \times 10^{-6} \mathrm{~s}}{\sqrt{1-\frac{\left(\frac{2}{3} c\right)^{2}}{c^{2}}}}
$$

$$
\Delta t=\frac{1.50 \times 10^{-6} \mathrm{~s}}{0.7454}
$$

$$
\Delta t=2.01 \times 10^{-6} \mathrm{~s}
$$

$L=L_{0} \sqrt{1-\frac{V^{2}}{c^{2}}}$
$L=(402 \mathrm{~m}) \sqrt{1-\frac{\left(\frac{2}{3} c\right)^{2}}{c^{2}}}$
$L=(402 \mathrm{~m})(0.7454)$
$L=300 \mathrm{~m}$

- If the distance was smaller, then Captain Quick could make it to the bracket before the fuse burned out.

$$
\begin{aligned}
& \Delta t=\frac{L}{V} \\
& \Delta t=\frac{300 \mathrm{~m}}{2.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}} \\
& \Delta t=1.50 \times 10^{-6} \mathrm{~s} \text { or } 1.50 \mu \mathrm{~s}
\end{aligned}
$$

This thought experiment illustrates that two ideas go hand in hand. If two observers are moving relative to each other, then a time dilation from one observer's point of view will be balanced by a corresponding length contraction from the other observer's point of view.

In the box below, $L_{0}$ represents the proper length, which is the length as measured by an observer at rest relative to the object or event and $L$ is the contracted length seen by the moving observer.

## LENGTH CONTRACTION

The contracted length is the product of the proper length and the expression, square root of one minus the velocity of the moving reference frame squared divided by the speed of light squared.

$$
L=L_{0} \sqrt{1-\frac{v^{2}}{c^{2}}}
$$

## Quantity

contracted length
proper length
velocity of the moving reference frame speed of light

$$
\begin{array}{cl}
\text { Symbol } & \text { SI unit } \\
L & \mathrm{~m} \text { (metres) } \\
L_{\mathrm{o}} & \mathrm{~m} \text { (metres) } \\
V & \frac{\mathrm{~m}}{\mathrm{~s}} \text { (metres per second) } \\
& \frac{\mathrm{m}}{\mathrm{~s}} \text { (metres per second) } \\
C & \text { m }
\end{array}
$$

## Unit Analysis

$$
\begin{aligned}
& \mathrm{m}=\frac{\mathrm{m}}{\sqrt{1-\frac{\left(\frac{n}{s}\right)^{2}}{\left(\frac{m}{s}\right)^{2}}}}=\mathrm{m}
\end{aligned}
$$

Note: Length contraction applies only to lengths measured parallel to the direction of the velocity. Lengths measured perpendicular to the velocity are not affected.

This thought experiment seems to yield strange results that go against common experience. However, the results explain a phenomenon involving a tiny particle called the "mu meson" (or muon). This particle has a lifetime of $2.2 \times 10^{-6} \mathrm{~s}$ and is formed about $1.0 \times 10^{4} \mathrm{~m}$ above the surface of Earth, speeding downward at about 0.998 c . At that speed (according to classical mechanics), it should travel only about 660 m before decaying into other particles, but it is observed in great numbers at Earth's surface. The relativistic explanation is that the muon's lifetime as measured by Earth-based observers has been dilated as follows.

$$
\begin{aligned}
& \Delta t=\frac{\Delta t_{0}}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& \Delta t=\frac{2.2 \times 10^{-6} \mathrm{~s}}{\sqrt{1-\frac{(0.998 \mathrm{c})^{2}}{c^{2}}}} \\
& \Delta t=\frac{2.2 \times 10^{-6} \mathrm{~s}}{0.0632} \\
& \Delta t=3.5 \times 10^{-5} \mathrm{~s}
\end{aligned}
$$

The distance travelled becomes

$$
\begin{aligned}
& \Delta d=v \Delta t \\
& \Delta d=(0.998)\left(3.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}\right)\left(3.5 \times 10^{-5} \mathrm{~s}\right) \\
& \Delta d=1.0 \times 10^{4} \mathrm{~m}
\end{aligned}
$$

