12.1 Newton's Law of Universal Gravitation

SECTION OUTCOMES

- Explain Kepler's laws.
- Describe Newton's law of universal gravitation.
- Apply Newton's law of universal gravitation quantitatively.

KEY TERMS

- Tychonic system
- Kepler's laws
- law of universal gravitation

In previous science courses, you learned about the Ptolemaic system for describing the motion of the planets and the Sun. The system developed by Ptolemy (151–127 B.C.E.) was very complex because it was geocentric. That is, it placed Earth at the centre of the universe. In 1543, Nicholas Copernicus (1473–1543) proposed a much simpler, heliocentric system for the universe in which Earth and all of the other planets revolved around the Sun. The Copernican system was rejected by the clergy, however, because the religious belief system at the time placed great importance on humans and Earth as being central to a physically perfect universe. You probably remember learning that the clergy put Galileo Galilei (1564–1642) on trial for supporting the Copernican system.

Have you ever heard of the **Tychonic system**? A famous Danish nobleman and astronomer, Tycho Brahe (1546–1601), proposed a system, shown in Figure 12.1, that was intermediate between the Ptolemaic and Copernican systems. In Brahe's system, Earth is still and is the centre of the universe; the Sun and Moon revolve around Earth, but the other planets revolve around the Sun. Brahe's system captured the interest of many scientists, but never assumed the prominence of either the Ptolemaic or Copernican systems. Nevertheless, Tycho Brahe contributed a vast amount of detailed, accurate information to the field of astronomy.

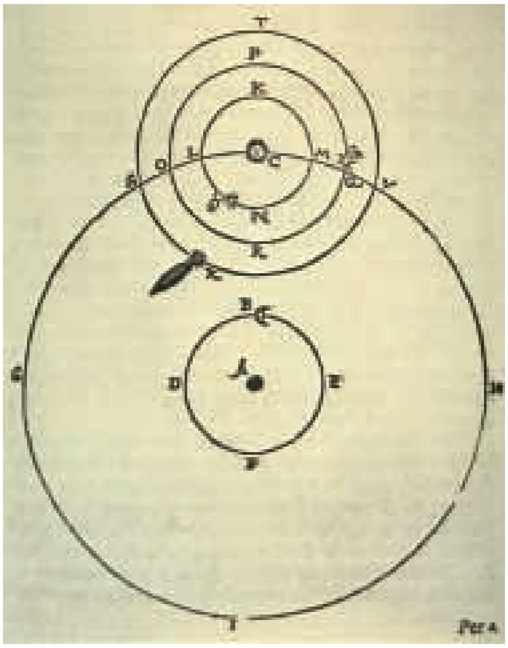


Figure 12.1 The Tychonic universe was acceptable to the clergy, because it maintained that Earth was the centre of the universe. The system was somewhat satisfying for scientists, because it was simpler than the Ptolemaic system.

Laying the Groundwork for Newton

Astronomy began to come of age as an exact science with the detailed and accurate observations of Tycho Brahe. For more than 20 years, Brahe kept detailed records of the positions of the planets and stars. He catalogued more than 777 stars and, in 1572, discovered a new star that he named "Nova." Brahe's star was one of very few supernovae ever found in the Milky Way galaxy.

In 1577, Brahe discovered a comet and demonstrated that it was not an atmospheric phenomenon as some scientists had believed, but rather that its orbit lay beyond the Moon. In addition to making observations and collecting data, Brahe designed and built the most accurate astronomical instruments of the day (see Figure 12.2). In addition, he was the first astronomer to make corrections for the refraction of light by the atmosphere.

History Link

Tycho Brahe was a brilliant astronomer who led an unusual and tumultuous life. At age 19, he was involved in a duel with another student and part of his nose was cut off. For the rest of his life, Brahe wore an artificial metal nose.

Figure 12.2 Brahe's observatory in Hveen, Denmark, contained gigantic instruments that, without magnification, were precise to 1/30 of a degree.

In 1600, Brahe invited Kepler to be one of his assistants. Brahe died suddenly the following year, leaving all of his detailed data to Kepler. With this wealth of astronomical data and his ability to perform meticulous mathematical analyses, Kepler discovered three empirical relationships that describe the motion of the planets. These relationships are known today as **Kepler's laws**.

