14.1 Laws of Force

SECTION OUTCOMES

- Define and describe the concepts related to electric, gravitational, and magnetic fields.
- Analyze and compare Coulomb's law and Newton's law of universal gravitation.
- Apply quantitatively Coulomb's law and Newton's law of universal gravitation.
- Collect, analyze, and interpret data from experiments on charged particles.

KEY TERMS

- inverse square law
- electrostatic force
- torsion balance
- Coulomb's law
- Coulomb's constant

In Chapter 5, you gained experience in applying the laws of motion of Sir Isaac Newton (1642–1727) and analyzing the motion of many types of objects. The two forces that you encounter most frequently are the forces of gravity and friction. In many cases, you have also dealt with an applied force, in which one object or person exerted a force on another. In this unit, you will focus on the nature of the forces themselves.

Gravity and the Inverse Square Law

Several astronomers and other scientists before Newton developed the concept that the force of gravity obeyed an **inverse square law**. In other words, the magnitude of the force of gravity between two masses is proportional to the inverse of the square of the distance separating their centres: $F \propto \frac{1}{r^2}$. It was Newton, though, who verified the relationship.

Figure 14.1 The centripetal force that keeps the Moon in its orbit is the gravitational force between Earth and the Moon.

Newton reasoned that, since the Moon is revolving around Earth with nearly circular motion, the gravitational force between Earth and the Moon must be providing the centripetal force. His reasoning was similar to the following.

- Write the equation for centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

- Write the equation for speed.

$$v = \frac{\Delta d}{\Delta t}$$

- The Moon travels the circumference of an orbit in one period. Therefore, its speed is

$$\Delta d = 2\pi r = 2\pi(3.84 \times 10^8\ \text{m}) = 2.41 \times 10^9\ \text{m}$$

$$T = 2.36 \times 10^6\ \text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2.41 \times 10^9\ \text{m}}{2.36 \times 10^6\ \text{s}} = 1.02 \times 10^3\ \frac{\text{m}}{\text{s}}$$

- The centripetal acceleration of the Moon is therefore

$$a_c = \frac{v^2}{r} = \frac{\left(1.02 \times 10^3\ \frac{\text{m}}{\text{s}}\right)^2}{3.84 \times 10^8\ \text{m}} = 2.71 \times 10^{-3}\ \frac{\text{m}}{\text{s}^2}$$

- If the force of gravity decreases with the square of the distance between the centre of Earth and the centre of the Moon, then the acceleration due to gravity should also decrease. Write the inverse square relationships and divide the first by the second. Since the relationships have the same quantity (acceleration) the proportionality constants would be the same. Therefore, the ratios can be set equal to each other.

$$a_{g(\text{Moon})} \propto \frac{1}{r^2_{\text{E-Moon}}}$$

$$g \propto \frac{1}{r^2_{\text{E}}}$$

$$\frac{a_{g(\text{Moon})}}{g} = \frac{\frac{1}{r^2_{\text{E-Moon}}}}{\frac{1}{r^2_{\text{E}}}} = \frac{r^2_{\text{E}}}{r^2_{\text{E-Moon}}}$$

- Solve for the acceleration due to gravity at the location of the Moon. Insert the value of g and the distances.

$$a_{c(\text{Moon})} = \frac{g r^2_{\text{E}}}{r^2_{\text{E-Moon}}}$$

$$a_{c(\text{Moon})} = \frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(6.38 \times 10^6\ \text{m})^2}{(3.84 \times 10^8\ \text{m})^2}$$

$$a_{c(\text{Moon})} = 2.71 \times 10^{-3}\ \frac{\text{m}}{\text{s}^2}$$

The values of acceleration due to gravity that were calculated in two completely different ways are in full agreement. The centripetal acceleration of the Moon in orbit is exactly what you would expect it to be if that acceleration was provided by the force of gravity and if the force of gravity obeyed an inverse square law.

The force of gravity exerts its influence over very long distances and is the same in all directions, suggesting that the influence extends outward like a spherical surface. The equation relating the surface area of a sphere to its radius is $A = 4\pi r^2$, or the area of a sphere increases as the square of the radius. You can relate the influence of the force of gravity with a portion of a spherical surface, A, at a distance r, as shown in Figure 14.2. When the distance doubles to $2r$, the area increases by a factor of 2^2, or four. When the distance increases to $4r$, the area of the sphere increases by a factor of 4^2, or 16. The influence of the force of gravity appears to be spreading out over the surface area of a sphere. How does this property of the force of gravity compare to the electromagnetic force?