At that speed, the muon's lifetime is so expanded (according to the observers on Earth) that the particle can reach the surface. On the other hand, the muon sees its own lifetime as unchanged, and from its frame of reference, Earth's surface is rushing toward it at 0.998 c . The distance it sees to Earth's surface is given by

$$
\begin{aligned}
& L=L_{0} \sqrt{1-\frac{V^{2}}{c^{2}}} \\
& L=\left(1.0 \times 10^{4} \mathrm{~m}\right) \sqrt{1-\frac{(0.998 c)^{2}}{c^{2}}} \\
& L=\left(1.0 \times 10^{4} \mathrm{~m}\right)(0.0632) \\
& L=632 \mathrm{~m}
\end{aligned}
$$

This reduced distance would take a shorter time, given by

$$
\begin{aligned}
& \Delta t=\frac{\Delta x}{v} \\
& \Delta t=\frac{632 \mathrm{~m}}{(0.998)\left(3.0 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}\right)} \\
& \Delta t=2.1 \times 10^{-6} \mathrm{~s}
\end{aligned}
$$

The muon therefore can reach Earth's surface before decaying.

## Which Is Correct?

The physicist standing on the surface of Earth claims that the lifetime of the muon is $3.5 \times 10^{-5} \mathrm{~s}$ and its height above Earth's surface is $1.0 \times 10^{4} \mathrm{~m}$. From the muon's point of view, however, its lifetime is $2.2 \times 10^{-6} \mathrm{~s}$ and its height is 632 m . Which is correct?

Both statements are correct. The value of any measurement is tied to the frame of reference in which that measurement is taken. Going from one inertial frame of reference to another will involve differences in the measurement of lengths and times. Normally, these differences are too small to be observed, but as relative speeds approach the speed of light, these differences become quite apparent.

## Gamma Saves Time

When solving problems involving relativistic speeds (speeds approaching the speed of light), you will often need to calculate the value of $\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$. Physicists have assigned the symbol gamma $(\gamma)$ to this value, or $\gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$. Using the $\gamma$ notation, the length and time equations become $\Delta t=\gamma \Delta t_{0}$ and $L=\frac{L_{0}}{\gamma}$.

## MODEL PROBLEM

## Relativistic Lengths

A spacecraft passes Earth at a speed of $2.00 \times 10^{\mathbf{8}} \mathbf{~ m} / \mathrm{s}$. If observers on Earth measure the length of the spacecraft to be 554 m , how long would it be according to its passengers?

## Frame the Problem

- Length appears to be shorter, or contracted, to the observer who is moving relative to the object being measured.
- The amount of length contraction that occurs is determined by the relative speeds of the reference frames of the two observers.


## Identify the Goal

The length of the spacecraft, $L_{0}$, as seen by its passengers

## Variables and Constants

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $V=2.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$ | $c=3.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$ | $L_{\mathrm{o}}$ |
| $L=554 \mathrm{~m}$ |  |  |

## Strategy

Calculate gamma.

Use the equation that describes

Solve.

## Calculations

$$
\begin{aligned}
& \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& \gamma=\frac{1}{\sqrt{1-\frac{\left(2.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{\left(3.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}}} \\
& \gamma=1.342
\end{aligned}
$$

## length contraction.

$$
\begin{aligned}
L & =\frac{L_{0}}{\gamma} \\
L_{0} & =L \gamma \\
L_{0} & =(554 \mathrm{~m})(1.342) \\
L_{0} & \cong 743 \mathrm{~m}
\end{aligned}
$$

The length of the spacecraft as seen by its passengers is 743 m .

## Validate

The proper length is expected to be longer than the contracted length, and it is.

