## Planetary and 12.2 Satellite Motion

A perfectly executed football or hockey pass is an amazing achievement. A fast-moving player launches an object toward the place where a fast-moving receiver will most likely be when the ball or puck arrives. The direction and force of the pass are guided by intuition and skill acquired from long practice.

Launching a spacecraft has the same objective, but extreme precision is required - the outcome is more critical than that of an incomplete pass. Scientists and engineers calculate every detail of the trajectories and orbits in advance. The magnitude and direction of the forces and the time interval for firing the rockets are analyzed and specified in minute detail. Even last-minute adjustments are calculated exhaustively. The process can be extremely complex, but it is based on principles that you have already studied - the dynamics of circular motion and the law of universal gravitation.


Figure 12.5 The steps in a typical Apollo lunar mission are: (1) lift off and enter Earth orbit, (2) leave Earth orbit, (3) release booster rocket, turn, and dock with lunar module that is stored between the booster rocket and the service module, (4) make a mid-course correction, (5) enter lunar orbit, (6) command module continues to orbit the Moon, while the lunar module descends to the lunar surface, carries out tasks, ascends, and reconnects with the command module, (7) leave lunar orbit, (8) eject lunar module, (9) mid-course correction, (10) eject service module, and (11) command module lands in the Pacific Ocean.

SECTION
OUTCOMES

- Use Newton's law of universal gravitation to explain planetary and satellite motion.
- Distinguish between scientific questions and technological problems in relation to orbital situations.


## K E Y

## TERMS

- geostationary orbit
- microgravity
- perturbation


## Newton's Mountain

The planets in our solar system appear to have been "orbiting" the Sun while they were forming. Great swirling dust clouds in space began to condense around a newly formed Sun until they finally became the planets. How, then, do artificial satellites begin orbiting Earth?

Soon after Newton formulated his law of universal gravitation, he began thought experiments about artificial satellites. He reasoned that you could put a cannon at the top of an extremely high mountain and shoot a cannon ball horizontally, as shown in Figure 12.6. The cannon ball would certainly fall toward Earth. If the cannon ball travelled far enough horizontally while it fell, however, the curvature of Earth would be such that Earth's surface would "fall away" as fast as the cannon ball fell.

You can determine how far the cannon ball will fall in one second by using the kinematic equation $\Delta d=v_{\mathrm{i}} \Delta t+\frac{1}{2} a \Delta t^{2}$. If a cannon ball had zero vertical velocity at time zero, in one second it would fall a distance $\Delta d=0+\frac{1}{2}\left(-9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(1 \mathrm{~s})^{2}=-4.9 \mathrm{~m}$. From the size and curvature of Earth, Newton knew that Earth's surface would drop by 4.9 m over a horizontal distance of 8 km .


Figure 12.6 The values shown here represent the distance that the cannon ball would have to go in one second in order to go into orbit.

Newton's reasoning was absolutely correct, but he did not account for air friction. Although the air is too thin to breathe easily on top of Mount Everest, Earth's highest mountain, it would still exert a large amount of air friction on an object moving at $8 \mathrm{~km} / \mathrm{s}$. If you could take the cannon to 150 km above Earth's surface, the atmosphere would be so thin that air friction would be negligible. Newton understood how to put an artificial satellite into orbit, but he did not have the technology.

Today, launching satellites into orbit is almost routine, but the scientists and engineers must still carefully select an orbit and perform detailed calculations to ensure that the orbit will fulfil the purpose of the satellite. For example, some weather satellites orbit over the Poles at a relatively low altitude in order to collect data in detail. Since a satellite is constantly moving in relation to a ground observer, the satellite receiver has to track the satellite continually so that it can capture the signals that the satellite is sending. In addition, the satellite is on the opposite side of Earth for long periods of time, so several receivers must be located around the globe to collect data at all times.

Communication satellites and some weather satellites travel in a geostationary orbit over the equator, which means that they appear to hover over one spot on Earth's surface at all times. Consequently, a receiver can be aimed in the same direction at all times and constantly receive a signal from the satellite. The following problem will help you to find out how these types of orbits are attained.

Arthur C. Clarke (1917-), scientist and science fiction writer, wrote a technical paper in 1945, setting out the principles of geostationary satellites for communications. Many scientists in the field at the time did not believe that it was possible. Today, geostationary orbits are sometimes called "Clarke orbits." Clarke also co-authored the book and movie 2001: A Space Odyssey.