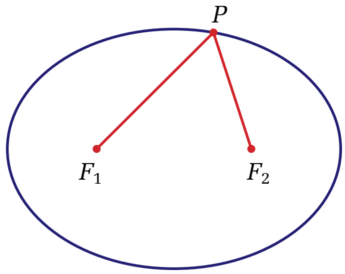
KEPLER'S LAWS

1. Planets move in elliptical orbits, with the Sun at one focus of the ellipse.
2. An imaginary line between the Sun and a planet sweeps out equal areas in equal time intervals.
3. The quotient of the square of the period of a planet's revolution around the Sun and the cube of the average distance from the Sun is constant and the same for all planets.

$\frac{r^3}{T^2} = k$ or $\frac{r_A^3}{T_A^2} = \frac{r_B^3}{T_B^2}$, where A and B are two planets.

Math Link

A circle is a special case of an ellipse. An ellipse is defined by two focuses and the relationship $\overline{F_1P} + \overline{F_2P} = k$, where k is a constant and is the same for every point on the ellipse. If the two focuses of an ellipse are brought closer and closer together until they are superimposed on each other, the ellipse becomes a circle.



Kepler's first law does not sound terribly profound, but he was contending not only with scientific observations of the day, but also with religious and philosophical views. For centuries, the perfection of "celestial spheres" was of extreme importance in religious beliefs. Ellipses were not considered to be "perfect," so many astronomers resisted accepting any orbit other than a "perfect" circle that fit on the surface of a sphere. However, since Kepler published his laws, there has never been a case in which the data for the movement of a satellite, either natural or artificial, did not fit an ellipse.

Kepler's second law is illustrated in Figure 12.3. Each of the shaded sections of the ellipse has an equal area. According to Kepler's second law, therefore, the planet moves along the arc of each section in the same period of time. Since the arcs close to the Sun are longer than the arcs more distant from the Sun, the planet must be moving more rapidly when it is close to the Sun.

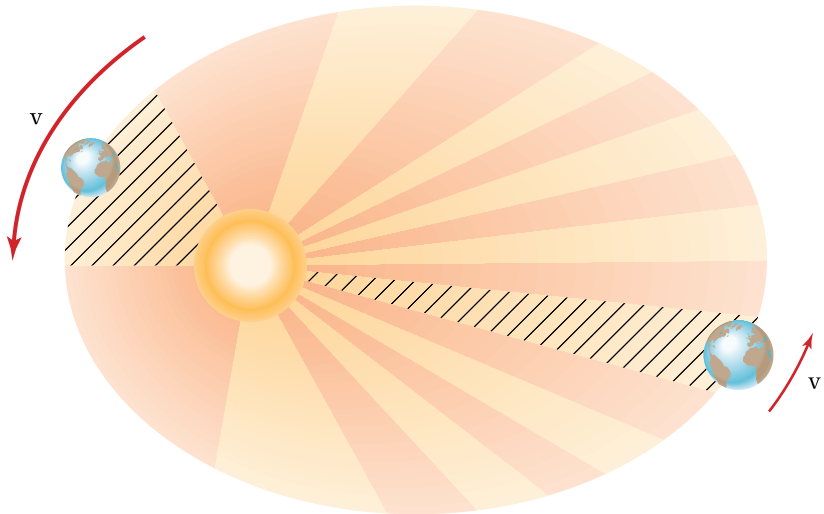


Figure 12.3 According to Kepler's second law, the same length of time was required for a planet to move along each of the arcs at the ends of the segments of the ellipse. Kepler could not explain why planets moved faster when they were close to the Sun than when they were farther away.

When Kepler published his third law, he had no way of knowing the significance of the constant in the mathematical expression $r^3/T^2 = k$. All he knew was that the data fit the equation. Kepler suspected that the Sun was in some way influencing the motion of the planets, but he did not know how or why this would lead to the mathematical relationship. The numerical value of the constant in Kepler's third law and its relationship to the interaction between the Sun and the planets would take on significance only when Sir Isaac Newton (1642–1727) presented his law of universal gravitation.