## PRACTICE PROBLEMS

4. An asteroid has a long axis of 725 km . A rocket passes by parallel to the long axis at a speed of 0.250 c . What will be the length of the long axis as measured by observers in the rocket?
5. An electron is moving at 0.95 c parallel to a metre stick. How long will the metre stick be in the electron's frame of reference?
6. A spacecraft passes a spherical space station. Observers in the spacecraft see the station's minimum diameter as 265 m and the maximum diameter as 325 m .
(a) How fast is the spacecraft travelling relative to the space station?
(b) Why does the station not look like a sphere to the observers in the spacecraft?

## PHYSICS FILE

Einstein's equations allow a particle to travel faster than light if it was already travelling faster than light when it was created. For such particles (called "tachyons"), the speed of light represents the slowest speed limit. Although the equations say that tachyons can exist, there is no evidence that they do. In fact, no one knows how they would interact with normal matter.

## The Universal Speed Limit

Calculation of expanded times and contracted lengths involve the expression $\sqrt{1-\frac{V^{2}}{c^{2}}}$. Since times and lengths are measurements, they must be represented by real numbers, so the value under the square root must be a positive real number. For this to be true, $\frac{v^{2}}{c^{2}}<1$. This implies that $v<c$. If $v$ approaches $c$, the value of gamma approaches infinity. Consider what happens to $\Delta t$ when $v$ approaches $c$ in $\Delta t=\frac{\Delta t_{0}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$. The denominator approaches zero.

Division of a non-zero real number by zero is undefined so an object's speed must be less than the speed of light.

This speed limit applies only to material objects. Obviously, light can travel at the speed of light. Also, once a light pulse has been slowed down by passing into a medium such as water, objects can travel faster through that medium than can the pulse. The blue glow (called "Cerenkov radiation") emanating from water in which radioactive material is being stored is created by highspeed electrons (beta particles) that are travelling through the water faster than the speed of light through water. This phenomenon is sometimes compared to sonic boom, in which particles (in the form of a jet airplane) are travelling faster than the speed of sound in air.


Figure 17.12 The blue glow from this storage pool in a nuclear generating station comes from particles that are travelling through the water faster than the speed of light through water.

### 17.2 Section Review

1. K/U (a) Explain what is meant by an inertial frame of reference.
(b) Would a rotating merry-go-round be an inertial frame of reference? Give reasons for your answer.
2. K/U Explain the meaning of the terms "proper length" and "proper time."
3. (1) An arrow and a pipe have exactly the same length when lying side by side on a table. The arrow is then fired at a relativistic speed through the pipe, which is still lying
on the table. Determine whether there is a frame of reference in which the arrow can
(a) be completely inside the pipe with extra pipe at each end (b) overhang the pipe at each end Give reasons for your answers.
4. K/U Explain the meaning of the terms "length contraction" and "time dilation."
5. C Explain why the results of the Michelson-Morley experiment were so important.
6. K/U (a) In the diagram, two stars (A and B) are equidistant from a planet $(\mathrm{P})$ and are at rest relative to that planet. They both explode into novas at the same time, according to an observer on the planet. From the point of view of passengers in a rocket ship travelling past at relativistic speeds, however, which star went nova first? Give reasons for your answer.
(b) Where could the observer stand on the planet in order to see both stars at the same time?

7. K/O Part (A) of the diagram shows a star (S) located at the midpoint between two planets (A) and (B), which are at rest relative to the star. The star explodes into a supernova.
(a) In the frame of reference of the planets, which planet saw the supernova first? Give reasons for your answer.
(b) A spacecraft is passing by as shown in part (B). In its frame of reference, the star and planets are moving as shown in part (C). In the spacecraft frame of reference, which planet saw the supernova first? Give reasons for your answer.

A

8. (a) Imagine that you are riding along on a motorcycle at $22 \mathrm{~m} / \mathrm{s}$ and throw a ball ahead of you with a speed of $35 \mathrm{~m} / \mathrm{s}$.

