14.3 Fields and Potential Energy

SECTION OUTCOMES

- Define and describe the concepts and units related to electric and gravitational fields.
- Apply the concept of electric potential energy and compare the characteristics of electric potential energy with those of gravitational potential energy.

KEY TERMS

- electric potential difference
- equipotential surface

Did you know that it takes almost 10 t of fuel for a large passenger jet to take off? It is hard to even imagine the amount of energy required for a rocket or space shuttle to lift off. How do the engineers and scientists determine these values?

Figure 14.14 The energy to hurl this spacecraft into orbit comes from the chemical potential energy of the fuel.

MISCONCEPTION

Gravity and Orbiting Spacecraft

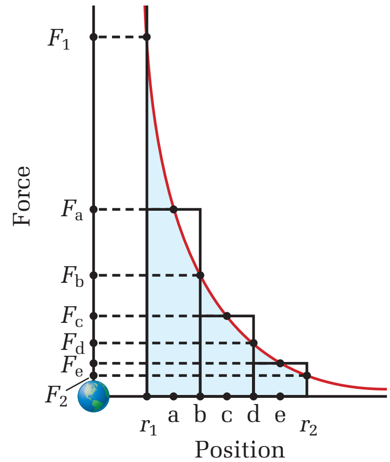
Many people believe that gravity does not act on orbiting spacecraft. In fact, a satellite such as the International Space Station *Freedom* still has about 80% of its initial weight. The impression of weightlessness comes from the fact that the weight is being used to hold the space station in its orbit. If there was no weight, it would simply continue to move off into space.

Work for Lift-Off

One way to determine the amount of energy needed to carry out a particular task is to determine the amount of work that you would have to do. When a spacecraft is lifting off from Earth, the force against which it must do work is the force of gravity.

In Chapter 12, Universal Gravitation, you learned that the equation for the gravitational force is $F_g = G\frac{m_1 m_2}{r^2}$. When working with a planet and a small object, physicists often use M for the planet and m for the small object. You can then write the equation as $F_g = G\frac{Mm}{r^2}$. In Chapter 6, you learned how to find the amount of work done by finding the area under the curve of a force versus position graph. If you were an engineer working for the space program, you would want a much more accurate value before you launched a spacecraft. In the following derivation, you will develop a general expression for the area under the curve of F_g versus r from position r_1 to r_2. This area will be the amount of work needed to raise an object such as a spacecraft of mass m from a distance r_1 to a distance r_2 from the centre of a planet of mass M.

- Draw a graph of gravitational force versus position, where the origin of the graph lies at the centre of the planet.
- Choose points r_1 and r_2. Divide the axis between r_1 and r_2 into six equal spaces and label the end point "a" through "e."
- Draw three rectangles with heights F_a, F_c, and F_e.



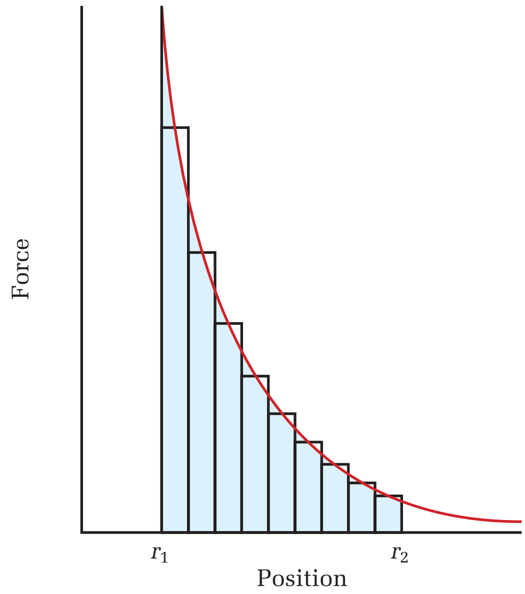
- A first rough estimate of the total work done to move m from r_1 to r_2 will be the sum of the areas of the rectangles.

$$W_{total} = W_e + W_c + W_a$$
$$W_{total} = F_a(b - r_1) + F_c(d - b) + F_e(r_2 - d)$$

- You could simplify this equation if you could express the forces in terms of the points on the curve at the ends of the rectangles, instead of the centre. For example, how can you express F_a in terms of F_1 and F_b? Clearly, F_a is not the average or arithmetic mean of F_1 and F_b, because the curve is an exponential curve. However, it can be accurately expressed as the *geometric* mean, which is expressed as $\sqrt{F_1F_b}$. Substitute the geometric mean of each value for force into the equation for work. Notice that in the last step, all intermediate terms have cancelled each other and only the first and last terms remain.

$$W_{total} = \sqrt{F_1F_b}(b - r_1) + \sqrt{F_bF_d}(d - b) + \sqrt{F_dF_2}(r_2 - d)$$
$$W_{total} = \sqrt{\frac{GMm}{r_1{}^2}\cdot\frac{GMm}{b^2}}(b - r_1) + \sqrt{\frac{GMm}{b^2}\cdot\frac{GMm}{d^2}}(d - b) + \sqrt{\frac{GMm}{d^2}\cdot\frac{GMm}{r_2{}^2}}(r_2 - d)$$
$$W_{total} = \frac{GMm}{r_1b}(b - r_1) + \frac{GMm}{bd}(d - b) + \frac{GMm}{dr_2}(r_2 - d)$$
$$W_{total} = GMm\left(\frac{b - r_1}{r_1b} + \frac{d - b}{bd} + \frac{r_2 - d}{dr_2}\right)$$
$$W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{b} + \frac{1}{b} - \frac{1}{d} + \frac{1}{d} - \frac{1}{r_2}\right)$$
$$W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- At first consideration, this result would appear to be a rough estimate. However, consider the fact that you could make as many rectangles as you want. Examination of the figure on the right shows that as the number of rectangles increases, the sum of their areas becomes very close to the true area under the curve. If you drew an infinite number of rectangles, your result would be precise. Now, analyze the last two mathematical steps above. No matter how many rectangles you drew, all of the intermediate terms would cancel and the result would be exactly the same as the result above. In this case, the result above is not an approximation but is, in fact, exact.