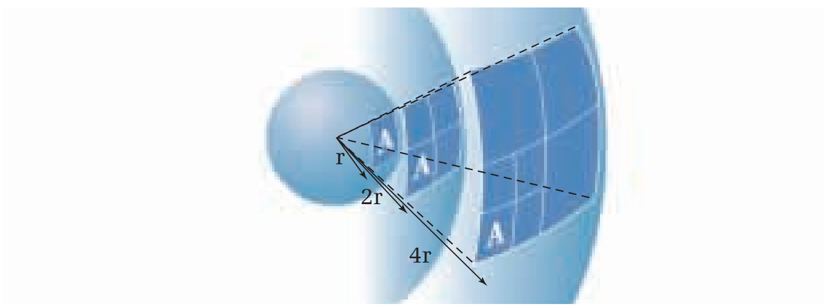


Figure 14.2 The intensity of physical phenomena that obey inverse square laws can be compared to the spreading out of the surface of a sphere.

COURSE CHALLENGE: SCANNING TECHNOLOGIES

Contact versus Non-Contact

Action at a distance — something that might have seemed magical to you as a child — lies at the heart of several cutting-edge technologies. Refer to your e-book for suggestions about non-contact interactions for a Course Challenge project.

INVESTIGATION 14-A

The Nature of the Electrostatic Force

TARGET SKILLS

- Hypothesizing
- Performing and recording
- Analyzing and interpreting

In this investigation, you will use pith balls to quantitatively analyze the electrostatic force of repulsion.

Problem

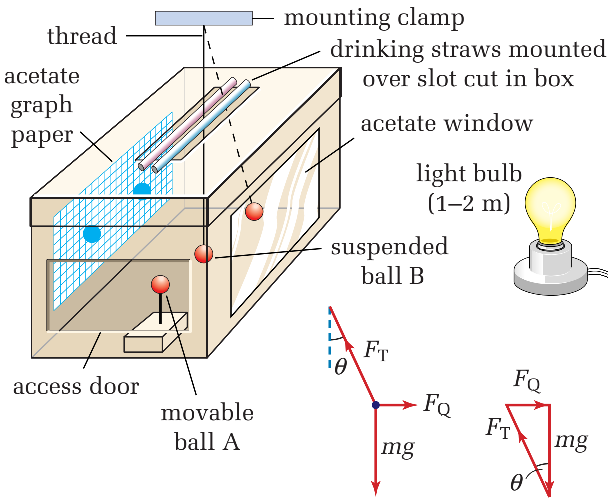
What is the relationship between electrostatic force and the distance of separation between two charged pith balls?

Equipment

- electronic balance
- clear straight filament lamp
- razor knife
- pith ball on thread
- pith ball mounted on wooden base
- acetate graph paper
- clear acetate sheet
- ebonite and fur
- cardboard shoe box
- two drinking straws

Procedure

CAUTION Be careful when using any sharp cutting object.



1. Cut rectangular holes in the front, rear, and side of the box and a slit on top, as shown.
2. Mount the clear acetate in the front hole and the acetate graph sheet in the rear. Mount the drinking straws on either side of the slit on top.
3. Poke the free end of the thread attached to pith ball B up between the drinking straws and mount on a clamp above. Ensure that the thread hangs vertically.
4. Place the pith ball with the wooden base (A) inside the box. Record the rest positions of both pith balls on the acetate grid.
5. Rub the ebonite with fur and reach in and charge both pith balls. Adjust the height of the mount of pith ball B so that it is level with pith ball A. Record the position of both pith balls.
6. Move pith ball A toward pith ball B several times. Adjust the mount of pith ball B each time to keep B level with A. For each trial, read and record the positions of both pith balls.
7. Measure the mass of a large number of balls and take an average to find the mass of one.