Universal Gravitation

Typically in research, the scientist makes some observations that lead to an hypothesis. The scientist then tests the hypothesis by planning experiments, accumulating data, and then comparing the results to the hypothesis. The development of Newton's law of universal gravitation happened in reverse. Brahe's data and Kepler's analysis of the data were ready and waiting for Newton to use to test his hypothesis about gravity.

Newton was not the only scientist of his time who was searching for an explanation for the motion, or orbital dynamics, of the planets. In fact, several scientists were racing to see who could find the correct explanation first. One of those scientists was astronomer Edmond Halley (1656–1742). Halley and others, based on their calculations, had proposed that the force between the planets and the Sun decreased with the square of the distance between a planet and the Sun. However, they did not know how to apply that concept to predict the shape of an orbit.

Halley decided to put the question to Newton. Halley first met Newton in 1684, when he visited Cambridge. He asked Newton what type of path a planet would take if the force attracting it to the Sun decreased with the square of the distance from the Sun. Newton quickly answered, "An elliptical path." When Halley asked him how he knew, Newton replied that he had made that calculation many years ago, but he did not know where his calculations were. Halley urged Newton to repeat the calculations and send them to him.

Three months later, Halley's urging paid off. He received an article from Newton entitled "De Motu" ("On Motion"). Newton continued to improve and expand his article and in less than three years, he produced one of the most famous and fundamental scientific works: *Philosophiae Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Philosophy*). The treatise contained not only the law of universal gravitation, but also Newton's three laws of motion.

Possibly, Newton was successful in finding the law of universal gravitation because he extended the concept beyond the motion of planets and applied it to all masses in all situations. While other scientists were looking at the motion of planets, Newton was watching an apple fall from a tree to the ground. He reasoned that the same attractive force that existed between the Sun and Earth was also responsible for attracting the apple to Earth. He also reasoned that the force of gravity acting on a falling object was proportional to the mass of the object. Then, using his own third law of action-reaction forces, if a falling object such as an apple was attracted to Earth, then Earth must also be attracted to the apple, so the force of gravity must also be proportional to the mass of Earth. Newton therefore proposed that *the force of gravity between any two objects is proportional to the product of their*

History Link

Sir Edmond Halley, the astronomer who prompted Newton to publish his work on gravitation, is the same astronomer who discovered the comet that was named in his honour — Halley's Comet. Without Halley's urging, Newton might never have published his famous *Principia*, greatly slowing the progress of physics.

masses and inversely proportional to the square of the distance between their centres — the **law of universal gravitation**. The mathematical equation for the law of universal gravitation is given in the following box.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

The force of gravity is proportional to the product of the two masses that are interacting and inversely proportional to the square of the distance between their centres.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Quantity	Symbol	SI unit
force of gravity	F_g	N (newtons)
first mass	m_1	kg (kilograms)
second mass	m_2	kg (kilograms)
distance between the centres of the masses	r	m (metres)
universal gravitational constant	G	$\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$ (newton · metre squared per kilogram squared)

Unit Analysis

$$\text{newton} = \left(\frac{\text{newton} \cdot \text{metre}^2}{\text{kilogram}^2}\right)\left(\frac{\text{kilogram} \cdot \text{kilogram}}{\text{metre}^2}\right)$$

$$\left(\frac{\text{N} \cdot \cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)\left(\frac{\cancel{\text{kg}} \cdot \cancel{\text{kg}}}{\cancel{\text{m}^2}}\right) = \text{N}$$

Note: The value of the universal gravitational constant is

$$G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}.$$

Conceptual Problem

- You have used the equation $F_g = mg$ many times to calculate the weight of an object on Earth's surface. Now, you have learned that the weight of an object on Earth's surface is $F_g = G\frac{m_E m_o}{r_{E\text{-}o}^2}$, where m_E is the mass of Earth, m_o is the mass of the object, and $r_{E\text{-}o}$ is the distance between the centres of Earth and the object. Explain how the two equations are related. Express g in terms of the variables and constant in Newton's law of universal gravitation.

