11.2 Uniform Circular Motion

Have you ever ridden on a ride like the one shown in the photograph? From a distance, it might not look exciting, but the sensations could surprise you.

Everyone lines up around the outer edge and the ride slowly begins to turn. Not very exciting yet, but soon, the ride is spinning quite fast and you feel as though you are being pressed tightly against the wall. The rotations begin to make you feel disoriented and your stomach starts to feel a little queasy. Then, suddenly, the floor drops away, but you stay helplessly "stuck" to the wall. Just as you realize that you are not going to fall, the entire ride begins to tilt. At one point during each rotation, you find yourself looking toward the ground, which is almost directly in front of you. You do not feel as though you are going to fall, though, because you are literally stuck to the wall.

Figure 11.5 If this ride stopped turning, the people would start to fall. What feature of circular motion prevents people from falling when the ride is in motion and they are facing the ground?

What is unique about moving in a circle that allows you to apparently defy gravity? What causes people on the Round Up to stick to the wall? As you study this section, you will be able to answer these questions and many more.

Centripetal Acceleration

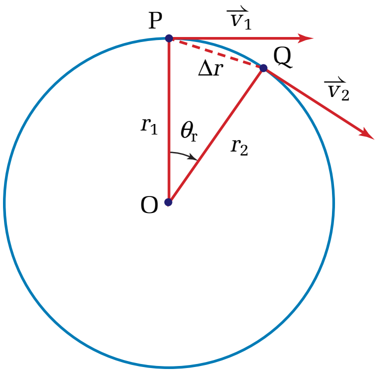
Amusement park rides are only one of a very large number of examples of circular motion. Motors, generators, vehicle wheels, fans, air in a tornado or hurricane, or a car going around a curve are other examples of circular motion. When an object is moving in a circle and its speed — the magnitude of its velocity — is

SECTION OUTCOMES

- Analyze, predict, and explain uniform circular motion.
- Explain forces involved in uniform circular motion in horizontal and vertical planes.
- Investigate relationships between period and frequency of an object in uniform circular motion.

KEY TERMS

- uniform circular motion
- centripetal acceleration
- centripetal force



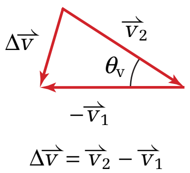


Figure 11.6 The direction of the change in velocity is found by defining the vector $-\vec{v}_1$ and then adding $\vec{v}_2$ and $-\vec{v}_1$. Place the tail of $-\vec{v}_1$ at the tip of $\vec{v}_2$ and draw the resultant vector, $\Delta\vec{v}$, from the tail of $\vec{v}_2$ to the tip of $-\vec{v}_1$.

constant, it is said to be moving with **uniform circular motion**. The direction of the object's velocity is always tangent to the circle. Since the direction of the motion is always changing, the object is always accelerating.

Figure 11.6 shows the how the velocity of the object changes when it is undergoing uniform circular motion. As an object moves from point P to point Q, its velocity changes from $\vec{v}_1$ to $\vec{v}_2$. Since the direction of the acceleration is the same as the direction of the *change* in the velocity, you need to find $\Delta\vec{v}$ or $\vec{v}_2 - \vec{v}_1$. Vectors and $\vec{v}_1$ and $\vec{v}_2$ are subtracted graphically under the circle. To develop an equation for centripetal acceleration, you will first need to show that the triangle OPQ is similar to the triangle formed by the velocity vectors, as shown in the following points.

- $r_1 = r_2$ because they are radii of the same circle. Therefore, triangle OPQ is an isosceles triangle.
- $|\vec{v}_1| = |\vec{v}_2|$ because the speed is constant. Therefore, the triangle formed by $-\vec{v}_1$, $\vec{v}_2$, and $\Delta\vec{v}$ is an isosceles triangle.
- $r_1 \perp \vec{v}_1$ and $r_2 \perp \vec{v}_2$ because the radius of a circle is perpendicular to the tangent to the point where the radius contacts the circle.
- $\theta_r = \theta_v$ because the angle between corresponding members of sets of perpendicular lines are equal.
- Since the angles between the equal sides of two isosceles triangles are equal, the triangles are similar.

Now use the two similar triangles to find the magnitude of the acceleration. Since the derivation involves only magnitudes, omit vector notations.