## MODEL PROBLEM

## Geostationary Orbits

At what velocity and altitude must a satellite orbit in order to be geostationary?

## Frame the Problem

- A satellite in a geostationary orbit must remain over the same point on Earth at all times.
- To be geostationary, the satellite must make one complete orbit in exactly the same time that Earth rotates on its axis. Therefore, the period must be 24 h .
- The period is related to the velocity of the satellite.
- The velocity and altitude of the satellite are determined by the amount of centripetal force that is causing the satellite to remain on a circular path.
- Earth's gravity provides the centripetal force for satellite motion.
- The values for the mass and radius of Earth are listed in Appendix B, Physical Constants and Data.


## Identify the Goal

(a) The velocity, $v$, of a geostationary satellite
(b) The altitude, $h$, of a geostationary satellite

## Variables and Constants

## Known

Implied
Orbit is geostationary.

$$
\begin{array}{lll}
T=24 \mathrm{~h} & m_{\mathrm{E}}=5.98 \times 10^{24} \mathrm{~kg} & r \text { (radius of satellite's orbit) } \\
G=6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}} & r_{\mathrm{E}}=6.38 \times 10^{6} \mathrm{~m} & h
\end{array}
$$

## Strategy

To find the velocity, start by setting the gravitational force equal to the centripetal force.
Simplify the expression.

## Calculations

$$
\begin{aligned}
& F_{\mathrm{g}}=F_{\mathrm{c}} \\
& G \frac{m_{\mathrm{E}} m_{\mathrm{s}}}{r^{2}}=\frac{m_{\mathrm{s}} v^{2}}{r} \\
& G \frac{m_{\mathrm{E}}}{r}=v^{2} \\
& v=\frac{\Delta d}{\Delta t} \\
& \Delta d=2 \pi r \\
& \Delta t=T \\
& v=\frac{2 \pi r}{T} \\
& r=\frac{v T}{2 \pi}
\end{aligned}
$$

To eliminate $r$ from the equation, use the equation for the definition of velocity and solve for $r$. Recall that the period, $T$, is known.

Substitute the expression for $r$ into the equation for $v$ above and solve for $v$.

$$
\begin{aligned}
& G \frac{m_{\mathrm{E}}}{\frac{v T}{2 \pi}}=v^{2} \\
& v^{2}=\frac{2 \pi G m_{\mathrm{E}}}{v T} \\
& v^{3}=\frac{2 \pi G m_{\mathrm{E}}}{T} \\
& V=\sqrt[3]{\frac{2 \pi G m_{\mathrm{E}}}{T}}
\end{aligned}
$$

Convert $T$ to SI units.

$$
\mathrm{T}=(24 \mathrm{~h})\left(\frac{60 \mathrm{~min}}{\mathrm{~h}}\right)\left(\frac{60 \mathrm{~s}}{\mathrm{~min}}\right)=8.64 \times 10^{4} \mathrm{~s}
$$

Substitute numerical values and solve.

$$
\begin{aligned}
& v=\sqrt[3]{\frac{2 \pi\left(6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{8.64 \times 10^{4} \mathrm{~s}}} \\
& v=\sqrt[3]{2.9006 \times 10^{10} \frac{\mathrm{~m}^{3}}{\mathrm{~s}^{3}}} \\
& v=3.0724 \times 10^{3} \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v \cong 3.07 \times 10^{3} \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

(a) The orbital velocity of the satellite is about $3.07 \times 10^{3} \mathrm{~m} / \mathrm{s}$, which is approximately $11000 \mathrm{~km} / \mathrm{h}$.

To find the altitude of the satellite, substitute the value for velocity into the expression above that you developed to find $r$ in terms of $v$.

$$
\begin{aligned}
r & =\frac{v T}{2 \pi} \\
r & =\frac{\left(3.0724 \times 10^{3} \frac{\mathrm{~m}}{\mathrm{~s}}\right)\left(8.64 \times 10^{4} \mathrm{~s}\right)}{2 \pi} \\
r & =4.2249 \times 10^{7} \mathrm{~m} \\
r & =r_{\mathrm{E}}+h \\
h & =r-r_{\mathrm{E}} \\
h & =4.2249 \times 10^{7} \mathrm{~m}-6.38 \times 10^{6} \mathrm{~m} \\
h & =3.5869 \times 10^{7} \mathrm{~m} \\
h & \cong 3.59 \times 10^{7} \mathrm{~m}
\end{aligned}
$$

(b) The altitude of all geostationary satellites must be $3.59 \times 10^{7} \mathrm{~m}$, or $3.59 \times 10^{4} \mathrm{~km}$, above Earth's surface.