MODEL PROBLEM

Weighing an Astronaut

A 65.0 kg astronaut is walking on the surface of the Moon, which has a mean radius of 1.74×10^3 km and a mass of 7.35×10^{22} kg. What is the weight of the astronaut?

Frame the Problem

- The weight of the astronaut is the gravitational force on her.
- The relationship $F_g = mg$, where $g = 9.81\ \frac{m}{s^2}$, *cannot* be used in this problem, since the astronaut is not on Earth's surface.
- The law of universal gravitation applies to this problem.

Identify the Goal

The gravitational force, F_g, on the astronaut

Variables and Constants

Known	Implied	Unknown
$m_M = 7.35 \times 10^{22}$ kg	$G = 6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$	F_g
$m_a = 65.0$ kg		
$r = 1.74 \times 10^3$ km (1.74×10^6 m)		

Strategy

Apply the law of universal gravitation.

Substitute the numerical values and solve.

Calculations

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = \left(6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}\right)\frac{(7.35 \times 10^{22}\ kg)(65.0\ kg)}{(1.74 \times 10^6\ m)^2}$$

$$F_g = 105.25\ N$$

$$F_g \cong 105\ N$$

The weight of the astronaut is approximately 105 N.

Validate

Weight on the Moon is known to be much less than that on Earth. The astronaut's weight on the Moon is about one sixth of her weight on Earth ($65.0\ kg \times 9.81\ \frac{m}{s^2} \cong 638\ N$), which is consistent with this common knowledge.

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PRACTICE PROBLEMS

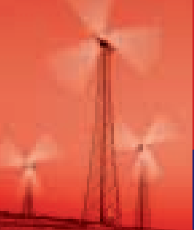
1. Find the gravitational force between Earth and the Sun. (See Appendix B, Physical Constants and Data.)
2. Find the gravitational force between Earth and the Moon. (See Appendix B, Physical Constants and Data.)
3. How far apart would you have to place two 7.0 kg bowling balls so that the force of gravity between them would be 1.25×10^{-4} N? Would it be possible to place them at this distance? Why or why not?
4. Find the gravitational force between the electron and the proton in a hydrogen atom if they are 5.30×10^{-11} m apart. (See Appendix B, Physical Constants and Data.)
5. On Venus, a person with mass 68 kg would weigh 572 N. Find the mass of Venus from this data, given that the planet's radius is 6.31×10^{6} m.
6. In an experiment, an 8.0 kg lead sphere is brought close to a 1.5 kg mass. The gravitational force between the two objects is 1.28×10^{-8} N. How far apart are the centres of the objects?
7. The radius of the planet Uranus is 4.3 times the radius of earth. The mass of Uranus is 14.7 times Earth's mass. How does the gravitational force on Uranus' surface compare to that on Earth's surface?
8. Along a line connecting Earth and the Moon, at what distance from Earth's centre would an object have to be located so that the gravitational attractive force of Earth on the object was equal in magnitude and opposite in direction from the gravitational attractive force of the Moon on the object?

Gravity and Kepler's Laws

The numerical value of *G*, the universal gravitational constant, was not determined experimentally until more than 70 years after Newton's death. Nevertheless, Newton could work with concepts and proportionalities to verify his law.

Newton had already shown that the inverse square relationship between gravitational force and the distance between masses was supported by Kepler's first law — that planets follow elliptical paths.

Kepler's second law showed that planets move more rapidly when they are close to the Sun and more slowly when they are farther from the Sun. The mathematics of elliptical orbits in combination with an inverse square relationship to yield the speed of the planets is somewhat complex. However, you can test the concepts graphically by completing the following investigation.