- The ratios of the corresponding sides of similar triangles are equal. There is no need to distinguish between the sides r_1 and r_2 or v_1 and v_2, because the radii are equal and the magnitudes of the velocities are equal.
$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$
- The object travelled from point P to point Q in the time interval Δt. Therefore, the magnitude of the object's displacement along the arc from P to Q is
$$\Delta d = v\Delta t$$
- The length of the arc from point P to point Q is almost equal to Δr. As the angle becomes very small, the lengths become more nearly identical.
$$\Delta r = v\Delta t$$
- Substitute this value of Δr into the first equation.
$$\frac{v\Delta t}{r} = \frac{\Delta v}{v}$$

- Divide both sides of the equation by Δt. $\frac{v}{r} = \frac{\Delta v}{v \Delta t}$
- Recall the definition of acceleration. $a = \frac{\Delta v}{\Delta t}$
- Substitute a into the equation for $\frac{\Delta v}{\Delta t}$. $\frac{v}{r} = \frac{a}{v}$
- Multiply both sides of the equation by v. $a = \frac{v^2}{r}$

The magnitude of the acceleration of an object moving with uniform circular motion is $a = v^2/r$. To determine its direction, again inspect the triangle formed by the velocity vectors in Figure 11.6. The acceleration is changing constantly, so imagine a vector $\vec{v}_2$ as close to $\vec{v}_1$ as possible. The angle θ is extremely small. In this case, $\Delta\vec{v}$ is almost exactly perpendicular to both $\vec{v}_1$ and $\vec{v}_2$. Since $\vec{v}_1$ and $\vec{v}_2$ are tangent to the circle and therefore are perpendicular to the associated radii of the circle, the acceleration vector points directly toward the centre of the circle.

Describing the acceleration vector in a typical Cartesian coordinate system would be extremely difficult, because the direction is always changing and, therefore, the magnitude of the x- and y-components would always be changing. It is much simpler to specify only the magnitude of the acceleration, which is constant for uniform circular motion, and to note that the direction is always toward the centre of the circle. To indicate this, physicists speak of a "centre-seeking acceleration" or **centripetal acceleration**, which is denoted as a_c, without a vector notation.

CENTRIPETAL ACCELERATION

Centripetal acceleration is the quotient of the square of the velocity and the radius of the circle.

$$a_c = \frac{v^2}{r}$$

Quantity	Symbol	SI unit
centripetal acceleration	a_c	$\frac{m}{s^2}$ (metres per second squared)
velocity (magnitude)	v	$\frac{m}{s}$ (metres per second)
radius (of circle)	r	m (metres)

Unit Analysis

$$\frac{\text{metre}}{\text{second}^2} = \frac{\left(\frac{\text{metre}}{\text{second}}\right)^2}{\text{metre}} \qquad \frac{\left(\frac{m}{s}\right)^2}{m} = \frac{\frac{m^{\not 2}}{s^2}}{\not m} = \frac{m}{s^2}$$

Note: The direction of the centripetal acceleration is always along a radius pointing toward the centre of the circle.

Math Link

Mathematicians have developed a unique system for defining components of vectors such as force, acceleration, and velocity for movement on curved paths, even when the magnitude of the velocity is changing. Any curve can be treated as an arc of a circle. So, instead of using the x- and y-components of the typical Cartesian coordinate system, the vectors are divided into tangential and radial components. The tangential component is the component of the vector that is tangent to the curved path at the point at which the object is momentarily located. The radial component is perpendicular to the path and points to the centre of the circle defined by the arc or curved section of the path. Radial components are the same as centripetal components.

Centripetal Force

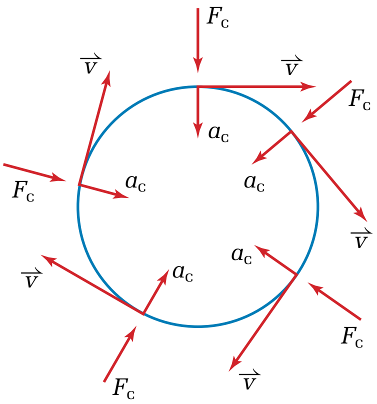


Figure 11.7 A force acting perpendicular to the direction of the velocity is always required in order for any object to move continuously along a circular path.

According to Newton's laws of motion, an object will accelerate only if a force is exerted on it. Since an object moving with uniform circular motion is always accelerating, there must always be a force exerted on it in the same direction as the acceleration, as illustrated in Figure 11.7. If at any instant the force is withdrawn, the object will stop moving along the circular path and will proceed to move with uniform motion, that is, in a straight line that is tangent to the circular path on which it had been moving.

Since the force causing a centripetal acceleration is always pointing toward the centre of the circular path, it is called a **centripetal force**. The concept of centripetal force differs greatly from that of other forces that you have encountered. It is not a type of force such as friction or gravity. It is, instead, a force that is *required* in order for an object to move in a circular path.