## Validate

A velocity of $11000 \mathrm{~km} / \mathrm{h}$ seems extremely fast to us, but the satellite is circling Earth once per day, so the velocity is reasonable.
Check the units for velocity. $\sqrt[3]{\frac{\frac{N \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2} \mathrm{~kg}}}{\mathrm{~s}}}=\sqrt[3]{\frac{\mathrm{N} \cdot \mathrm{m}^{2} \cdot \mathrm{~kg}}{\mathrm{~s} \cdot \mathrm{~kg}^{2}}}=\sqrt[3]{\frac{\frac{\mathrm{kg} \cdot \mathrm{m}^{3}}{\mathrm{~s}^{2}}}{\mathrm{~s} \cdot \mathrm{~kg}}}=\sqrt[3]{\frac{\mathrm{m}^{3}}{\mathrm{~s}^{3}}}=\frac{\mathrm{m}}{\mathrm{s}}$

## PRACTICE PROBLEMS

15. The polar-orbiting environmental satellites (POES) and some military satellites orbit at a much lower level in order to obtain more detailed information. POES complete an Earth orbit 14.1 times per day. What are the orbital speed and the altitude of POES?
16. The International Space Station orbits at an altitude of approximately 226 km . What is its orbital speed and period?
17. (a) The planet Neptune has an orbital radius around the Sun of about $4.50 \times 10^{12} \mathrm{~m}$. What are its period and its orbital speed?
(b) Neptune was discovered in 1846. How many orbits has it completed since its discovery?

NASA operates two polar-orbiting environmental satellites (POES) designed to collect global data on cloud cover; surface conditions such as ice, snow, and vegetation; atmospheric temperatures; and moisture, aerosol, and ozone distributions.



- The space station orbits at an altitude of approximately 226 km . Find the radius of its orbit by adding the altitude to Earth's radius.
- Use Newton's law of universal gravitation to find the astronaut's weight.


## "Weightlessness" in Orbit

You have probably seen pictures of astronauts in a space capsule, space shuttle, or space station similar to Figure 12.7. The astronauts appear to float freely in the spacecraft and they describe the condition as being "weightless." Is a 65 kg astronaut in a space station weightless? Check it out, using the following calculation.

Figure 12.7 If an astronaut in a spacecraft dropped an apple, it would fall toward Earth, but it would not look as though it was falling.

$$
\begin{aligned}
& r=r_{\mathrm{E}}+h \\
& r=6.38 \times 10^{6} \mathrm{~m}+226 \mathrm{kin}\left(\frac{1000 \mathrm{~m}}{\mathrm{kth}}\right) \\
& r=6.606 \times 10^{6} \mathrm{~m} \\
& F_{\mathrm{g}}=G \frac{m_{1} m_{2}}{r^{2}} \\
& F_{\mathrm{g}}=\left(6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}\right) \frac{\left(5.98 \times 10^{24} \mathrm{~kg}\right)(65 \mathrm{~kg})}{\left(6.606 \times 10^{6} \mathrm{~m}\right)^{2}} \\
& F_{\mathrm{g}}=594 \mathrm{~N}
\end{aligned}
$$

The astronaut's weight on Earth would be $F_{g}=m g=(65 \mathrm{~kg})\left(9.81 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}\right) \cong 638 \mathrm{~N}$. There is very little difference between the astronaut's weight on Earth and in the space station. Why do astronauts appear "weightless" in space? Think back to your calculations of apparent weight in Chapter 5, Newton's Laws. When the imaginary elevator that you were riding in was falling with an acceleration of $9.81 \mathrm{~m} / \mathrm{s}^{2}$, the scale you were standing on read zero newtons. Your apparent weight was zero because you, the scale, and the elevator were falling with the same acceleration. You and the scale were not exerting any force on each other.

The same situation exists in the space station and all orbiting spacecraft all of the time. As in the case of Newton's cannon ball, everything falls to Earth with the same acceleration, but Earth is "falling away" equally as fast. You could say that a satellite is "falling around Earth." Some physicists object to the term "weightlessness" because, as you saw, there is no such condition. NASA coined the term microgravity to describe the condition of apparent weightlessness.