INVESTIGATION 12-A

Orbital Speed of Planets

TARGET SKILLS
- Modelling concepts
- Analyzing and interpreting
- Communicating results

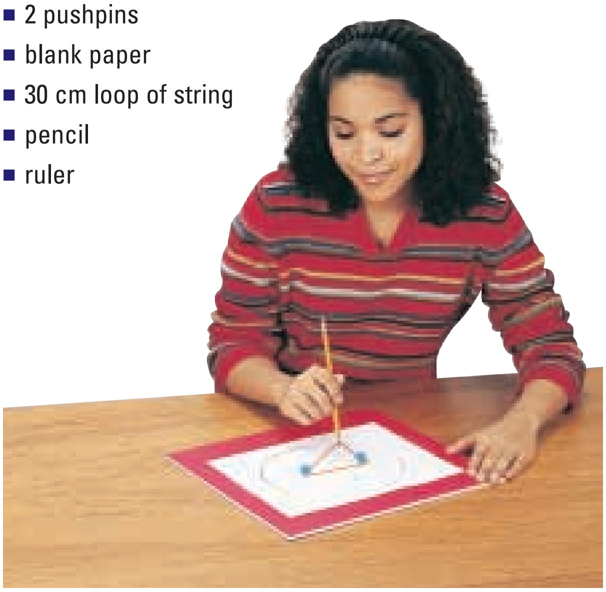
Can you show diagrammatically that a force directed along the line between the centres of the Sun and a planet would cause the planet's speed to increase as it approached the Sun and decrease as it moved away? If you can, you have demonstrated that Kepler's second law supports Newton's proposed law of universal gravitation.

Problem

How does a force that follows an inverse square relationship affect the orbital speed of a planet in an elliptical orbit?

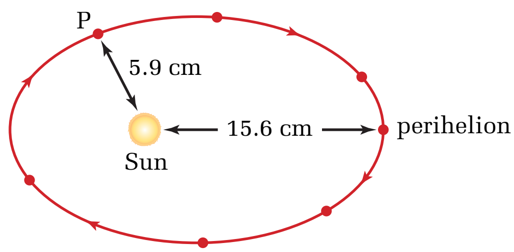
Equipment

- corkboard or large, thick piece of cardboard
- 2 pushpins
- blank paper
- 30 cm loop of string
- pencil
- ruler



Procedure

1. Place the paper on the corkboard or cardboard. Insert two pushpins into the paper about 8 to 10 cm apart.
2. Loop the string around the pushpins, as shown in the illustration. With your pencil, pull the string so that it is taut and draw an ellipse by pulling the string all the way around the pushpins.
3. Remove the string and pushpins and label one of the pinholes "Sun."
4. Choose a direction around the elliptical orbit in which your planet will be moving. Make about four small arrowheads on the ellipse to indicate the direction of motion of the planet.
5. Make a dot for the planet at the point that is most distant from the Sun (the perihelion). Measure and record the distance on the paper from the perihelion to the Sun. From that point, draw a 1 cm vector directed straight toward the Sun.
6. This vector represents the force of gravity on the planet at that point: $F_{g(per)} = 1$ unit. ($F_{g(per)}$ is the force of gravity when the planet is at perihelion.)
7. Select and label at least three more points on each side of the ellipse at which you will analyze the force and motion of the planet.
8. For each point, measure and record, on a separate piece of paper, the distance from the Sun to point P, as indicated in the diagram. Do not write on your diagram, because it will become too cluttered.



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9. Follow the steps in the table to see how to determine the length of the force vector at each point.

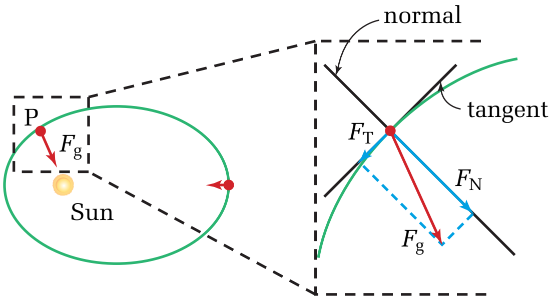
Procedure	Equation
■ The masses of the Sun and planet remain the same, so the value Gm_Sm_p is constant. Therefore, the expression F_gr^2 for any point on the orbit is equal to the same value.	$F_g = G\frac{m_Sm_p}{r^2}$ $F_gr^2 = Gm_Sm_p$
■ Consequently, you can set the expression F_gr^2 for any one point equal to F_gr^2 for any other point. Use the values at perihelion as a reference and set $F_{g(P)}r^2$ equal to $F_{g(peri)}r^2_{peri}$. Then solve for the $F_{g(P)}$.	$F_{g(peri)}\,r^2_{peri} = F_{g(P)}\,r^2_P$ $F_{g(P)} = \frac{F_{g(peri)}\,r^2_{peri}}{r^2_P}$
■ You can now find the relative magnitude of the gravitational force on the planet at any point on the orbit by substituting the magnitudes of the radii into the above equation. For example, the magnitude of the force at point P in step 8 is 6.99 units.	$F_{g(P)} = \frac{(1\text{ unit})(15.6\text{ cm})^2}{(5.9\text{ cm})^2}$ $F_{g(P)} = 6.99\text{ units}$