What will be the speed of that ball relative to the ground?
(b) If the velocity of the motorcycle relative to the ground is $V_{\mathrm{mg}}$, the velocity of the ball relative to the motorcycle is $V_{b m}$, and the velocity of the ball relative to the ground is $V_{b g}$, state the vector equation for calculating the velocity of the ball relative to the ground.
(c) Apply this formula to a situation in which the motorcycle is travelling at 0.60 c and the ball is thrown forward with a speed of 0.80 c . What is the speed of the ball relative to the ground? What is wrong with this answer?
(d) In the special theory of relativity, the formula for adding these velocities is

$$
V_{\mathrm{bg}}=\frac{V_{\mathrm{bm}}+v_{\mathrm{mg}}}{1+\frac{v_{\mathrm{bm}} \cdot v_{\mathrm{mg}}}{c^{2}}}
$$

- What does this formula predict for the answer to (c)?
- What does this formula predict for the answer to (a)?
- Imagine that you are travelling in your car at a speed of 0.60 c and you shine a light beam ahead of you that travels away from you at a speed of $c$. According to this formula, what would be the speed of that light beam relative to the ground?


## UNIT PROJECT PREP

How would the general public have received the new information in Einstein's special theory of relativity?

- Do you believe that at the turn of the twentieth century society had more or less faith in science than people do today? Why or why not?
- Dramatic events often steer thinking into new directions. Do you believe that Einstein was affected by any one particular event as he developed his theories?
- Are you able to link recent societal events with current changes in the direction of scientific research?


## Not Even the Sky's the Limit!

When a signal leaves a satellite or interplanetary space probe, a special code is embedded in it to give it a time-stamp. When the signal is picked up on Earth, that time-stamp is compared to a terrestrial clock. Subtracting the two gives the travel time between the satellite or space probe and the ground station. Since the signal travels at the speed of light, all you should need to do then is multiply the time by $c$ to determine the distance but it's not that simple.


Gravity is part of the problem. According to Einstein's general theory of relativity, clocks run more slowly in a gravitational field. The stronger the field is, the slower the clocks run. Clocks on board spacecraft and satellites run slightly faster in interplanetary space than they do near Earth. These timing differences result in distance measurement differences between what is observed from a ground station and from a spacecraft. The situation becomes even more complicated as the spacecraft dips into and out of the gravitational fields of planets that it encounters on its voyage.

A second problem results from the relative velocity between the spacecraft and the ground station. Einstein's special theory of relativity, discussed in detail in this chapter, describes how time intervals and distance measurements vary between inertial frames of reference that are in motion relative to each other. This relative velocity is continually changing as a result of the gravity of the Sun and planets and due to Earth's orbital and rotational velocities. Relativistic corrections numerical adjustments based on the theory of relativity - are an ongoing challenge in spacecraft instrument design.

There are "a whole suite of careers that utilize these things," Steve Lichten, manager of the Tracking Systems and Applications Section of NASA's Jet Propulsion Laboratory, says of relativity. Einsteinian physics is no longer the sole property of university researchers. Commercial satellite manufacturers must have an understanding of relativity in order for their products to work.

Theoreticians, engineers, and computer scientists must work together to help a spacecraft communicate with its ground station, so the companies that manufacture spacecraft and commercial satellites are always on the lookout for people with the necessary knowledge. Generally, an advanced graduate degree in engineering, physics, or mathematics is preferred, although a bachelor's degree in science with a demonstrated understanding of the concepts and techniques involved will go a long way.

So brush up your math skills and keep doing those thought experiments. Some day, they might take you to the stars!

## Going Further

1. Describe some examples of satellites that require extremely precise distance and time measurements. Explain why such precision is necessary for those satellites.
2. Many companies that manufacture satellites or equipment for use on satellites (including space stations) offer summer internship programs for interested students. Find out if any of these companies are located near you and call them. You might be able to get a head start on a great career!
3. Research the space probes, such as the one shown in the photograph, that are currently active. Explain why precise knowledge of time intervals and distances is of extreme importance to the operation of space probes.
[^0]
[^0]:    ## (D) Web Link

    www.mcgrawhill/ca/links/atlphysics
    For information about the NASA Jet Propulsion Laboratory's past, current, and planned space missions, go to the above Internet site and click on Web Links.