Analyze and Conclude

1. For each trial, use the rest positions and the final positions of the pith balls to determine the distance between A and B.
2. For each trial, use the lateral displacement of B, relative to its original rest position, to determine the electrostatic force acting on B. (Prove for yourself that $F_Q = mg\tan\theta$.)
3. Draw a graph with the electrostatic force on the vertical axis and the distance of separation between the charges on the horizontal axis. What does your graph suggest about the relationship between the electrostatic force and the distance of separation?
4. Calculate $1/r^2$ for each of your trials and plot a new graph of F versus $1/r^2$. Does your new graph provide evidence to back up the prediction you made in your original analysis? Discuss.

Electromagnetic Force

The exact nature of frictional forces and applied forces that are due to the electromagnetic force is very complex. How would anyone obtain fundamental information about such complex forces? Physicists start with the simplest cases of such forces, analyze these cases, and then extend them to more and more complex situations. The simplest case of an electromagnetic force is the electrostatic force between two stationary point charges.

Several scientists, including Daniel Bernoulli, Joseph Priestly, and Henry Cavendish, had proposed that the **electrostatic force** obeyed an inverse square relationship, based on a comparison with Newton's inverse square law of universal gravitation.

Coulomb's Experiment

French scientist Charles Augustin Coulomb (1736–1806) carried out experiments in 1785 similar to the investigation that you have just completed. Coulomb had previously developed a **torsion balance** for measuring the twisting forces in metal wires. He used a similar apparatus, shown in Figure 14.3, to analyze the forces between two charged pith balls.

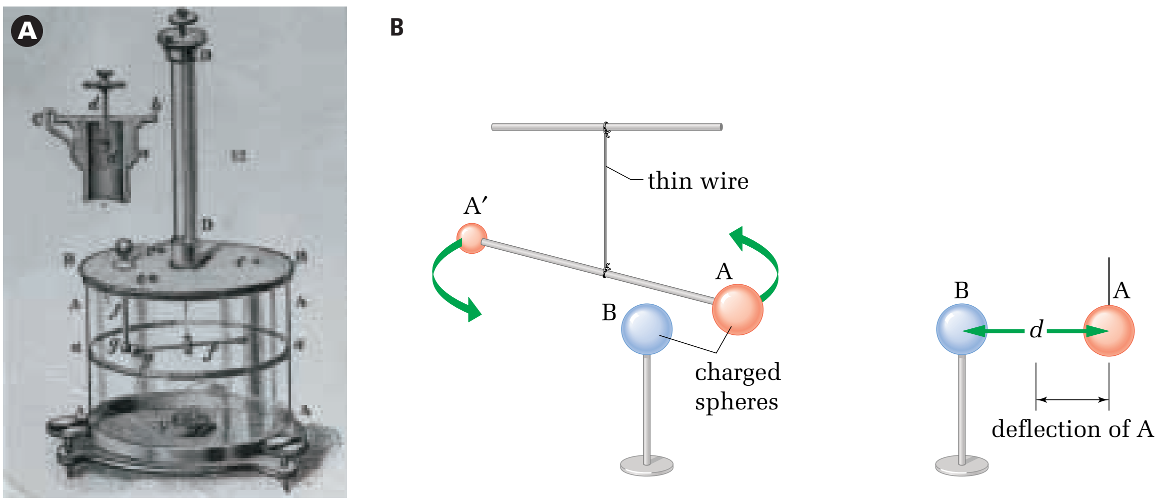


Figure 14.3 Coulomb's torsion balance **(A)** is simplified in **(B)**. Coulomb measured the force required to twist the thread a given angle. He then used this value to determine the force between the two pith balls.

Coulomb charged the two pith balls equally, placed them at precisely measured distances apart. Observing the angle of deflection, he was able to determine the force acting between them for each distance of separation. He found that the electric force, F, varied inversely with the square of the distance between the centres of the pith balls $\left(F_Q \propto \frac{1}{r^2}\right)$.

To investigate the dependence of the force on the magnitude of the charge on the pith balls, Coulomb began with two identically charged pith balls and measured the force between them. He then touched a pith ball with a third identical but uncharged pith ball to reduce the amount of charge on the ball by half. He found that the force was now only one half the previous value. After several similar modifications of the charges, Coulomb concluded that the electric force varied directly with the magnitude of the charge on each pith ball ($F_Q \propto q_1 q_2$). The two proportion statements can be combined as one $\left(F_Q \propto \frac{q_1 q_2}{r^2}\right)$ and expressed fully as **Coulomb's law**.