Gravitational Potential Energy

In Chapter 7, Conservation of Energy and Momentum, you demonstrated that the change in the gravitational potential energy of an object was equal to the work done in raising the object from one height to another. That relationship ($W = mg\Delta h$) was the special case, where any change in height was very close to Earth's surface. Since you are now dealing with objects being launched into space, you cannot use the special case. You must consider the change in the force of gravity as the distance from Earth increases. Fortunately, however, you have already developed an expression for the amount of work required to lift an object from a distance r_1 to a distance r_2 from Earth's centre. Therefore, the result of your derivation is equal to the change in the gravitational potential energy between those two positions.

$$\Delta E_g = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

As you know, you must choose a reference point for all forms of potential energy. Earth's surface in no longer an appropriate reference, because you are measuring distances from Earth's centre to deep into space. Physicists have accepted the convention of assigning the reference or zero point for gravitational potential energy as an infinite distance from the centre of the planet or other celestial body that is exerting the gravitational force on the object of mass m. This is appropriate because at an infinite distance, the gravitational force goes to zero. You can now state that the gravitational potential energy of an object at a distance r_2 from Earth's centre is the amount of work required to move an object from an infinite distance, r_1, to r_2.

$$E_g = GMm\left(\frac{1}{\infty} - \frac{1}{r_2}\right)$$

$$E_g = GMm\left(0 - \frac{1}{r_2}\right)$$

$$E_g = -\frac{GMm}{r_2}$$

Since there is only one distance (r_2) in the equation, it is often written without a subscript. Notice, also, that the value is negative. This is simply a result of the arbitrary choice of an infinite distance for the reference position. You will discover as you work with the concept that this reference point was very carefully chosen.

GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy of an object is the negative of the product of the universal gravitational constant, the mass of the planet or celestial body, and the mass of the object, divided by the distance from the centre of the planet or celestial body.

$$E_g = -\frac{GMm}{r}$$

Quantity	Symbol	SI unit
gravitational potential energy	E_g	J (joules)
universal gravitational constant	G	$\frac{N \cdot m^2}{kg^2}$ (newton metres squared per kilograms squared)
mass of the planet or celestial body	M	kg (kilograms)
mass of the object	m	kg (kilograms)
distance from centre of planet or celestial body	r	m (metres)

Unit Analysis

$$\text{joule} = \frac{\frac{\text{newton} \cdot \text{metre}^2}{\text{kilogram}^2} \cdot \text{kilogram} \cdot \text{kilogram}}{\text{metre}}$$

$$J = \frac{\frac{N \cdot m^{\cancel{2}}}{\cancel{kg^2}} \cdot \cancel{kg} \cdot \cancel{kg}}{\cancel{m}} = N \cdot m = J$$

Note: Use of this equation implies that the reference or zero position is an infinite distance from the planet or celestial body.

It might seem odd that the potential energy is always negative. Since changes in energy are always of interest, however, these changes will be the same, regardless of the location of the zero level.

To illustrate this concept, consider the houses in the Loire Valley in France that are carved out of the face of limestone cliffs. To the person on the cobblestone street, everyone on floors A, B, and C in Figure 14.15(A) would have positive gravitational energy, due to their height above the street. However, to a person on floor B, those on floor A are at a negative height, and so have negative gravitational potential energy relative to them. At the same time, the person on floor B would consider that people on floor C would have a positive gravitational potential energy because they are higher up the cliff.

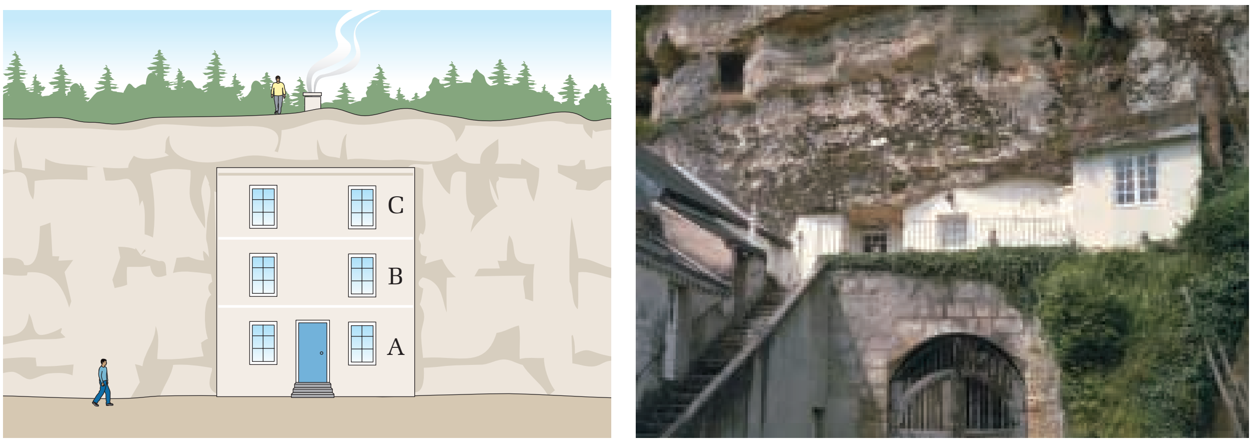


Figure 14.15 Some houses in the Loire Valley are carved out of limestone cliffs.

Naturally, the person standing on the roof beside the chimney would consider that everyone in the house had negative gravitational potential energy. All of the residents would agree, however, on the amount of work that it took to carry a chair up from floor A to floor C, so the energy change would remain the same, regardless of the observer's level. At the same time, a book dropped from a window in floor B would hit the ground with the same kinetic energy, regardless of the location of the zero level for gravitational potential energy.

Figure 14.16 is a graph of the gravitational potential energy of a 1.0 kg object as it moves away from Earth's surface. Since work must be done on that object to increase the separation, the object is often referred to as being in a gravitational potential energy "well."