A centripetal force can be supplied by any type of force. For example, as illustrated in Figure 11.8, gravity provides the centripetal force that keeps the Moon on a roughly circular path around Earth, friction provides a centripetal force that causes a car to move in a circular path on a flat road, and the tension in a string tied to a ball will cause the ball to move in a circular path when you twirl it around. In fact, two different types of force could act together to provide a centripetal force.

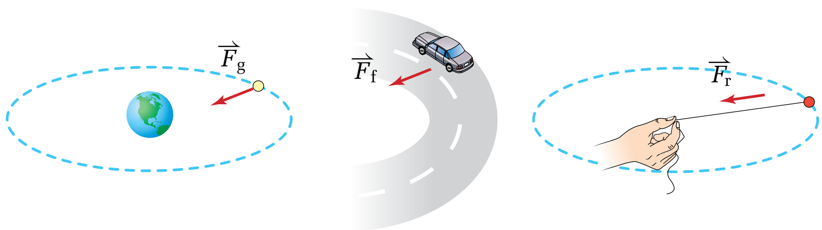


Figure 11.8 Any force that is directed toward the centre of a circle can provide a centripetal force.

You can determine the magnitude of a centripetal force required to cause an object to travel in a circular path by applying Newton's second law to a mass moving with a centripetal acceleration.

- Write Newton's second law. $\vec{F} = m\vec{a}$
- Write the equation describing centripetal acceleration. $a_c = \frac{v^2}{r}$
- Substitute into Newton's second law. Omit vector notations because the force and acceleration always point toward the centre of the circular path. $F_c = \frac{mv^2}{r}$

The equation for the centripetal force required to cause a mass m moving with a velocity v to follow a circular path of radius r is summarized in the following box.

CENTRIPETAL FORCE

The magnitude of the centripetal force is the quotient of the mass times the square of the velocity and the radius of the circle.

$$F_c = \frac{mv^2}{r}$$

Quantity	**Symbol**	**SI unit**
centripetal force	F_c	N (newtons)
mass	m	kg (kilograms)
velocity	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
radius of circular path	r	m (metres)

Unit Analysis

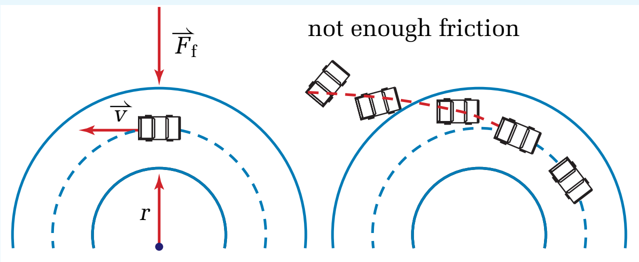
$$(\text{newtons}) = \left(\frac{\text{kilogram}\left(\frac{\text{metres}}{\text{second}}\right)^2}{\text{metres}}\right)$$

$$N = \frac{\text{kg}\left(\frac{\text{m}}{\text{s}}\right)^2}{\text{m}} = \frac{\text{kg}\frac{\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

MODEL PROBLEMS

Centripetal Force in a Horizontal and a Vertical Plane

1. A car with a mass of 2135 kg is rounding a curve on a level road. If the radius of curvature of the road is 52 m and the coefficient of friction between the tires and the road is 0.70, what is the maximum speed at which the car can make the curve without skidding off the road?



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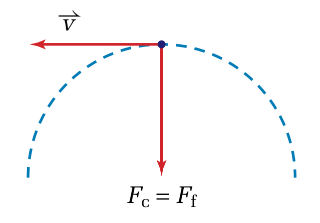
Frame the Problem



- Make a sketch of the motion of the car and the forces acting on it.
- The *force of friction* must provide a sufficient *centripetal force* to cause the car to follow the curved road.
- The magnitude of *force* required to keep the car on the road depends on the *velocity* of the car, its *mass*, and the *radius of curvature* of the road.
- Since r is in the denominator of the expression for centripetal force, as the *radius* becomes *smaller*, the amount of *force* required becomes *greater*.
- Since v is in the numerator, as the *velocity* becomes *larger*, the *force* required to keep the car on the road becomes *greater*.

Identify the Goal

The maximum speed, v, at which the car can make the turn

Variables and Constants

Known		Implied	Unknown	
$m = 2135$ kg	$\mu = 0.70$	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	F_f	F_N
$r = 52$ m			v	

Strategy

Set the frictional force equal to the centripetal force.

$$F_f = F_c$$
$$\mu F_N = \frac{mv^2}{r}$$

Since the car is moving on a level road, the normal force of the road is equal to the weight of the car. Substitute mg for F_N.

$$\mu mg = \frac{mv^2}{r}$$

Solve for the velocity.

$$v^2 = \mu \cancel{m} g\left(\frac{r}{\cancel{m}}\right)$$
$$v = \sqrt{\mu rg}$$

Substitute in the numerical values and solve.

$$v = \sqrt{(0.70)(52 \text{ m})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}$$
$$v = \sqrt{357.08 \frac{\text{m}^2}{\text{s}^2}}$$
$$v = 18.897 \frac{\text{m}}{\text{s}}$$
$$v \cong 19 \frac{\text{m}}{\text{s}}$$

If the car is going faster than 19 m/s, it will skid off the road.