Any proportionality can be written as an equality by including a proportionality constant. Although the value of the constant was not known until long after Coulomb's law was accepted, it is now known to be $8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$, in SI units.

The value of the proportionality constant in a vacuum is denoted k and known as the **Coulomb constant**. In fact, air is so close to "free space" — the early expression for a vacuum — that any effect on the value of the constant is beyond the number of significant digits that you will be using. For practical purposes, the Coulomb constant is often rounded to $9.00 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

Coulomb's law can now be written as $F_Q = k\frac{q_1 q_2}{r^2}$. The direction of the force is always along the line between the two point charges. Between charges of like sign, the force is repulsive; between charges of unlike sign, the force is attractive.

PHYSICS FILE

You can develop a sense of the meaning of the Coulomb constant by considering two charges that are carrying exactly one unit of charge, a coulomb, and located one metre apart. Substituting ones into Coulomb's law, you would discover that these two charges exert a force of 9.00×10^9 N on each other. This amount of force could lift about 50 000 railroad cars or 2 million elephants. Clearly, one coulomb is an exceedingly large amount of charge. Typical laboratory charges would be much smaller — in the order of μC or millionths of a coulomb.

COULOMB'S LAW

The magnitude of the electrostatic force between two point charges, q_1 and q_2, distance r apart, is directly proportional to the magnitudes of the charges and inversely proportional to the square of the distance between their centres.

$$F_Q = k\frac{q_1 q_2}{r^2}$$

Quantity	Symbol	SI unit
electrostatic force between charges	F_Q	N (newtons)
Coulomb's constant	k	$\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$ (newton · metres squared per coulomb squared)
electric charge on object 1	q_1	C (coulombs)
electric charge on object 2	q_2	C (coulombs)
distance between object centres	r	m (metres)

Unit Analysis

$$\text{newton} = \frac{(\text{newton})(\text{metre})^2}{(\text{coulomb})^2} \cdot \frac{(\text{coulomb})(\text{coulomb})}{(\text{metre})^2}$$

$$\frac{\mathrm{N} \cdot \cancel{\mathrm{m}^2}}{\cancel{\mathrm{C}^2}} \cdot \frac{\cancel{\mathrm{C}} \cdot \cancel{\mathrm{C}}}{\cancel{\mathrm{m}^2}} = \mathrm{N}$$

PHYSICS FILE

Note that not only does the proportionality constant have to validate the numerical relationship, it must also make the units match. Thus, the units for k are obtained by rearranging the Coulomb equation.

$$k = \frac{F \cdot r^2}{q_1 \cdot q_2}$$

$$\text{units of } k = \frac{(\text{force})(\text{distance})^2}{(\text{charge})(\text{charge})}$$

$$\text{units of } k = \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

Strictly speaking, the description of Coulomb's law given on the previous page is meant to apply to point charges. However, just as Newton was able to develop the mathematics (calculus) that proved that the mass of any spherical object can be considered to be concentrated at a point at the centre of the sphere for all locations outside the sphere, so it might also be proven that if charge is uniformly distributed over the surface of a sphere, then the value of the charge can be considered to be acting at the centre for all locations outside the sphere.

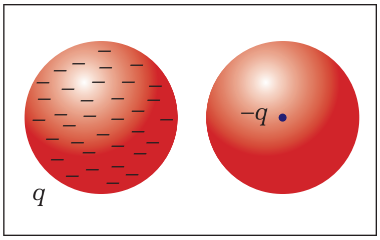


Figure 14.4 A uniformly charged sphere acts as if all of its charge is concentrated at its centre.

MODEL PROBLEM

Applying Coulomb's Law

A small sphere, carrying a charge of –8.0 μC, exerts an attractive force of 0.50 N on another sphere carrying a charge with a magnitude of 5.0 μC.

(a) What is the sign of the second charge?

(b) What is the distance of separation of the centres of the spheres?

Frame the Problem

- *Charged spheres* appear to be the same as *point charges* relative to any point *outside* of the sphere.
- Since *Coulomb's law* determines magnitudes only, signs are not used in the calculations.