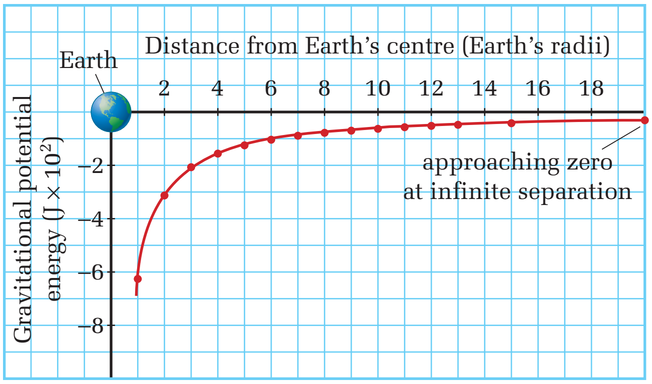


Figure 14.16 Since work must be done on the 1.0 kg object to move it away from Earth, although the gravitational potential energy is always negative, it is increasing (becoming less negative) as it retreats farther and farther from Earth.

Electric Potential Energy

As a thundercloud billows, rising ice crystals collide with falling hailstones. The hail strips electrons from the rising ice and the top of the cloud becomes predominantly positive, while the bottom is mostly negative. Negative charges in the lower cloud repel negative charges on the ground, inducing a positive region, or "shadow," on Earth below. Electric fields build and a spark ignites a cloud-to-ground lightning flash through a potential difference of hundreds of millions of volts.

The lightning bolt featured in Figure 14.17 dramatically demonstrates that when a charge is placed in an electric field, it *will* move. The potential to move implies the existence of stored energy.

Figure 14.17 Tremendous amounts of electric energy are "stored" in the electric fields created by the separation of charge between thunderclouds and the ground. This energy is often released in the "explosion" of a lightning bolt.

Previously, you derived an equation for the gravitational potential energy of one mass due to the presence of a central mass. You started the derivation by determining the amount of work that you would have to do on the first mass to move it from a distance r_1 to a distance r_2 from a central mass. Then you learned that physicists have agreed on a reference position that is assigned a value of zero gravitational potential energy. That distance is infinitely far from the central mass. In this application, an infinite distance means so far away that the magnitude of the force of gravity is negligible.

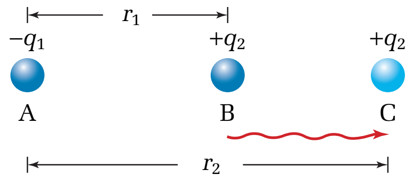


Figure 14.18 By doing work on charge q_2, you give it potential energy.

Physicists take the same approach in developing the concept of electric potential energy of a charge q_1 in the vicinity of another charge q_2 as shown in Figure 14.18. The change in electric potential energy of charge q_1 due to the presence of q_2, in moving q_1 from r_1 to r_2, is the work that you would have to do on the charge in moving it. In Figure 14.19, note the similarities in the equations for the force of gravity and the Coulomb force as well as the curves for force versus position.

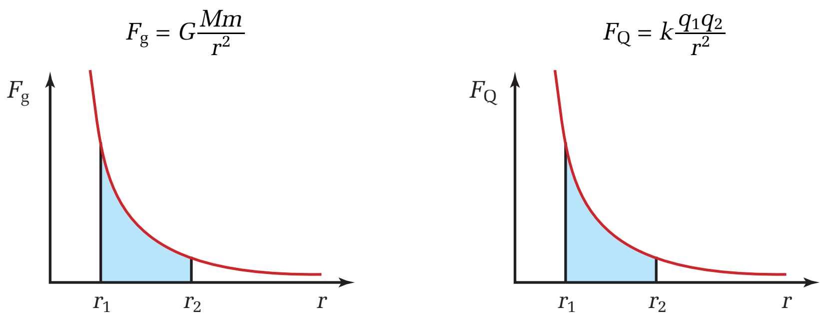


Figure 14.19 The Coulomb force and the force of gravity both follow inverse square relationships, so the curves of force versus position have exactly the same form.

Since the two equations and the two curves have identical mathematical forms, the result of the derivation of the change in the electric potential energy in moving a charge will be mathematically identical to the form of the change in the gravitational potential energy in moving a mass from position r_1 to position r_2.

$$\Delta E_g = \frac{GMm}{r_1} - \frac{GMm}{r_2} \qquad \Delta E_Q = \frac{kq_1q_2}{r_1} - \frac{kq_1q_2}{r_2}$$

When working with point charges, the choice of a reference position for electric potential energy is the same as that for gravitational potential energy — an infinite distance — so far apart that the force between the two charges is negligible. Therefore, the equations for potential energy have the same mathematical form, with one small difference: There is no negative sign in the equation for the electric potential energy.

$$E_g = -\frac{GMm}{r} \qquad\qquad E_Q = \frac{kq_1q_2}{r}$$

The negative sign is absent from the equation for electric potential energy, because the energy might be negative or positive, depending on the sign of the charges. If the charges have opposite signs, the Coulomb force between them is attractive. Consequently, if one charge moves from infinity to a distance r from the second charge, it does work and therefore has less potential energy. Less than zero is negative. If the charges have the same sign, you must do work on one charge to move it from infinity to a distance r from the second charge, and therefore it has positive potential energy. If you include the sign of the charges when using the equation for electric potential energy, the final sign will tell you whether the potential is positive or negative.

■ Two positive charges	$E_Q = \frac{k(+q_1)(+q_2)}{r}$ $E_Q > 0$	Both q_1 and q_2 are positive, so the charges have positive potential energy when they are a distance r apart.
■ Two negative charges	$E_Q = \frac{k(-q_1)(-q_2)}{r}$ $E_Q > 0$	Both q_1 and q_2 are negative, so the charges have positive potential energy when they are a distance r apart.
■ A positive and a negative charge.	$E_Q = \frac{k(+q_1)(-q_2)}{r}$ $E_Q < 0$	The product q_1q_2 is negative, so the charges have negative potential energy when they are a distance r apart.

A second difference between electric potential energy and gravitational potential energy is that the two interacting charges might be similar in magnitude. Therefore, either charge could be considered the stationary or central charge, or the "movable" charge. You could therefore consider the two charges to be a system, and refer to the electric potential energy of the system that results from the proximity of the two charges.

