17.1 Troubles with the Speed of Light

SECTION OUTCOMES

- State Einstein's two postulates for the special theory of relativity.
- Conduct thought experiments as a way of developing an abstract understanding of the physical world.
- Outline the historical development of scientific views and models of matter and energy.

KEY TERMS

- interferometer
- Lorentz-Fitzgerald contraction

Toward the end of the nineteenth century, many scientists felt that they were close to a complete understanding of the physical world. Newton's laws described motion. Maxwell's laws described radiant energy. The chemists were learning more and more about the behaviour of atoms. No one realized that their fundamental concepts of space, time, matter, and energy were seriously limited.

The Michelson-Morley Experiment

The first indication of a difficulty came from a critical experiment performed in 1881 by Albert Michelson (1852–1931), using an **interferometer**, an instrument he had devised for measuring wavelengths of light. In this experiment, he unsuccessfully attempted to detect the motion of Earth through the luminiferous ether, the substance that was then believed to be the medium through which light waves could travel through space. The apparent failure of Michelson's first experiment to find any such motion prompted many physicists to drop the ether concept.

Later, in 1887, Michelson and Edward Williams Morley (1838–1923) performed a refined version of the experiment, using an improved version of the interferometer. They reasoned that if light behaved like a sound wave or a wave on water, if you moved toward an oncoming beam of light, it would seem to approach you at a higher speed than if you were moving away from it. These different speeds would affect the interference pattern in the interferometer. By comparing interference patterns for light beams travelling perpendicular to each other, Michelson and Morley hoped to detect and measure the speed with which Earth passed through the ether.

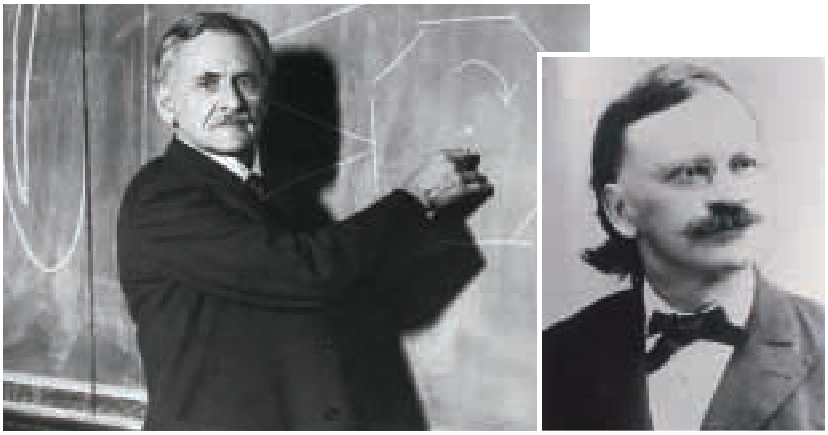

Figure 17.1 Albert Michelson (left) and Edward Morley used Michelson's interferometer (see Figure 17.2) to conduct an experiment that later became the foundation of Einstein's special theory of relativity.

To understand the basis of this experiment, consider the following scenario involving relative velocities. Two identical boats, X and Y, are about to travel in a stream. Boat Y will go straight across the stream and straight back. Boat X will travel the same distance downstream and then return to its starting point. Which boat will make the trip in the shortest time? Examine Figure 17.3 and then follow the steps below to determine the time required for boat Y to travel across the stream and back.

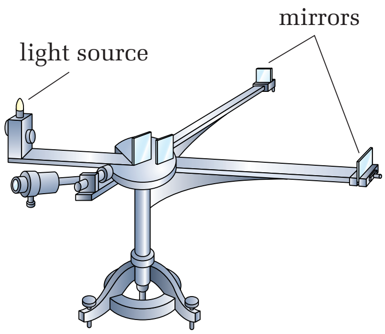

Figure 17.2 Michelson's first interferometer was designed to determine wavelengths of light. It should also be able to determine whether light travelling in directions perpendicular to each other travelled at different speeds.