Validate

A radius of curvature of 52 m is a sharp curve. A speed of 19 m/s is equivalent to 68 km/h, which is a high speed at which to take a sharp curve. The answer is reasonable. The units cancelled properly to give metres per second for velocity.

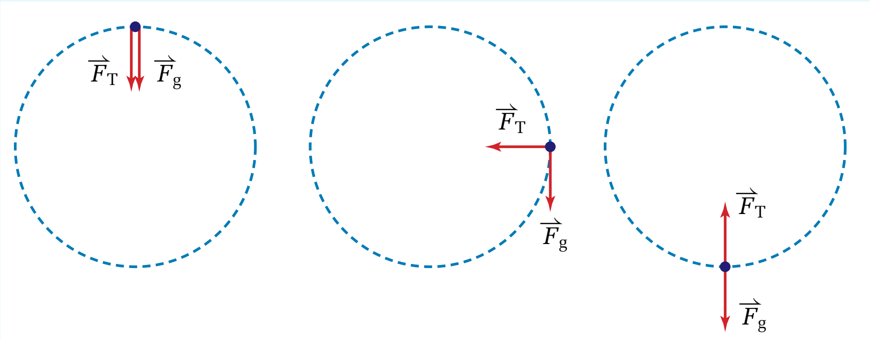
2. You are playing with a yo-yo with a mass of 225 g. The full length of the string is 1.2 m. You decide to see how slowly you can swing it in a vertical circle while keeping the string fully extended, even when the yo-yo is at the top of its swing.

(a) Calculate the minimum speed at which you can swing the yo-yo while keeping it on a circular path.

(b) At the speed that you determine in part (a), find the tension in the string when the yo-yo is at the side and at the bottom of its swing.

Frame the Problem

- Draw *free-body diagrams* of the yo-yo at the *top, bottom, and one side* of the swing.



- At the *top* of the swing, both *tension and the force of gravity* are acting *toward the centre* of the circle.
- If the required *centripetal force* is *less than the force of gravity*, the yo-yo will *fall* away from the circular path.
- If the required *centripetal force* is *greater than the force of gravity*, the *tension* in the string *will* have to *contribute* to the centripetal force.
- Therefore, the *smallest possible velocity* would be the case where the required *centripetal force* is exactly *equal* to the *force of gravity*.
- At the *side* of the swing, the force of *gravity is perpendicular* to the direction of the required centripetal force and therefore contributes *nothing*. The centripetal force must all be supplied by the tension in the string.
- At the *bottom* of the swing, the force of *gravity* is in the *opposite* direction from the required *centripetal force*. Therefore, the *tension* in the string must *balance* the force of *gravity* and *supply* the required *centripetal force*.

Identify the Goal

The minimum speed, v, at which the yo-yo will stay on a circular path
The tension, F_T, in the string when the yo-yo is at the side of its circular path
The tension, F_T, in the string when the yo-yo is at the bottom of its circular path

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Variables and Constants

Known	Implied	Unknown
$m = 225$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	v_{min}
$r = 1.2$ m		$F_{\text{T(side)}}$
		$F_{\text{T(bottom)}}$

Strategy / Calculations

Set the force of gravity on the yo-yo equal to the centripetal force and solve for the velocity.

$$F_g = F_c$$

$$mg = \frac{mv^2}{r}$$

$$\cancel{m}g\left(\frac{r}{\cancel{m}}\right) = v^2$$

$$v = \sqrt{gr}$$

Substitute numerical values and solve.

$$v = \sqrt{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.2\ \text{m})}$$

$$v = \sqrt{11.772\ \frac{\text{m}^2}{\text{s}^2}}$$

A negative answer has no meaning in this application.

$$v = \pm 3.431\ \frac{\text{m}}{\text{s}}$$

$$v \cong 3.4\ \frac{\text{m}}{\text{s}}$$

(a) The minimum speed at which the yo-yo can move is 3.4 m/s.

Set the force of tension in the string equal to the centripetal force. Insert numerical values and solve.

$$F_T = F_c$$

$$F_T = \frac{mv^2}{r}$$

$$F_T = \frac{(225\ \cancel{\text{g}})\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)\left(3.431\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\ \text{m}}$$

$$F_T = 2.207\ \frac{\text{kg}\frac{\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}}$$

$$F_T \cong 2.2\ \text{N}$$

(b): Side – When the yo-yo is at the side of its swing, the tension in the string is 2.2 N.