## PHYSICS \& TECHNOLOGY

## Simulating Microgravity

In preparation for space flights, astronauts benefit by practising movements in microgravity conditions. As you recall from your study of projectile motion, an object will follow a parabolic trajectory if the only force acting on it is gravity. Large jet aircraft can fly on a perfect parabolic path by exerting a force to overcome air friction. Thus, inside the aircraft, all objects move as though they were following a path determined only by gravity. Objects inside the aircraft, including people, exert no forces on each other, because they are all "falling" with the same acceleration. Astronauts can experience 20 s of microgravity on each parabolic trajectory.

Scientists have also found that certain chemical and physical reactions occur in a different way in microgravity. They believe that some manufacturing processes can be carried out more efficiently under these conditions - a concept that might lead to manufacturing in space. To test some of these reactions without going into orbit, researchers sometimes use drop towers, as shown in the photograph. A drop tower has a very long shaft that can be evacuated to eliminate air friction. A sample object or, sometimes, an entire experiment is dropped and the reaction can proceed for up to 10 s in microgravity conditions. These experiments provide critical information for future processing in space.


Analyze

1. Explain in detail why an airplane must use energy to follow an accurate parabolic trajectory, but everything inside the airplane appears to be weightless.
2. Do research to learn about some chemical or physical processes that might be improved by carrying them out in microgravity.

## Web Link

## www.mcgrawhill.ca/links/atlphysics

To learn more about experiments carried out in drop towers, go to the above Internet site and click on Web Links.

## Perturbing Orbits

In all of the examples that you have studied to this point, you have considered only perfectly circular or perfectly elliptical orbits. Such perfect orbits would occur only if the central body and satellite were totally isolated from all other objects. Since this is essentially never achieved, all orbits, such as those of planets around the Sun and moons and satellites around planets, are slightly distorted ellipses. For example, when an artificial satellite is between the Moon and Earth, the Moon's gravity pulls in an opposite direction to that of Earth's gravity. When a satellite is on the side of Earth opposite to the Moon, the Moon and Earth exert their forces in the same direction. The overall effect is a very slight change in the satellite's orbit. Engineers must take these effects into account.

In the solar system, each planet exerts a gravitational force on every other planet, so each planet perturbs the orbit of the other planets. In some cases, the effects are so small that they cannot be measured. Astronomers can, however, observe these perturbations in the paths of the planets when the conditions are right. In fact, in 1845, two astronomers in two different countries individually observed perturbations in the orbit of the planet Uranus. British astronomer and mathematician John Couch Adams (1819-1892) and French astronomer Urbain John Joseph Le Verrier (1811-1877) could not account for their observed perturbations of the planet's orbit, even by calculating the effects of the gravitational force of the other planets. Both astronomers performed detailed calculations and predicted both the existence and the position of a new, as yet undiscovered planet. In September of 1846, at the Berlin Observatory, astronomer J.G. Galle (1812-1910) searched the skies at the location predicted by the two mathematical astronomers. Having excellent star charts for comparison, Galle almost immediately observed the new planet, which is now called "Neptune."

About 50 years later, U.S. astronomer Percival Lowell (1855-1916) performed calculations on the orbits of both Neptune and Uranus, and discovered that these orbits were again perturbed, probably by yet another undiscovered planet. About 14 years after Lowell's death, astronomer Clyde Tombaugh (1906-1997), working in the Lowell Observatory, discovered the planet now called "Pluto."

Since the discovery of two planets that were predicted mathematically by the perturbations of orbits of known planets, several more predictions about undiscovered planets have been made. None have been discovered and most astronomers believe that no more planets exist in our solar system. The laws of Newton and Kepler, however, have provided scientists and astronomers with a solid foundation on which to explain observations and make predictions about planetary motion, as well as send space probes out to observe all of the planets in our solar system.

### 12.2 Section Review

1. C Explain Newton's thought experiment about "launching a cannon-ball satellite."
2. (I) Why must a geostationary satellite orbit over the equator? To answer that question, think about the point that is the centre of the orbit. If you launched a satellite that had a period of 24 hr , but it did not start out over the equator, what path would it follow? If you were at the spot on Earth
just below the point where the satellite started to orbit, how would the path of the satellite appear to you?
3. K/O What conditions create apparent weightlessness when an astronaut is in an orbiting spacecraft?
4. K/U How could you discover a planet without seeing it with a telescope?