MODEL PROBLEM

Electric Potential Energy

What is the electric potential energy stored between charges of +8.0 μC and +5.0 μC that are separated by 20.0 cm?

Frame the Problem

- Two *charges* are close together and therefore they exert a *force* on each other.
- *Work* must be done on or to the charges in order to bring them close to each other.
- Since work was done on a charge, it has positive *electric potential energy*.

Identify the Goal

The electric potential energy, E_Q, stored between the charges

Variables and Constants

Known	Implied	Unknown
$q_1 = 8.0 \times 10^{-6}$ C	$k = 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$	E_Q
$q_2 = 5.0 \times 10^{-6}$ C		
$r = 0.200$ m		

Strategy

Write the equation for electric potential energy between two charges.

$$E_Q = k\frac{q_1 q_2}{r}$$

Substitute numerical values and solve.

$$E_Q = \frac{\left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(+8.0 \times 10^{-6}\ \text{C})(+5.0 \times 10^{-6}\ \text{C})}{0.200\ \text{m}}$$

$$E_Q = +1.8\ \text{J}$$

Calculations

The electric potential energy stored in the field between the charges is +1.8 J.

Validate

Magnitudes seem to be consistent. The units cancel to give J:
$\frac{\text{N} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{C}^2}} \cdot \frac{\cancel{\text{C}} \cdot \cancel{\text{C}}}{\cancel{\text{m}}} = \text{N} \cdot \text{m} = \text{J}$. The sign is positive, indicating that the electric potential energy is positive. A positive sign is correct for like charges, because work was done *on* the charges to put them close each other.

PRACTICE PROBLEMS

38. Find the electric potential energy stored between charges of +2.6 μC and −3.2 μC placed 1.60 m apart.

39. Two identical charges of +2.0 μC are placed 10.0 cm apart in a vacuum. If they are released, what will be the final kinetic energy of each charged object (assuming that no other objects or fields interfere)?

40. How far apart must two charges of $+4.2 \times 10^{-4}$ C and -2.7×10^{-4} C be placed in order to have an electric potential energy with a magnitude of 2.0 J?

41. Two charges of equal magnitude, separated by a distance of 82.2 cm, have an electric potential energy of 2.64×10^{2} J. What are the signs and magnitudes of the two charges?

CAREERS IN PHYSICS

Seeing Inside Storms

Blizzards can cause traffic accidents. Hurricanes can cause flooding. Tornadoes can destroy houses. Often, advance warning of these and other severe storms helps prevent deaths and reduce damage. For example, radio announcements can warn motorists to stay off roads, and municipal authorities can prepare to deal with possible flooding.

Giving advance warning is part of Dr. Paul Joe's work. Dr. Joe, a radar scientist and cloud physicist, is based at Environment Canada's radar site in King City, north of Toronto. Radar — short for *ra*dio *d*etection *a*nd *r*anging — involves transmitting pulses of electromagnetic waves from an antenna. When objects such as snowflakes or raindrops interrupt these pulses, part of their electromagnetic energy is reflected back. A receiver picks up the reflections, converting them into a visible form and indicating a storm's location and intensity.

Dr. Paul Joe, radar scientist and cloud physicist

Conventional radar cannot detect a storm's internal motions, however. This is why, in recent years, Environment Canada has been improving its radar sites across the country by adding Doppler capability. This improved radar technology applies the Doppler effect: If an object is moving toward the radar, the frequency of its reflected energy is increased from the frequency of the energy that the radar is transmitting. If an object is moving away from the radar, the frequency of its reflected energy is decreased.

"This is the same effect we notice with a subway train," Dr. Joe explains. "As it approaches, we hear a higher-pitched sound than when it leaves."

On Dr. Joe's radar screen, the frequency shifts are visualized using colours. In general, blue means an object is approaching; red means it is receding. But it's not that simple. Doppler images are complex and difficult for conventional weather forecasters to interpret, and Dr. Joe is working on ways to make them simpler. He also specializes in nowcasting — forecasting weather for the near future; for example, within an hour. As part of the 2000 Olympics, he went to Sydney, Australia, to join other scientists in demonstrating nowcasting technologies.

"I have it great," says Dr. Joe. "I love using what I've learned in mathematics, physics, and meteorology to decipher what Mother Nature is telling us and warning people about what she might do. Using the radar network, I can be everywhere chasing storms and seeing inside them in cyberspace."

Going Further

Dr. Joe's field, known in general as meteorology, includes radar science, cloud physics, climatology, and hydrometeorology. Research one of these fields and prepare a two-page report for presentation to the class.

Web Link

www.mcgrawhill.ca/links/atlphysics

The Canadian Hurricane Centre site maintained on the Internet by Environment Canada has a wide variety of information about hurricanes. Just go to the above Internet site and click on **Web Links**.

Electric Potential Difference

In section 14.1, you learned how to describe the intensity of a field at any point in that field. You are now learning how to describe the potential energy of an object in a field relative to some arbitrarily assigned reference point. Physicists have concluded that it would be convenient to be able to describe electric fields in terms of potential energy in a more general way. They have defined the **electric potential difference** as the potential energy that a unit charge would have if it were placed at a particular point in a field. This concept allows you to describe the condition of a point in a field without placing a charge at that point.

You have just derived an equation for the electric potential energy of a point charge, relative to infinity, a distance r from another point charge that can be considered as having created the field. For this case, you can find the electric potential difference between that point and infinity by considering the charge q_1 as the charge creating an electric field and q_2 as a unit charge.