Set the centripetal force equal to the vector sum of the force of tension in the string and the gravitational force. Solve for the force due to the tension in the string.

$$F_c = F_T + F_g$$

$$\frac{mv^2}{r} = F_T - mg$$

$$F_T = \frac{mv^2}{r} + mg$$

Substitute numerical values and solve.

$$F_T = \frac{(225\ \cancel{\text{g}})\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)\left(3.431\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\ \text{m}} + (225\ \cancel{\text{g}})\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$

$$F_T = 2.207\ \frac{\frac{\text{kg}\cdot\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}} + 2.207\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

$$F_T = 4.414\ \text{N}$$

$$F_T \cong 4.4\ \text{N}$$

(b): Bottom – When the yo-yo is at the bottom of its swing, the tension in the string is 4.4 N.

Validate

The force of gravity (weight) of the yo-yo is 2.2 N. At the top of the swing, the weight supplies the entire centripetal force and the speed of the yo-yo is determined by this value. At the side of the swing, the tension must provide the centripetal force and the problem was set up so that the centripetal force had to be equal to the weight of the yo-yo, or 2.2 N. At the bottom of the swing, the tension must support the weight (2.2 N) and, in addition, provide the required centripetal force (2.2 N). You would therefore expect that the tension would be twice the weight of the yo-yo. The units cancel properly to give newtons for force.

PRACTICE PROBLEMS

15. A boy is twirling a 155 g ball on a 1.65 m string in a horizontal circle. The string will break if the tension reaches 208 N. What is the maximum speed at which the ball can move without breaking the string?

16. An electron (mass 9.11×10^{-31} kg) orbits a hydrogen nucleus at a radius of 5.3×10^{-11} m at a speed of 2.2×10^{6} m/s. Find the centripetal force acting on the electron. What type of force supplies the centripetal force?

17. A stone of mass 284 g is twirled at a constant speed of 12.4 m/s in a vertical circle of radius 0.850 m. Find the tension in the string **(a)** at the top and **(b)** at the bottom of the revolution. **(c)** What is the maximum speed the stone can have if the string will break when the tension reaches 33.7 N?

18. You are driving a 1654 kg car on a level road surface and start to round a curve at 77 km/h. If the radius of curvature is 129 m, what must be the frictional force between the tires and the road so that you can safely make the turn?

19. A stunt driver for a movie needs to make a 2545 kg car begin to skid on a large, flat, parking lot surface. The force of friction between his tires and the concrete surface is 1.75×10^{4} N and he is driving at a speed of 24 m/s. As he turns more and more sharply, what radius of curvature will he reach when the car just begins to skid?

Centripetal Force versus Centrifugal Force

You learned in Chapter 5 that a centrifugal force is a fictitious force. Now that you have learned about centripetal forces, you can understand more clearly why a centrifugal force is classed as fictitious.

Analyze the motion of and the force on a person who is riding the Round Up. Imagine that Figure 11.9 is a view of the Round Up ride from above and at some instant you are at point A on the ride. At that moment, your velocity ($\vec{v}$) is tangent to the path of the ride. If no force was acting on you at all, you would soon be located at point B. However, the solid cylindrical structure of the ride exerts a normal force on you, pushing you to point C. There is no force pushing you outward, just a centripetal force pushing you toward the centre of the circular ride.

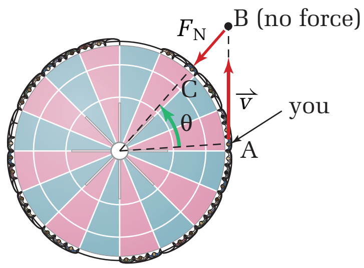


Figure 11.9 Assume that the Round Up ride is rotating at a constant speed and you are at point A. After a short time interval, in the *absence* of a force acting on you, you would move to point B, radially outward from point C. A centripetal force is required to change the direction of your velocity and place you at point C.

PROBEWARE

www.mcgrawhill.ca/links/atlphysics

If your school has probeware equipment, visit the Internet site above and follow the links for an in-depth activity on circular motion.

Describing Rotational Motion

When an object is constantly rotating, physicists sometimes find it more convenient to describe the motion in terms of the frequency — the number of complete rotations per unit time — or the period — the time required for one complete rotation — instead of the velocity of the object. You can express the centripetal acceleration and the centripetal force in these terms by finding the relationship between the magnitude of the velocity of an object in uniform circular motion and its frequency and period.