10. Calculate the length of the force vector from each of the points that you have selected on your orbit.

11. On your diagram, draw force vectors from each point directly toward the Sun, making the lengths of the vectors equal to the values that you calculated in step 10.

12. At each point at which you have a force vector, draw a very light pencil line tangent to the ellipse. Then, draw a line that is perpendicular (normal) to the tangent line.

13. Graphically draw components of the force vector along the tangent (F_T) and normal (F_N) lines, as shown in the diagram.



Analyze and Conclude

1. The tangential component of the force vector (F_T) is parallel to the direction of the velocity of the planet when it passes point P. What effect will the tangential component of force have on the velocity of the planet?

2. The normal component of the force vector (F_N) is perpendicular to the direction of the velocity of the planet when it passes point P. What effect will the normal component of force have on the velocity of the planet?

3. Analyze the change in the motion of the planet caused by the tangential and normal components of the gravitational force at each point where you have drawn force vectors. Be sure to note the direction of the velocity of the planet as you analyze the effect of the components of force at each point.

4. Summarize the changes in the velocity of the planet as it makes one complete orbit around the Sun.

5. The force vectors and components that you drew were predictions based on Newton's law of universal gravitation. How well do these predictions agree with Kepler's observations as summarized in his second law? Would you say that Kepler's data supports Newton's predictions?

Kepler's third law simply states that the ratio r^3/T^2 is constant and the same for each planet orbiting the Sun. At first glance, it would appear to have little relationship to Newton's law of universal gravitation, but a mathematical analysis will yield a relationship. To keep the mathematics simple, you will consider only circular orbits. The final result obtained by considering elliptical orbits is the same, although the math is more complex. Follow the steps below to see how Newton's law of universal gravitation yields the same ratio as given by Kepler's third law.

- Write Newton's law of universal gravitation, using m_S for the mass of the Sun and m_p for the mass of a planet.

$$F_g = G\frac{m_S m_p}{r^2}$$

- Since the force of gravity must provide a centripetal force for the planets, set the gravitational force equal to the required centripetal force.

$$G\frac{m_S m_p}{r^2} = \frac{m_p v^2}{r}$$

  Simplify the equation.

$$G\frac{m_S}{r} = v^2$$

- Since Kepler's third law includes the period, T, as a variable, find an expression for the velocity, v, of the planet in terms of its period.

$$v = \frac{\Delta d}{\Delta t}$$

$$\Delta d = 2\pi r$$

$$\Delta t = T$$

  A planet travels a distance equal to the circumference of the orbit during a time interval equal to its period.

$$v = \frac{2\pi r}{T}$$

- Substitute the expression for the velocity of the planet into the above equation.

$$G\frac{m_S}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$G\frac{m_S}{r} = \frac{4\pi^2 r^2}{T^2}$$

- Multiply each side of the equation by $\frac{r}{4\pi^2}$.

$$\left(G\frac{m_S}{\cancel{r}}\right)\left(\frac{\cancel{r}}{4\pi^2}\right) = \left(\frac{\cancel{4\pi^2} r^2}{T^2}\right)\left(\frac{r}{\cancel{4\pi^2}}\right)$$

$$\frac{r^3}{T^2} = \frac{Gm_S}{4\pi^2}$$

As you can see, Newton's law of universal gravitation indicates not only that the ratio $\frac{r^3}{T^2}$ is constant, but also that the constant is $\frac{Gm_S}{4\pi^2}$. All of Kepler's laws, developed prior to the time when Newton did his work, support Newton's law of universal gravitation. Kepler had focussed only on the Sun and planets, but

Newton proposed that the laws applied to all types of orbital motion, such as moons around planets. Today, we know that all of the artificial satellites orbiting Earth, as well as the Moon, follow Kepler's laws.