- The definition of electric potential difference between a point and the reference point is $$V = \frac{E_Q}{q_2}$$
- Substitute the expression for the difference in electric potential energy of charge q_2 between the reference at infinity and the distance r from the charge q_1 due to the presence of q_1. $$V = \frac{\frac{kq_1q_2}{r}}{q_2}$$
- Since only one q, the charge creating the field, remains in the expression, there is no need for a subscript. $$V = \frac{kq}{r}$$

ELECTRIC POTENTIAL DIFFERENCE DUE TO A POINT CHARGE

The electric potential difference between any point in the field surrounding a point charge and the reference point at infinity is the product of Coulomb's constant and the electric charge divided by the distance from the centre of the charge to the point.

$$V = k\frac{q}{r}$$

Quantity	Symbol	SI unit
electric potential difference	V	V (volts)
Coulomb's constant	k	$\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$ (newton metres squared per coulombs squared)
electric charge	q	C (coulombs)
distance	r	m (metres)

Unit Analysis

$$\frac{\mathrm{N \cdot m^{\cancel{2}} \cdot \cancel{C}}}{\mathrm{C^{\cancel{2}} \cdot \cancel{m}}} = \frac{\mathrm{N \cdot m}}{\mathrm{C}} = \frac{\mathrm{J}}{\mathrm{C}} = \mathrm{V}$$

Note: Electric potential difference is a scalar quantity.

PHYSICS FILE

Physicists often use the phrase, potential at a point, when they are referring to the potential difference between that point and the reference point an infinite distance away. It is not incorrect to use the phrase as long as you understand its meaning.

Problems involving electric potential difference can be extended, as can those involving electric field, to situations in which several source charges create an electric field. Since electric potential is a scalar quantity, the electric potential difference created by each individual charge is first calculated, being careful to use the correct sign, and then these scalar quantities are added algebraically.

You can go one step further and describe the electric potential difference between two points, P_1 and P_2, within a field. To avoid confusion, this quantity is symbolized ΔV and the relationship is written as follows.

$$\Delta V = V_2 - V_1$$

Always keep in mind that V_1 and V_2 represent the electric potential difference between point 1 and infinity, and point 2 and infinity — a location so far away that the field is negligible. The following sample problems will help you to clarify these concepts in your mind.

MODEL PROBLEMS

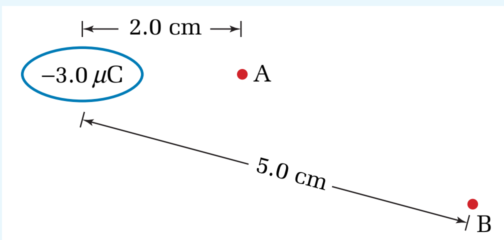
Calculations Involving Electric Potential Difference

1. **A small sphere with a charge of −3.0 μC creates an electric field.**

(a) Calculate the electric potential difference at point A, located 2.0 cm from the source charge, and at point B, located 5.0 cm from the same source charge.

(b) What is the potential difference between A and B?

(c) Which point is at the higher potential?



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Frame the Problem

- A *charged sphere* creates an *electric field*.
- At *any point* in the field, you can describe an *electric potential difference* between *that point* and a location an *infinite distance* away.
- Electric *potential difference* is a *scalar* quantity and depends only on the distance from the source charge and not the direction.
- The *potential difference* between *two points* is the *algebraic* difference between the individual potential differences of the points.

Identify the Goal

The electric potential difference, V, at each point
The electric potential difference, ΔV, between the two points
The point at a higher potential

Variables and Constants

Known	**Implied**	**Unknown**
$q = -3.0 \times 10^{-6}$ C	$k = 9.0 \times 10^{9}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}$	V_{A}
$d_{\text{A}} = 2.0 \times 10^{-2}$ m		V_{B}
$d_{\text{B}} = 5.0 \times 10^{-2}$ m		ΔV

Strategy

Use the equation for the electric potential difference at a point a distance r from a point charge.

Substitute numerical values and solve.

Calculations

$$V_{\text{A}} = k\frac{q}{d_{\text{A}}}$$

$$V_{\text{A}} = \left(9.0 \times 10^{9}\ \frac{\text{N}\cdot\text{m}}{\text{C}^2}\right)\left(\frac{-3.0 \times 10^{-6}\ \text{C}}{2.0 \times 10^{-2}\ \text{m}}\right)$$

$$V_{\text{A}} = -1.35 \times 10^{6}\ \text{V}$$

$$V_{\text{A}} \cong -1.4 \times 10^{6}\ \text{V}$$

$$V_{\text{B}} = k\frac{q}{d_{\text{B}}}$$

$$V_{\text{B}} = \left(9.0 \times 10^{9}\ \frac{\text{N}\cdot\text{m}}{\text{C}^2}\right)\left(\frac{-3.0 \times 10^{-6}\ \text{C}}{5.0 \times 10^{-2}\ \text{m}}\right)$$

$$V_{\text{B}} = -5.4 \times 10^{5}\ \text{V}$$

(a) The electric potential difference is -1.4×10^{6} V at point A, and -5.4×10^{5} V at point B.

Use algebraic subtraction to determine the potential difference between the two points.

$$\Delta V = V_{\text{B}} - V_{\text{A}}$$

$$\Delta V = (-5.4 \times 10^{5}\ \text{V}) - (-1.35 \times 10^{6}\ \text{V})$$

$$\Delta V = 8.1 \times 10^{5}\ \text{V}$$

(b) The electric potential difference, ΔV, between points A and B is 8.1×10^{5} V.

Analyze the algebraic result and validate by considering the path of a positive test charge.

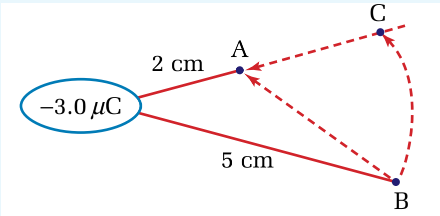
Algebraically, since $(V_B - V_A) > 0$, V_B is at the higher potential.

A positive test charge placed at point A would have to be dragged against the electric forces to get it to point B, which again places point B at the higher potential.

(c) Point B is at the higher potential.