- Write the definition of velocity. Since period and frequency are scalar quantities, omit vector notations.

$$v = \frac{\Delta d}{\Delta t}$$

- The distance that an object travels in one rotation is the circumference of the circle.

$$\Delta d = 2\pi r$$

- The time interval for one cycle is the period, T.

$$\Delta t = T$$

- Substitute the distance and period into the equation for velocity, v.

$$v = \frac{2\pi r}{T}$$

- Substitute the above value for v into the equation for centripetal acceleration, a, and simplify.

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c = \frac{\frac{4\pi^2 r^{\not 2}}{T^2}}{\not r}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

- Substitute the above value for a into the equation for centripetal force and simplify.

$$F_c = ma_c$$

$$F_c = m\left(\frac{4\pi^2 r}{T^2}\right)$$

$$F_c = \frac{4\pi^2 mr}{T^2}$$

- The frequency is the inverse of the period.

$$f = \frac{1}{T} \text{ or } T = \frac{1}{f}$$

- Substitute the above value for the period into the equation for centripetal acceleration and simplify.

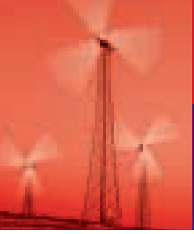
$$a_c = \frac{4\pi^2 r}{\left(\frac{1}{f}\right)^2}$$

$$a_c = 4\pi^2 rf^2$$

- Substitute the above value for acceleration into the equation for the centripetal force.

$$F_c = m(4\pi^2 rf^2)$$

$$F_c = 4\pi^2 mrf^2$$



INVESTIGATION 11-B

Verifying the Circular Motion Equation

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Communicating results

You have seen the derivation of the equation for circular motion and solved problems by using it. However, it is always hard to accept a theoretical concept until you test it for yourself. In this investigation, you will obtain experimental data for uniform circular motion and compare your data to the theory.

Problem

How well does the equation describe actual experimental results?

Equipment

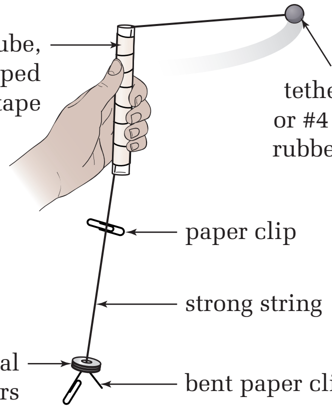
- laboratory balance
- force probeware or stopwatch
- ball on the end of a strong string
- glass tube (15 cm long with fire-polished ends, wrapped in tape)
- metre stick
- 12 metal washers
- tape
- paper clips

CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people and equipment while doing this activity.

Procedure

Alternative A: Using Traditional Apparatus

1. Measure the mass of the ball.
2. Choose a convenient radius for swinging the ball in a circle. Use the paper clip or tape as a marker, as shown in the diagram at the top of the next column, so you can keep the ball circling within your chosen radius.
3. Measure the mass of one washer.
4. Fasten three washers to the free end of the string, using a bent paper clip to hold them in place. Swing the string at a velocity that will maintain the chosen radius. Measure the time for several revolutions and use it to calculate the period of rotation.



5. Calculate the gravitational force on the washers (weight), which creates tension in the string. This force provides the centripetal force to keep the ball moving on the circular path.
6. Repeat for at least four more radii.

Alternative B: Using Probeware

1. Measure the mass of the ball.
2. Attach the free end of the string to a swivel on a force probe, as shown in the diagram on the next page.
3. Set the software to collect force-time data approximately 50 times per second. Start the ball rotating at constant velocity, keeping the radius at the proper value, and collect data for at least 10 revolutions.

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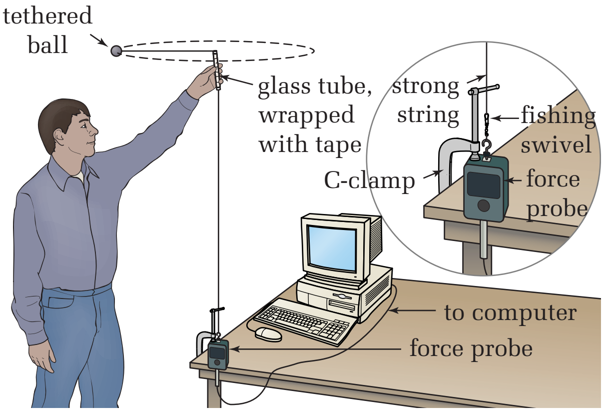
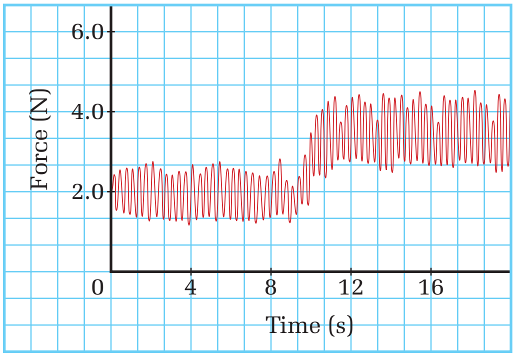
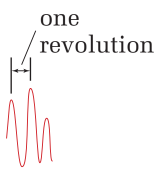


4. Examination of the graph will show regular variations from which you can calculate the period of one revolution, as well as the average force.