Identify the Goal

The sign, ±, and separation distance, r, of the charges

Variables and Constants

Known		Implied	Unknown
$q_1 = -8.0 \times 10^{-6}$ C	$F = 0.50$ N	$k = 9.0 \times 10^{9} \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$	r
$\lvert q_2\rvert = 5.0 \times 10^{-6}$ C			

Strategy

Since the spheres are uniformly charged, they can be treated as point charges and Coulomb's law can be applied.

Calculations

$$F = k\frac{q_1 q_2}{r^2}$$

$$r^2 = \frac{kq_1q_2}{F}$$

$$r = \sqrt{\frac{kq_1q_2}{F}}$$

Only the positive root is chosen to represent the distance in this situation

$$r = \pm\sqrt{\frac{\left(9.0 \times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(8.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{5.0 \times 10^{-1}\ \text{N}}}$$

$$r = \pm 0.84853\ \text{m}$$

$$r \cong 0.85\ \text{m}$$

(a) Since the force is attractive, the second charge must be positive.

(b) The distance between the centres of the charges is 0.85 m.

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Validate

Charges in the microcoulomb range are expected to exert moderate forces on each other.

PRACTICE PROBLEMS

1. Calculate the electrostatic force between charges of $-2.4\ \mu C$ and $+5.3\ \mu C$, placed 58 cm apart in a vacuum.
2. The electrostatic force of attraction between charges of $+4.0\ \mu C$ and $-3.0\ \mu C$ is 1.7×10^{-1} N. What is the distance of separation of the charges?
3. Two identically charged objects exert a force on each other of 2.0×10^{-2} N when they are placed 34 cm apart. What is the magnitude of the charge on each object?
4. Two oppositely charged objects exert a force of attraction of 8.0 N on each other. What will be the new force of attraction if the objects are moved to a distance four times their original distance of separation?
5. Two identical objects have charges of $+6.0\ \mu C$ and $-2.0\ \mu C$, respectively. When placed a distance d apart, their force of attraction is 2.0 N. If the objects are touched together, then moved to a distance of separation of $2d$, what will be the new force between them?

QUICK LAB
Graphical Analysis of Coulomb's Law

TARGET SKILLS
- **Analyzing and interpreting**
- **Communicating results**

In this Quick Lab, you will use sample data to gain practice with the inverse square dependence of the electrostatic force between two point charges on the distance between them. Two equally charged, identical small spheres are placed at measured distances apart and the force between them is determined by using a torsion balance. Prepare a table similar to the one shown here, in which to record your data.

1. Draw a graph of force versus distance for this data. What is the shape of this graph?
2. Rearrange the distance data (use the third column in your table) and draw a graph that shows the relationship as a linear one (refer to Skill Set 4, Mathematical Modelling and Curve Straightening).
3. Measure the slope of the straight line.
4. Using the known value of Coulomb's constant ($k = 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$), calculate the value of the original charge on the spheres.

Force ($\times 10^2$ N)	Distance between centres (cm)	
5.63	1.2	
2.50	1.8	
1.30	2.5	
0.791	3.2	
0.383	4.6	
0.225	6.0	

The Nature of Electric, Magnetic, and Gravitational Forces

All forces, including electrostatic forces, are vector quantities and obey the laws of vector addition. The equation describing Coulomb's law uses only scalar quantities, with the understanding that the direction of the force always lies along the line joining the centre of the two charges. However, when one charge experiences a force from more than one other charge, the direction must be resolved.

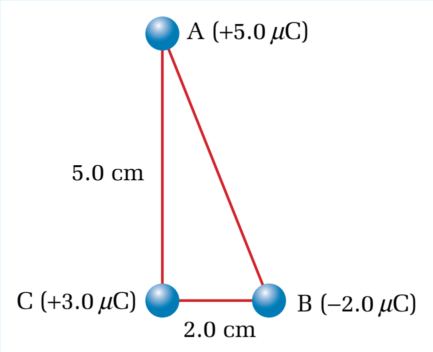
ELECTRONIC LEARNING PARTNER

Go to your Electronic Learning Partner to enhance your knowledge of Coulomb's law.