Mass of the Sun and Planets

Have you ever looked at tables that contain data for the mass of the Sun and planets and wondered how anyone could "weigh" the Sun and planets or determine their masses? English physicist and chemist Henry Cavendish (1731–1810) realized that if he could determine the universal gravitational constant, G, he could use the mathematical relationship in Kepler's third law to calculate the mass of the Sun. A brilliant experimentalist, Cavendish designed a torsion balance, similar to the system in Figure 12.4, that allowed him to measure G.

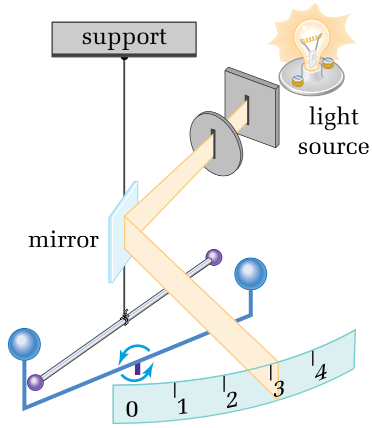


Figure 12.4 In the torsion balance that Cavendish designed and used, the spheres were made of lead. The small spheres were about 5 cm in diameter and were attached by a thin but rigid rod about 1.83 m long. The large spheres were about 20 cm in diameter.

A torsion balance can measure extremely small amounts of the rotation of a wire. First, the torsion balance must be calibrated to determine the amount of force that causes the wire to twist by a specific amount. Then, the large spheres are positioned so that the bar supporting them is perpendicular to the rod supporting the small spheres. In this position, the large spheres are exerting equal gravitational attractive forces on each of the small spheres. The system is in equilibrium and the scale can be set to zero. The large spheres are then moved close to the small spheres and the amount of twisting of the wire is determined. From the amount of twisting and the calibration, the mutual attractive force between the large and small spheres is calculated.

Using his torsion balance, Cavendish calculated the value of G to be $6.75 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. The best-known figure today is $6.672\ 59 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. Cavendish's measurement was within approximately 1% of the correct value. As Cavendish did, you can now calculate the mass of the Sun and other celestial bodies.

History Link

Henry Cavendish was a very wealthy and brilliant man, but he also was very reclusive. He was rarely seen in public places, other than at scientific meetings. His work was meticulous, yet he published only a very small part of it. After his death, other scientists discovered his notebooks and finally published his results. Cavendish had performed the same experiments and obtained the same results for some experiments that were later done by Coulomb, Faraday, and Ohm, who received the credit for the work.

PHYSICS FILE

The value of G shows that the force of gravity is extremely small. For example, use unit amounts of each of the variables and substitute them into Newton's law of universal gravitation. You will find that the mutual attractive force between two 1 kg masses that are 1 m apart is $6.672\ 59 \times 10^{-11}$ N.

MODEL PROBLEM

The Mass of the Sun

Find the mass of the Sun, using Earth's orbital radius and period of revolution.

Frame the Problem

- Kepler's third law, combined with Newton's law of universal gravitation, yields an equation that relates the period and orbital radius of a satellite to the mass of the body around which the satellite is orbiting.
- Earth orbits the Sun once per year.
- Let R_E represent the radius of Earth's orbit around the Sun. This value can be found in Appendix B, Physical Constants and Data.

Identify the Goal

The mass of the Sun, m_S

Variables and Constants

Known	Implied	Unknown
Sun	$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	m_S
	$T = 365.25$ days	
	$R_{E(orbit)} = 1.49 \times 10^{11}$ m	

Strategy / Calculations

Write Kepler's third law, using the constant derived from Newton's law of universal gravitation.

$$\frac{r^3}{T^2} = \frac{Gm_S}{4\pi^2}$$

Solve for the mass of the Sun.