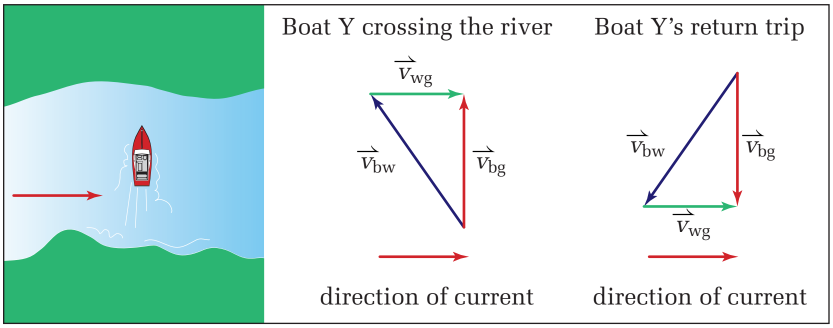

Figure 17.3 Since boat Y must go directly across the stream, the driver must angle the boat upstream while crossing either way perpendicular to the current.

- Define the symbols.

$\vec{v}_{bw}$: velocity of the boat relative to the water
$\vec{v}_{wg}$: velocity of the water relative to the ground
$\vec{v}_{bg}$: velocity of the boat relative to the ground
L : distance travelled along each leg of the trip
Δt : total time for the trip

- Write the definition for velocity and solve it for the time interval.

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$
$$\Delta t = \frac{\Delta \vec{d}}{\vec{v}}$$

- Use vector addition to find the magnitude of the velocity of the boat relative to the ground. Notice in Figure 17.3 that this velocity is the same for both legs of the trip.

$$(v_{bw})^2 = (v_{bg})^2 + (v_{wg})^2$$
$$(v_{bg})^2 = (v_{bw})^2 - (v_{wg})^2$$
$$v_{bg} = \sqrt{(v_{bw})^2 - (v_{wg})^2}$$

- Substitute the total length of the trip ($2L$) and the magnitude of the velocity into the expression for the time interval to find the time required for boat Y to make the round trip.

$$\Delta t_Y = \frac{2L}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}$$

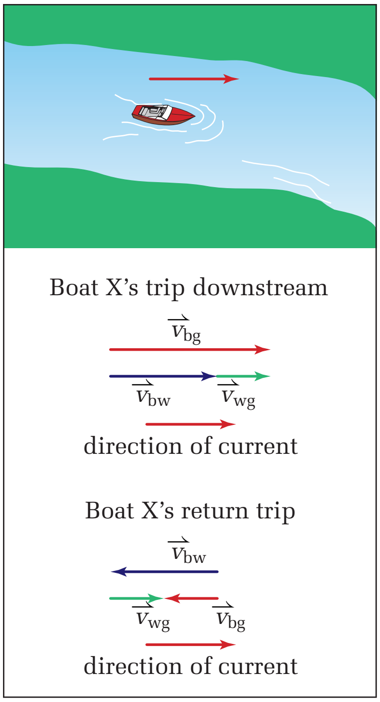

Figure 17.4 Boat X travels with the current when it is going downstream and against the current on its return trip.

Study Figure 17.4 to determine the velocities of boat X as it makes its trip downstream and back. Then, follow the steps below that determine the time for boat X to make the trip.

- Since the direction of the velocities of boat X and of the stream are in one dimension, the magnitudes can be added algebraically.

  Trip downstream: $v_{bg} = v_{bw} + v_{wg}$
  Trip upstream: $v_{bg} = v_{bw} - v_{wg}$

- Use the equation for the time interval in terms of displacement and velocity to write the time interval for boat X to travel downstream.

$$\Delta t_{down} = \frac{L}{v_{bw} + v_{wg}}$$

- Write the time interval for boat X to travel back upstream.

$$\Delta t_{up} = \frac{L}{v_{bw} - v_{wg}}$$

- To find the total time for boat X to make the round trip, add the time intervals for the two directions.

$$\Delta t_X = \frac{L}{v_{bw} + v_{wg}} + \frac{L}{v_{bw} - v_{wg}}$$

- Find a common denominator and simplify.

$$\Delta t_X = \frac{L(v_{bw} - v_{wg}) + L(v_{bw} + v_{wg})}{(v_{bw} + v_{wg})(v_{bw} - v_{wg})}$$

$$\Delta t_X = \frac{Lv_{bw} - Lv_{wg} + Lv_{bw} + Lv_{wg}}{(v_{bw})^2 - (v_{wg})^2}$$

- The time required for boat X to travel downstream and return is

$$\Delta t_X = \frac{2Lv_{bw}}{(v_{bw})^2 - (v_{wg})^2}$$

So, did boat Y or boat X complete the trip more quickly? You can find this out by dividing Δt_X by Δt_Y.