MODEL PROBLEM

Multiple Charges

Three charges, A (+5.0 μC), B (–2.0 μC), and C (+3.0 μC), are arranged at the corners of a right triangle as shown. What is the net force on charge C?



Frame the Problem

- *Charges* A and B both exert a *force* on C.
- Although A and B exert forces on each other, these forces have no effect on the forces that they exert on C.
- The *net force* on charge C is the *vector sum* of the *two forces* exerted by charges A and B.
- The *forces* exerted by A and B are related to the magnitude of the *charges* and the *distance* between the charges, according to *Coulomb's law*.

Identify the Goal

The net force, $\vec{F}_{\text{net}}$, on charge C

Variables and Constants

Known

$q_A = +5.0 \times 10^{-6}$ C

$q_B = -2.0 \times 10^{-6}$ C

$q_C = +3.0 \times 10^{-6}$ C

$r_{AC} = 5.0 \times 10^{-2}$ m

$r_{BC} = 2.0 \times 10^{-2}$ m

Implied

$k = 9.0 \times 10^{9} \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}$

Unknown

$\vec{F}_{\text{net}}$

Strategy

Use Coulomb's law to find the magnitude of the forces acting on C.

Let F_{AC} represent the magnitude of the force of charge A on charge C.

Calculations

$$F_{AC} = k\frac{q_A q_C}{r^2}$$

$$F_{AC} = \left(9.0 \times 10^{9} \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{(5.0 \times 10^{-6}\ \text{C})(3.0 \times 10^{-6}\ \text{C})}{(0.050\ \text{m})^2}$$

$$F_{AC} = 54\ \text{N}$$

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Let F_{BC} represent the magnitude of the force of charge B on charge C (attraction).

$$F_{BC} = k\frac{q_B q_C}{r^2}$$

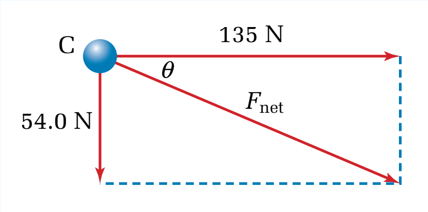
$$F_{BC} = \left(9.0 \times 10^9 \ \frac{N \cdot m^2}{C^2}\right)\frac{(2.0 \times 10^{-6}\ C)(3.0 \times 10^{-6}\ C)}{(0.020\ m)^2}$$

$$F_{BC} = 135\ N$$

Since charges A and C are both positive, the force will be repulsive and point directly downward on C.

Since B and C are oppositely charged, the force will be attractive and will point directly to the right of C.

Draw a diagram of the forces on charge C.



Use the Pythagorean theorem to calculate the magnitude of F_{net}.

$$F_{net}^2 = (135\ N)^2 + (54.0\ N)^2$$

$$F_{net}^2 = 21\ 141\ N^2$$

$$F_{net} = 145.39\ N$$

$$F_{net} \cong 1.5 \times 10^2\ N$$

Use the definition of the tangent function to find the angle, θ.

$$\tan\theta = \frac{54.0}{135}$$

$$\tan\theta = 0.40$$

$$\theta = \tan^{-1} 0.40$$

$$\theta = 21.8^\circ$$

$$\theta \cong 22^\circ$$

The net force on charge C is 1.5×10^2 N at an angle of 22° clockwise from the horizontal.

Validate

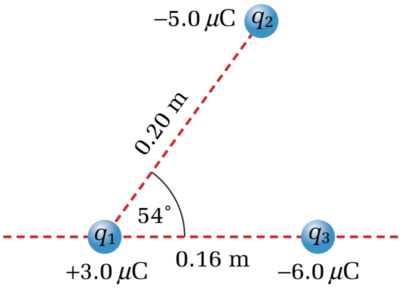
The magnitude and direction of the net force are consistent with the orientation of the three charges.

PRACTICE PROBLEMS

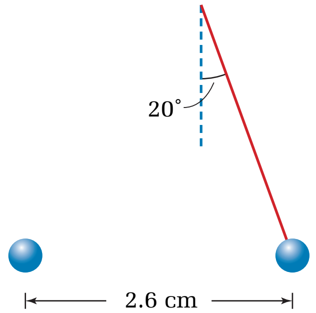
6. A single isolated proton is fixed on a surface. Where must another proton be located in relation to the first in order that the electrostatic force of repulsion would just support its weight?