Validate

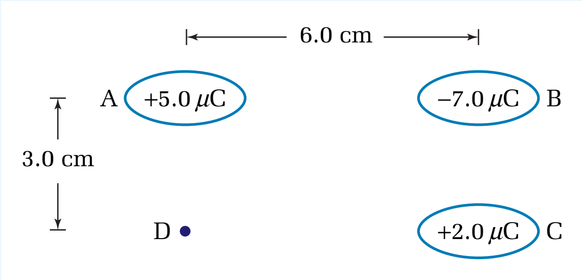
The more distant point has a smaller magnitude potential, but its negative sign makes it a higher value. The analysis with a positive test charge validates the statement of higher potential.



Note: The diagram shows two possible paths a test charge could take in moving from B to A. If the test charge followed the path BCA, no work would be done between B and C, because the force would be perpendicular to the path.

The only segment of the path where work is done by a positive test charge, and therefore the electric potential energy changes, is from C to A, parallel to the direction in which the force is acting.

2. The diagram shows three charges, A (+5.0 μC), B (−7.0 μC), and C (+2.0 μC), placed at three corners of a rectangle. Point D is the fourth corner. What is the electric potential difference between point D and the reference at infinity?



Frame the Problem

- There is an *electric potential difference* between point D, and the reference point due to each of the separate charges.
- The separate values of potential difference can be calculated and then *added algebraically*.

Identify the Goal

The electric potential difference, V, between point D and the reference at infinity

Variables and Constants

Known	Implied	Unknown
$q_A = 5.0\ \mu C$	$k = 9.0 \times 10^9\ \dfrac{N \cdot m^2}{C^2}$	$V_{\text{at D}}$
$q_B = -7.0\ \mu C$	$d_{CD} = 6.0$ cm	
$q_C = 2.0\ \mu C$		
$d_{AB} = 6.0$ cm		
$d_{AD} = 3.0$ cm		

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Strategy / **Calculations**

Calculate d_{BD}, using the Pythagorean theorem. Choose the positive value as a measure of the real distance.

$$d_{BD}^2 = (6.0\text{ cm})^2 + (3.0\text{ cm})^2$$
$$d_{BD}^2 = 45\text{ cm}^2$$
$$d_{BD} = \pm\sqrt{45\text{ cm}^2}$$
$$d_{BD} = \pm 6.7\text{ cm}$$

Calculate the contribution of each charge to the potential difference at point D independently.

$$V_{\text{A at D}} = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{+5.0 \times 10^{-6}\text{ C}}{0.030\text{ m}}\right) = 1.5 \times 10^6\text{ V}$$
$$V_{\text{B at D}} = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{-7.0 \times 10^{-6}\text{ C}}{0.067\text{ m}}\right) = -9.4 \times 10^5\text{ V}$$
$$V_{\text{C at D}} = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{+2.0 \times 10^{-6}\text{ C}}{0.060\text{ m}}\right) = 3.0 \times 10^5\text{ V}$$

Calculate the net potential difference at point D by adding the separate potential differences algebraically.

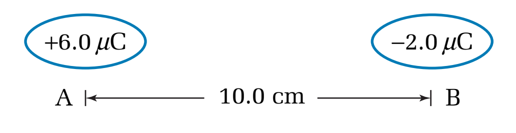
$$V_{\text{at D}} = (1.5 \times 10^6\text{ V}) + (-9.4 \times 10^5\text{ V}) + (3.0 \times 10^5\text{ V})$$
$$V_{\text{at D}} = 8.6 \times 10^5\text{ V}$$

The electric potential difference at point D is 8.6×10^5 V.

Validate

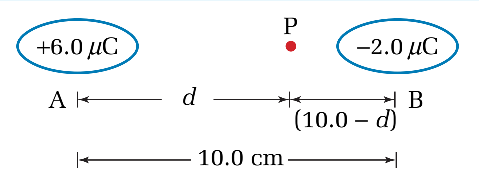
The electric potential difference contributed by A is expected to be stronger, due to its closer proximity and average charge.

3. A charge of +6.0 μC at point A is separated 10.0 cm from a charge of –2.0 μC at point B. At what locations on the line that passes through the two charges will the total electric potential be zero?



Frame the Problem

- The *total electric potential* due to the combination of charges is the *algebraic sum* of the electric potential due to *each point* alone.
- Draw a diagram and assess the likely position.
- Let the points be designated a distance d to the right of point A, and set the absolute magnitudes of the potential equal to each other. This allows for two algebraic scenarios.



Identify the Goal

The location of the point of zero total electric potential

Variables and Constants

Known	Implied	Unknown
$q_A = +6.00 \times 10^{-6}$ C	$k = 9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$	d at zero total electric potential
$q_B = -2.00 \times 10^{-6}$ C		
$d_{AB} = 10.0 \times 10^{-2}$ m		

Strategy

For the potentials to cancel algebraically, the point cannot be to the left of point A, which would be closer to the larger positive charge and could not be balanced by the potential of the negative charge. That leaves two locations: one between points A and B, and one to the right of point B, where the smaller distance to the negative charge balances the smaller value of that charge.

Calculations

$$|V_{\text{due to A}}| = |V_{\text{due to B}}|$$

Scenario 1

$$k\frac{q_A}{d} = k\frac{q_B}{(0.10 - d)}$$

$$q_A(0.10 - d) = q_B(d)$$

$$0.10q_A - q_A d = q_B d$$

$$0.10q_A = d(q_A + q_B)$$

$$d = \frac{0.10q_A}{q_A + q_B}$$

$$d = \frac{(0.10\ \text{m})(6.0\ \mu\text{C})}{6.0\ \mu\text{C} + (-2.0\ \mu\text{C})}$$

$$d = \frac{0.60\ \text{m} \cdot \cancel{\mu\text{C}}}{4.0\ \cancel{\mu\text{C}}}$$

$$d = 0.15\ \text{m}$$

Scenario 2

$$k\frac{q_A}{d} = -k\frac{q_B}{(0.10 - d)}$$

$$q_A(0.10 - d) = -q_B(d)$$

$$0.10q_A - q_A d = -q_B d$$

$$0.10q_A = d(q_A - q_B)$$

$$d = \frac{0.10q_A}{q_A - q_B}$$

$$d = \frac{(0.10\ \text{m})(6.0\ \mu\text{C})}{6.0\ \mu\text{C} - (-2.0\ \mu\text{C})}$$

$$d = \frac{0.60\ \text{m} \cdot \cancel{\mu\text{C}}}{8.0\ \cancel{\mu\text{C}}}$$

$$d = 0.075\ \text{m}$$

The points of zero potential are 7.5 cm to the right of point A and 5.0 cm to the right of point B. (Note: 15 cm to the right of A is the same as 5 cm to the right of B.)