- Divide Δt_X by Δt_Y.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{\dfrac{2v_{bw}L}{(v_{bw})^2 - (v_{wg})^2}}{\dfrac{2L}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}}$$

- Simplify.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{v_{bw}}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}$$

- Divide the numerator and denominator by v_{bw} and simplify.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{1}{\sqrt{1 - \frac{(v_{wg})^2}{(v_{bw})^2}}}$$

Since the denominator is less than one, the ratio is greater than one; thus, Δt_X is greater than Δt_Y — boat Y was faster.

Math Link

Normally, taking a square root results in both positive and negative roots. However, since both time intervals were measured forward from a common starting point, they must both be positive, so the ratio must also be positive.

In the Michelson-Morley experiment, the speed of light through the luminiferous ether, usually represented by c, is equivalent to the speed of a boat through water. The speed of the water relative to the ground is equivalent to the speed of the ether relative to Earth. Because motion is relative, it is also the speed of Earth relative to the ether. If this speed is represented by v, the time ratio can be written as

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This means that light that is moving back and forth parallel to the motion of Earth should take longer to complete the trip than light that is moving back and forth perpendicular to the motion of Earth.

Language Link

The "ether" to which the text refers is not the chemical form of ether. It stems from the Latin word *aether* and was thought to be a highly rarefied medium through which light and other electromagnetic waves travelled. The word "ethereal" comes from this concept.

Conceptual Problems

- As you study the Michelson-Morley experiment, you will find similarities to this problem. Keep your answers in mind as you read further. Suppose two identical boats can travel at 5.0 m/s relative to the water. A river is flowing at 3.0 m/s. Boat Y travels 1.00×10^2 m straight across the river and then the same distance back. Boat X travels 1.00×10^2 m upstream and then returns the same distance.
  (a) Which boat makes the trip in the shortest time?
  (b) How much sooner does it arrive than the other boat?
- Imagine that both of the two identical boats in the previous problem headed out from the same point at the same time. The river flows due east. Boat Y travelled 1.00×10^2 m[NW] relative to its starting point on shore and then returned straight to its starting point. Boat X travelled 1.00×10^2 m[NE] relative to its starting point on shore and then returned straight to its starting point. Which boat will make the trip in the shortest time? Hint: Sketch the vector diagrams for each case. You might not have to do any calculations.

Michelson's Interferometer

In Michelson's interferometer, a light beam is split into two beams as it passes through the beam splitter, such as a half-silvered mirror. Beam X continues straight on, while beam Y reflects at right angles to its original path. The beams reflect from mirrors and recombine as they once again pass through the beam splitter. Since the two beams do not travel precisely the same distance before they recombine, they interfere with each other as they head toward the telescope. This combination produces an interference pattern that can be observed with the telescope. Anything that