$$\left(\frac{r^3}{T^2}\right)\left(\frac{4\pi^2}{G}\right) = \left(\frac{\cancel{G}m_S}{\cancel{4\pi^2}}\right)\left(\frac{\cancel{4\pi^2}}{\cancel{G}}\right)$$

$$m_S = \left(\frac{4\pi^2}{G}\right)\left(\frac{r^3}{T^2}\right)$$

Convert the period into SI units.

$$365.25\ \cancel{\text{days}}\left(\frac{24\ \cancel{h}}{\cancel{\text{day}}}\right)\left(\frac{60\ \cancel{\text{min}}}{\cancel{h}}\right)\left(\frac{60\ s}{\cancel{\text{min}}}\right) = 3.1558 \times 10^7\ s$$

Substitute the numerical values into the equation and solve.

$$m_S = \left(\frac{4\pi^2}{6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}}\right)\frac{(1.49 \times 10^{11}\ m)^3}{(3.1558 \times 10^7\ s)^2}$$

$$m_S = 1.9660 \times 10^{30}\ kg$$

$$m_S \cong 1.97 \times 10^{30}\ kg$$

The mass of the Sun is approximately 1.97×10^{30} kg.

Validate

The Sun is much more massive than any of the planets.
The value sounds reasonable.

Check the units: $\left(\frac{1}{\frac{N \cdot m^2}{kg^2}}\right)\left(\frac{m^3}{s^2}\right) = \left(\frac{kg^2}{N \cdot \cancel{m^2}}\right)\left(\frac{\cancel{m^3}}{s^2}\right) = \left(\frac{kg^{\cancel{2}}}{\frac{\cancel{kg} \cdot \cancel{m}}{\cancel{s^2}} \cdot \cancel{m^2}}\right)\left(\frac{\cancel{m^3}}{\cancel{s^2}}\right) = kg$.

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PRACTICE PROBLEMS

9. Jupiter's moon Io orbits Jupiter once every 1.769 days. Its average orbital radius is 4.216×10^8 m. What is Jupiter's mass?

10. Charon, the only known moon of the planet Pluto, has an orbital period of 6.387 days at an average distance of 1.9640×10^7 m from Pluto. Use Newton's form of Kepler's third law to find the mass of Pluto from this data.

11. Some weather satellites orbit Earth every 90.0 min. How far above Earth's surface is their orbit? (Hint: Remember that the centre of the orbit is the centre of Earth.)

12. How fast is the moon moving as it orbits Earth at a distance of 3.84×10^5 km?

13. On each of the *Apollo* lunar missions, the command module was placed in a very low, approximately circular orbit above the Moon. Assume that the average height was 60.0 km above the surface and that the Moon's radius is 7738 km.

(a) What was the command module's orbital period?

(b) How fast was the command module moving in its orbit?

14. A star at the edge of the Andromeda galaxy appears to be orbiting the centre of that galaxy at a speed of about 2.0×10^2 km/s. The star is about 5×10^9 AU from the centre of the galaxy. Calculate a rough estimate of the mass of the Andromeda galaxy. Earth's orbital radius (1 AU) is 1.49×10^8 km.

Newton's law of universal gravitation has stood the test of time and the extended limits of space. As far into space as astronomers can observe, celestial bodies move according to Newton's law. As well, the astronauts of the crippled *Apollo 13* spacecraft owe their lives to the dependability and predictability of the Moon's gravity. Although Albert Einstein (1879–1955) took a different approach in describing gravity in his general theory of relativity, most calculations that need to be made can use Newton's law of universal gravitation and make accurate predictions.

12.1 Section Review

1. K/U Explain the meaning of the term "empirical" as it applies to empirical equations.

2. K/U What did Tycho Brahe contribute to the development of the law of universal gravitation?

3. K/U Describe how Newton used each of the following phenomena to support the law of universal gravitation.

(a) the orbit of the moon

(b) Kepler's third law

4. K/U How did Newton's concepts about gravity and his development of the law of universal gravitation differ from the ideas of other scientists and astronomers who were attempting to find a relationship that could explain the motion of the planets?

5. K/U Describe the objective, apparatus, and results of the Cavendish experiment.

6. C Explain how you can "weigh" a planet.

7. I Suppose the distance between two objects is doubled and the mass of one is tripled. What effect does this have on the gravitational force between the objects?