Validate

The electric potentials due to point A at the two points are

$$(9.0 \times 10^9)\left(\frac{+6.0 \times 10^{-6}}{0.075}\right) = +7.2 \times 10^5\ \text{V} \text{ and } (9.0 \times 10^9)\left(\frac{+6.0 \times 10^{-6}}{0.15}\right) = +3.6 \times 10^5\ \text{V}$$

The electric potentials due to point B at the two points are

$$(9.0 \times 10^9)\left(\frac{-2.0 \times 10^{-6}}{0.025}\right) = -7.2 \times 10^5\ \text{V} \text{ and } (9.0 \times 10^9)\left(\frac{-2.0 \times 10^{-6}}{0.050}\right) = -3.6 \times 10^5\ \text{V}$$

In both locations, the potentials due to points A and B add algebraically to zero.

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PRACTICE PROBLEMS

42. Find the electric field due to a point charge of 4.2×10^{-7} C at a point 2.8 cm from the charge.

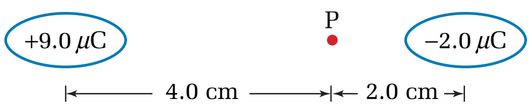
43. How far from a positive point source of 8.2 C will the electric potential difference be 5.0 V? (**Note:** 8.2 C is a very large charge!)

44. The electric potential difference due to a point charge is 4.8 V at a distance of 4.2 cm from the charge. What will be the electric potential energy of the system if a second charge of +6.0 μC is placed at that location?

45. The electric potential difference at a distance of 15 mm from a point charge is –2.8 V. What is the magnitude and sign of the charge?

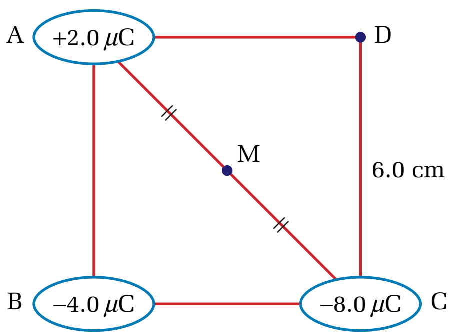
46. Point charges of +8.0 μC and –5.0 μC, respectively, are placed 10.0 cm apart in a vacuum. At what location along the line between their centres will the electric potential difference be zero?

47. What is the potential difference at point P situated between the charges +9.0 μC and –2.0 μC, as shown in the diagram.

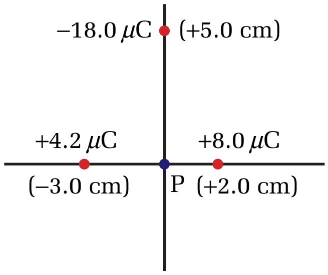


48. Point X has an electric potential difference of +4.8 V and point Y has a potential difference of –3.2 V. What is the electric potential difference, ΔV, between them?

49. Charges of +2.0 μC, –4.0 μC, and –8.0 μC are placed at three vertices of a square, as shown in the diagram. Calculate the electric potential difference at M, the midpoint of the diagonal AC.



50. The diagram shows three small charges located on the axes of a Cartesian coordinate system. Calculate the potential difference at point P.



51. Two charges are placed at the corners of a square. One charge, +4.0 μC, is fixed to one corner and another, –6.0 μC, is fixed to the opposite corner. What charge would need to be placed at the intersection of the diagonals of the square in order to make the potential difference zero at each of the two unoccupied corners?

52. Point A has an electric potential difference of +6.0 V. When a charge of 2.0 C is moved from point B to point A, 8.0 J of work are done on the charge. What was the electric potential difference of point B?

53. The potential difference between points X and Y is 12.0 V. If a charge of 1.0 C is released from the point of higher potential and allowed to move freely to the point of lower potential, how many joules of kinetic energy will it have?

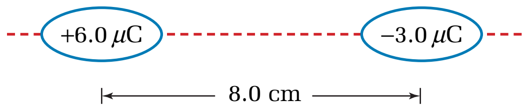
54. Identical charges of +2.0 μC are placed at the four vertices of a square of sides 10.0 cm. What is the potential difference between the point at the intersection of the diagonals and the midpoint of one of the sides of the square?

55. **(a)** If 6.2×10^{-4} J of work are required to move a charge of 3.2 nC (one nanocoulomb = 10^{-9} coulombs) from point B to point A in an electric field, what is the potential difference between A and B?

(b) How much work would have been required to move a 6.4 nC charge instead?

(c) Which point is at the higher electric potential? Explain.

56. Two different charges are placed 8.0 cm apart, as shown in the diagram. Calculate the location of the two positions along a line joining the two charges, where the electric potential is zero.

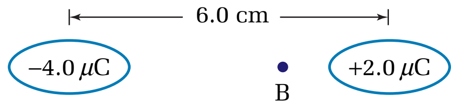


57. A charge of +8.2 nC is 10.0 cm to the left of a charge of −8.2 nC. Calculate the locations of three points, all of which are at zero electric potential.

58. A charge of −6.0 μC is located at the origin of a set of Cartesian coordinates. A charge of +8.0 μC. is 8.0 cm above it. What are the coordinates of the points at which the potential is zero?

59. A charge of +4.0 μC is 8.0 cm to the left of a point that has zero potential. Calculate three possible values for the magnitude and location of a second charge causing the potential to be zero.

60. Calculate the location of point B in the diagram below so that its electric potential is zero.



Conceptual Problem

- In practice problem 56, do you think there could be locations (other than along a line joining the two charges) where the electric potential difference could be the same, but not zero? Explain.