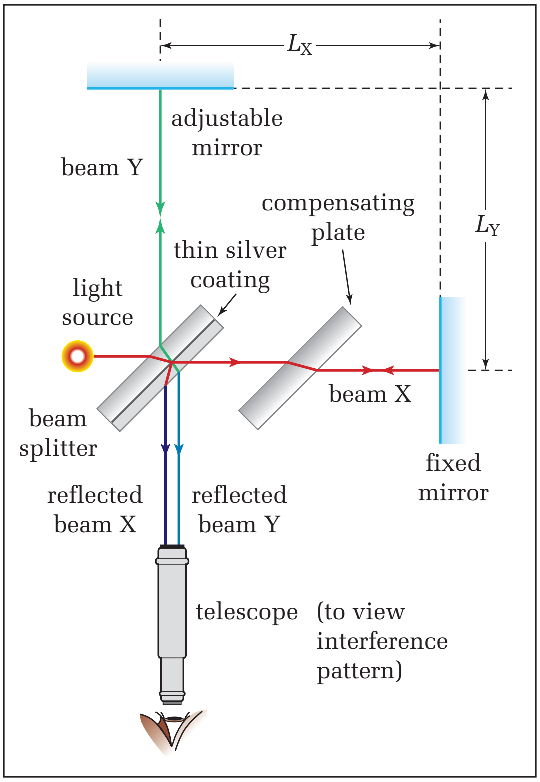

Figure 17.5 If the two beams (X and Y) are not in phase, they will interfere with each other, producing a pattern that can be seen in the telescope.

changes the time of travel of the two beams, such as moving the adjustable mirror even a small distance, produces obvious changes in the interference pattern.

When Michelson and Morley used the interferometer, they assumed that if beam X was parallel to the direction in which the planet was travelling, then that beam would take longer to reach the telescope than beam Y. This would produce a certain interference pattern. However, if the apparatus was rotated through 90˚, beam Y would lag behind. During the rotation, the interference pattern should change as the arrival time for each beam changed. Their hope was to measure this change and use it to measure the speed of Earth through the ether. The relationship between the motion of Earth and two perpendicular interferometer positions is shown in Figure 17.6.

It was an elegant experiment, and yet it seemed to be a disaster. The interference pattern refused to change. This lack of change, or nul result, greatly discouraged the two experimenters and was a source of puzzlement for other physicists. Could it be that Earth really did not move at all relative to the ether? This did not make sense, since Earth obviously orbited the Sun. Did Earth drag the luminiferous ether along with it? This did not seem likely, since that would affect the appearance of stars as seen from Earth.

One guess, which in a sense paved the way for the relativity answer, was that objects that moved through the ether were compressed, just as a spring could be compressed if it was pushed lengthwise through oil. This contraction would cause a shortening of lengths in the direction of motion, thus reducing the time required for the light to make the round trip. In this way, both light beam X and light beam Y would always arrive at the telescope at the same time. This hypothesis was known as the **Lorentz-Fitzgerald contraction**.

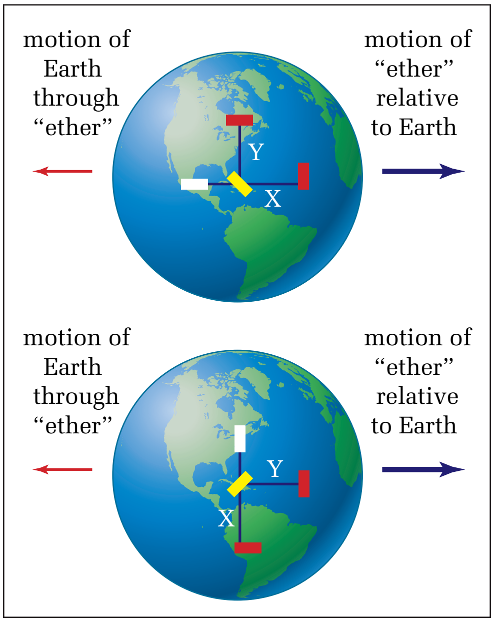

Figure 17.6 Arrival times for the light beams were expected to change when the interferometer was rotated by 90˚, but this did not happen.

The Theoretical Speed of Light

The strange results of the Michelson-Morley experiment remained a mystery for nearly two decades. Then, in 1905, the explanation came with Albert Einstein's publication of his special theory of relativity. He had developed this theory while considering the propagation of electromagnetic waves, as described by James Clerk Maxwell. Maxwell's equations showed how electromagnetic waves would spread out from accelerated charges.

In the early 1870s, Maxwell realized that a changing magnetic field could induce a changing electric field and that the changing electric field could in turn induce a changing magnetic field. Most importantly, he realized that these mutually inducing fields could spread out through space with a speed given by $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, where c represents the speed at which the fields spread out through space (the speed of light in a vacuum), ε_0 represents the electric permittivity of free space ($\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$), and μ_0 represents the magnetic permeability of free space ($\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$).