5. Repeat for at least five different radii.

Analyze and Conclude

1. For each radius, calculate and record in your data table the velocity of the ball. Use the period and the distance the ball travels in one revolution (the circumference of its circular path).
2. For each radius, calculate and record in your data table $\frac{mv^2}{r}$.
3. Graph F_c against $\frac{mv^2}{r}$. Each radius will produce one data point on your graph.
4. Draw the best-fit line through your data points. How can you tell from the position of the points whether the relationship being tested, $F_c = \frac{mv^2}{r}$, actually describes the data reasonably well?
5. Calculate the slope of the line. What does the slope tell you about the validity of the mathematical relationship?
6. Identify the most likely sources of error in the experiment. That is, what facet of the experiment might have been ignored, even though it could have a significant effect on the results?

Apply and Extend

Based on the experience you have gained in this investigation and the theory that you have learned, answer the following questions about circular motion. Support your answers in each case by describing how you would experimentally determine the answer to the question and how you would use the equations to support your answer.

7. How is the required centripetal force affected when everything else remains the same but the frequency of rotation increases?
8. How is the required centripetal force affected when everything else remains the same but the period of rotation increases?
9. If the radius of the circular path of an object increases and the frequency remains the same, how will the centripetal force change?
10. How can you keep the velocity of the object constant while the radius of the circular path decreases?

Banked Curves

Have you ever wondered why airplanes tilt or bank so much when they turn, as the airplanes in the photograph are doing? Now that you have learned that a centripetal force is required in order to follow a curved path or turn, you probably realize that banking the airplane has something to do with creating a centripetal force. Land vehicles can use friction between the tires and the road surface to obtain a centripetal force, but air friction (or drag) acts opposite to the direction of the motion of the airplane and cannot act perpendicular to the direction of motion. What force could possibly be used to provide a centripetal force for an airplane?

When an airplane is flying straight and horizontally, the design of the wings and the flow of air over them creates a lift force (L) that keeps the airplane in the air, as shown in Figure 11.11. The lift must be equal in magnitude and opposite in direction to the weight of the airplane in order for the airplane to remain on a level path. When an airplane banks, the lift force is still perpendicular to the wings. The vertical component of the lift now must balance the gravitational force, while the horizontal component of the lift provides a centripetal force. The free-body diagram on the right-hand side of Figure 11.11 helps you to see the relationship of the forces more clearly.

Figure 11.10 When an airplane follows a curved path, it must tilt or bank to generate a centripetal force.

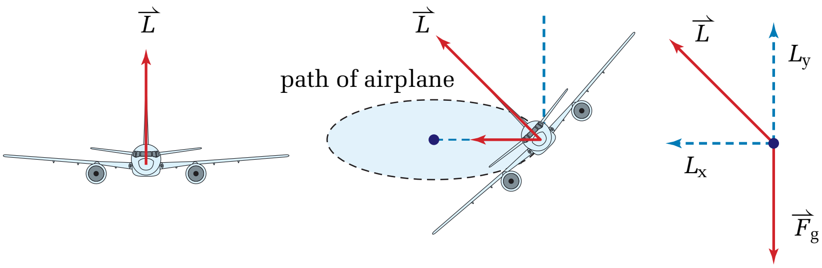


Figure 11.11 When a pilot banks an airplane, the forces of gravity and lift are not balanced. The resultant force is perpendicular to the direction that the airplane is flying, thus creating a centripetal force.

Cars and trucks can use friction as a centripetal force. However, the amount of friction changes with road conditions and can become very small when the roads are icy. As well, friction causes wear and tear on tires and causes them to wear out faster. For these reasons, the engineers who design highways where speeds are high and large centripetal forces are required incorporate another source of a centripetal force — banked curves. Banked curves on a road function in a way that is similar to the banking of airplanes.

Figure 11.12 shows you that the normal force of the road on a car provides a centripetal force when the road is banked, since a normal force is always perpendicular to the road surface.