7. Three charged objects are located at the vertices of a right triangle. Charge A (+5.0 μC) has Cartesian coordinates (0,4); charge B (–5.0 μC) is at the origin; charge C (+4.0 μC) has coordinates (5,0), where the coordinates are in metres. What is the net force on each charge?

8. The diagram shows three charges situated in a plane. What is the net electrostatic force on q_1?



9. The diagram below shows two pith balls, equally charged and each with a mass of 1.5 g. While one ball is suspended by a thread, the other is brought close to it and a state of equilibrium is reached. In that situation, the two balls are separated by 2.6 cm and the thread attached to the suspended ball makes an angle of 20˚ with the vertical. Calculate the charge on each of the pith balls.



10. Two 2.0 g spheres are attached to each end of a silk thread 1.20 m long. The spheres are given identical charges and the midpoint of the thread is then suspended from a point on the ceiling. The spheres come to rest in equilibrium, with their centres 15 cm apart. What is the magnitude of the charge on each sphere?

Comparing Forces

Although magnetic forces and electrostatic forces are related and both fit into the category of electromagnetic forces, the strength of a magnetic force cannot be defined in the same way as electrostatic and gravitational forces. The reason for the difference is that magnetic monopoles, if they exist, have never been detected in spite of the efforts of physicists. Where there is a north pole, you will also find a south pole. Nevertheless, Coulomb was able to approximate isolated magnetic monopoles by measuring the forces between the poles of very long, thin magnets.

If one pole of a long, thin bar magnet is placed in the vicinity of one pole of another long, thin bar magnet, Coulomb's magnetic force law states: The magnetic force F between one pole of magnetic strength p_1 and another pole of magnetic strength p_2 is inversely proportional to the square of the distance r between them, or $F \propto \frac{p_1 p_2}{r^2}$. It is not possible, however, to find a proportionality constant, because it is not possible to define a unit for p, a magnetic monopole.

You have seen that the three different types of forces — electrostatic, gravitational, and magnetic — all exhibit some form of an inverse square distance relationship. Are there any significant differences that you should note?

Probably the greatest difference between gravitational and electromagnetic forces is the strength. Gravitational forces are much weaker than electrostatic and magnetic forces. For example, you do not see uncharged pith balls, nor demagnetized iron bars, moving toward each other under the action of their mutual gravitational attraction.

In summary, the similarities and differences among electrostatic, gravitational, and magnetic forces are listed in Table 14.1.

Table 14.1 Differences among Electrostatic, Gravitational, and Magnetic Forces

Electrostatic force	Gravitational force	Magnetic force
■ can be attractive or repulsive ■ demonstrates an inverse square relationship in terms of distance ■ depends directly on the unit property (charge) ■ law easily verified using point charges (or equivalent charged spheres)	■ can only be attractive ■ demonstrates an inverse square relationship in terms of distance ■ depends directly on the unit property (mass) ■ law easily verified using point masses (or solid spheres) ■ magnitude of the force is much weaker than electrostatic or magnetic force	■ can be attractive or repulsive ■ demonstrates an inverse square relationship in terms of distance (between isolated poles) ■ depends directly on the unit property (pole strength) ■ law cannot be verified using magnetic monopoles as they have never been detected (must be simulated using long, thin magnets or thin, magnetized wire)

14.1 Section Review

1. K/U What is meant by the statement that Coulomb "quantified" the electric force?

2. K/U In what way did Coulomb determine the dependence of the electrostatic force on different variables?

3. C Explain the similarities and differences between the Coulomb experiment for charge and the Cavendish experiment on mass.

4. C Explain how, in one sense, Coulomb's law is treated as a scalar relationship, but on the other hand, its vector properties must always be considered.

5. K/U State some similarities and some differences between the gravitational force and the electrostatic force.

6. MC Research the role of electrostatic charge in technology and write a brief report on your findings. Examples could include photocopiers and spray-painting equipment.

7. I By what factor would the electrostatic force between two charges change under the following conditions?
   (a) The distance is tripled.
   (b) Each of the charges is halved.
   (c) Both of the above changes are made.