This formula yields a speed for electromagnetic radiation through space of 3.00×10^8 m/s. This was a major triumph in the field of theoretical physics, since it predicted the speed of light in terms of basic properties involving the behaviour of electric and magnetic fields in space. In addition, there was now no necessity for assuming the existence of luminiferous ether — magnetic and electric fields can exist in space without such a medium.

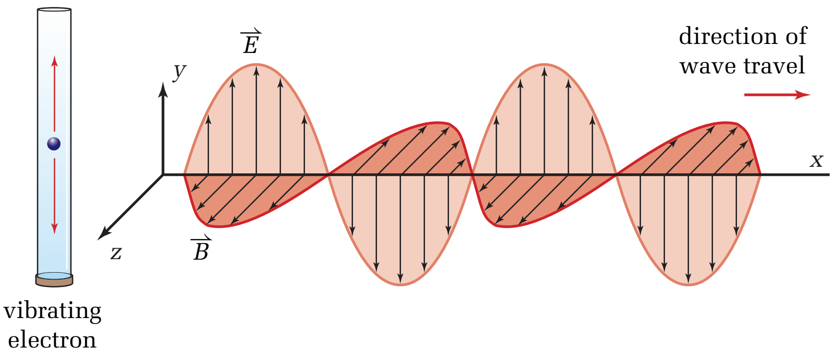

Figure 17.7 In this diagram, $\vec{E}$ represents the electric field, while $\vec{B}$ represents the magnetic field.

Einstein, however, was puzzled by an apparent inconsistency in this equation. It did not indicate any particular frame of reference. The laws of physics are expected to be valid in any inertial frame of reference. However, quantities such as speed and velocity could appear to be different from different frames of reference. For example, a race car can be seen to travel at a high speed relative to spectators in the stands. However, it might have zero velocity relative to another race car.

PHYSICS FILE

Toward the end of Michelson's life, Einstein praised him publicly for his ground-breaking experiments, which provided the first experimental confirmation for the special theory of relativity.

PHYSICS FILE

Electric permittivity is related to the Coulomb constant (k): $\varepsilon_0 = \frac{1}{4\pi k}$. Magnetic permeability comes from the expression for the strength of the magnetic field in the vicinity of a current-carrying conductor. The equation for the magnetic field, $\vec{B}$, is $\vec{B} = \frac{\mu_0 I}{2\pi r}$, where I is the current in the wire in amperes and r is the radial distance from the wire.

Apparently, there was no specified frame of reference for the speed of light in Maxwell's equation. This implied that the speed of light (and, in fact, of all members of the electromagnetic spectrum) through a vacuum should be seen as being the same in any inertial frame of reference. Einstein realized that this was indeed the case and announced his special theory of relativity.

Figure 17.8 In any race, relative velocity is all that counts.

The Special Theory of Relativity

Einstein based his special theory of relativity on two postulates.

1. All physical laws must be equally valid in all inertial frames of reference.
2. The speed of light through a vacuum will be measured to be the same in all inertial frames of reference.

The first statement had been accepted since the time of Galileo and Newton. The second one was a radical departure from the common understanding of the basics of physics, so it took scientists a long time to accept it. Eventually it was accepted, though, and the special theory of relativity is now considered to be one of the principal scientific triumphs of the twentieth century.

17.1 Section Review

1. K/U Make sketches of the velocity vectors identical to those in Figures 17.3 and 17.4 on pages 743 and 744. Label the vectors as though they represented the velocities of light through the ether, the ether relative to Earth, and light relative to Earth.

2. MC Show that the units used in Maxwell's equation for the speed of light simplify to metres per second. Note that $1\ T = 1\ N/A \cdot m$.

3. I In the interferometer shown in Figure 11.5, how far in wavelengths would the adjustable mirror have to move so that the interference pattern would return to its initial appearance? Hint: Review the interference relationships found in Chapter 9.

4. K/U What caused physicists to assume that space was filled with a medium that they called the "luminiferous ether"?

5. MC **(a)** State the two basic postulates of the special theory of relativity.

   (b) Explain why the constancy of the speed of a light beam, as seen from different inertial frames of reference, seems to be wrong. Try to use commonplace examples to make your point.