Equipotential Surfaces

The quantities of gravitational potential energy, electric potential energy, and electric potential difference are all scalar quantities. Although it is rarely used, there is also a quantity called "gravitational potential difference," which is defined as gravitational potential energy per unit mass. It is expressed mathematically as $V_g = \frac{E_g}{m} = -\frac{GM}{r}$. Since these are scalar quantities, the direction from the charge or mass that is creating the field does not affect the values. If you connected all of the points that are equidistant from a point mass or an isolated point charge, they would have the same potential difference and they would be creating a spherical surface. Such a surface, illustrated in Figure 14.20, is called an **equipotential surface**.

Geography Link

The equipotential lines around a system of charges could be compared to the contour lines on topographical maps. Since these contour lines represent identical heights above sea level, they also represent points that have the same gravitational energy per unit mass, and so are equipotential lines.

$$E_g = mgh$$

$$\frac{E_g}{m} = gh$$

$$V_g \propto h$$

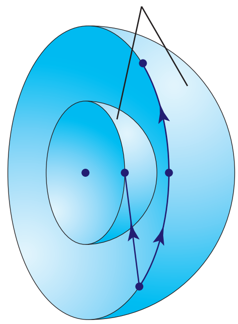


Figure 14.20 The spherical shells could represent equipotential surfaces either for a gravitational field around a point mass (or spherical mass) or for an electric field around an isolated point charge. In cross section, the equipotential spherical surfaces appear as concentric circles.

You will recall that the work done per unit charge in moving that charge from a potential V_1 to a potential V_2 is $\frac{W}{q} = V_2 - V_1$. Since, on an equipotential surface, $V_1 = V_2$, the work done must be zero. In other words, no work is required to move a charge or mass around on an equipotential surface, and the electric or gravitational force does no work on the charge or mass. Consequently, a field line must have no component along the equipotential surface. An equipotential surface must be perpendicular to the direction of the field lines at all points. Figure 14.21 shows the electric field lines and equipotential surfaces for pairs of point charges.

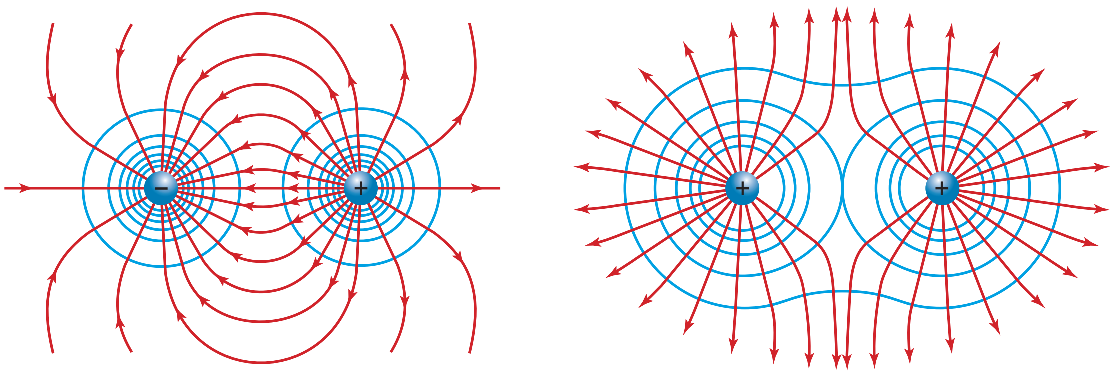


Figure 14.21 The field lines for these electric dipoles are shown in red and the cross section of the equipotential surfaces are in blue. Notice that *field lines are always perpendicular to equipotential surfaces*.

Conceptual Problem

- Could the barometric lines on a weather map be considered to be equipotential lines? Explain.

CAREERS IN PHYSICS

"Good Seeing"

The first time Dr. Neil de Grasse Tyson looked at the Moon through binoculars, he knew he wanted to be a scientist. Today, he is an astrophysicist doing research into dwarf galaxies and the "bulge" at the centre of the Milky Way. At a recent high school reunion, his former class-mates voted him the one with the "coolest job."

Astrophysicists such as Dr. Tyson study the physical properties and behaviour of celestial bodies. They make observations using optical telescopes that generate visible images of stars and other celestial bodies by means of concave mirrors. Using computers, they record the images and then examine them in detail.

To find the kinds of images they seek, astrophysicists often need to travel. One of Canada's best optical observation sites is the Dominion Astrophysical Observatory (DAO) near Victoria, B.C. The Victoria area generally has clear nights and stable weather, which make for what astrophysicists call "good seeing." A more recently developed facility is the Canada-France-Hawaii Telescope (CFHT) on the extinct Hawaiian volcano Mauna Kea. Mauna Kea, with its dark skies and super-sharp star images, is the northern hemisphere's best site for optical observations.

Mauna Kea is also one site for a new optical-infrared telescope project. Called the Gemini project, it has another site on Cerro Pachon, a mountain in central Chile. Together, the twin Gemini telescopes give astrophysicists total coverage of both the northern and southern hemispheres' skies. That's "good seeing!"

Going Further

Research one of these: the Dominion Astrophysical Observatory (DAO), the Canada-France-Hawaii Telescope (CFHT), or the Gemini Project.

14.3 Section Review

1. K/U What are the differences in the data required to calculate the gravitational potential energy of a system and the electric potential energy?

2. K/U How does the amount of work done relate to the electric potential difference between two points in an electric field?

3. MC Research and briefly report on the use of electric potential differences in medical diagnostic techniques such as electrocardiograms.

4. MC Research and report on the role played by electric potential differences in the transmission of signals in the human nervous system.

5. C Can an equipotential surface in the vicinity of two like charges have a potential of zero? Explain the reason for your answer.

6. I Investigate Internet sites that use computer programs to draw the electric field lines near a variety of charge systems. Prepare a portfolio of various patterns.

7. K/U How could you draw in the equipotential surfaces associated with the patterns obtained in question 6?

8. I **(a)** Why do you think atomic physicists tend to speak of the electrons in atoms as having "binding energy"?
   (b) Investigate the use of the term "potential well" to describe the energy state of atoms.