You can use the following logic to develop an equation relating the angle of banking to the speed of a vehicle rounding a curve. Since an angle is a scalar quantity, omit vector notations and use only magnitudes. Assume that you wanted to know what angle of banking would allow a vehicle to move around a curve with a radius of curvature r at a speed v, without needing any friction to supply part of the centripetal force.

- Since a car does not move in a vertical direction, the vertical component of the normal force must be equal in magnitude to the force of gravity.

$$F_N \cos\theta = F_g$$
$$F_N \cos\theta = mg$$

- The horizontal component of the normal force must supply the centripetal force.

$$F_N \sin\theta = F_c$$
$$F_N \sin\theta = \frac{mv^2}{r}$$

- Divide the second equation by the first and simplify.

$$\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$
$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{v^2}{r}}{g}$$
$$\tan\theta = \frac{v^2}{rg}$$

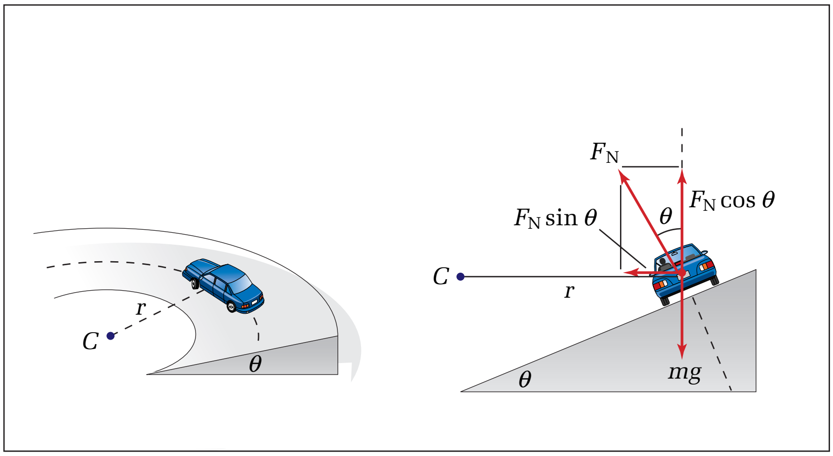
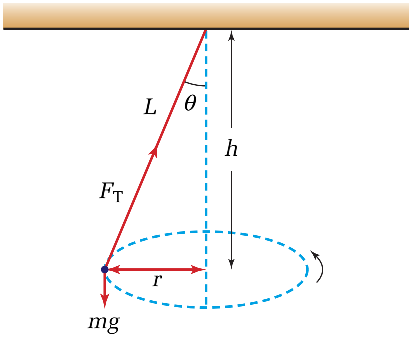


Figure 11.12 When you look at a cross section of a car rounding a curve, you can see that the only two forces in a vertical plane that are acting on the car are the force of gravity and the normal force of the road. The resultant force is horizontal and perpendicular to the direction in which the car is moving. This resultant force supplies a centripetal force that causes the car to follow a curved path.

Notice that the mass of the vehicle does not affect the amount of banking that is needed to drive safely around a curve. A semitrailer and truck could take a curve at the same speed as a motorcycle without relying on friction to supply any of the required centripetal force. Apply what you have learned about banking to the following problems.

Conceptual Problem

- A conical pendulum swings in a circle, as shown in the diagram. Show that the form of the equation relating the angle that the string of the pendulum makes with the vertical to the speed of the pendulum bob is identical to the equation for the banking of curves. The pendulum has a length L, an angle θ with the vertical, a force of tension F_T in the string, a weight mg, and swings in a circular path of radius r. The plane of the circle is a distance h from the ceiling from which the pendulum hangs.



MODEL PROBLEM

Banked Curves and Centripetal Force

Canadian Indy racing car driver Paul Tracy set the speed record for time trials at the Michigan International Speedway (MIS) in the year 2000. Tracy averaged 378.11 km/h in the time trials. The ends of the 3 km oval track at MIS are banked at 18.0° and the radius of curvature is 382 m.

(a) **At what speed can the cars round the curves without needing to rely on friction to provide a centripetal force?**

(b) **Did Tracy rely on friction for some of his required centripetal force?**

Frame the Problem

- The *normal force* of a *banked curve provides* a *centripetal force* to help cars turn without requiring an excessive amount of friction.
- For a given *radius of curvature* and *angle of banking*, there is *one speed* at which the normal force provides precisely the amount of centripetal force that is needed.

Identify the Goal

(a) The speed, v, for which the normal force provides exactly the required amount of centripetal force for driving around the curve

(b) Whether Tracy needed friction to provide an additional amount of centripetal force

Variables and Constants

Known	Implied	Unknown
$r = 382\ \text{m}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	v
$\theta = 18.0^\circ$		
$v_{PT} = 378.11\ \frac{\text{km}}{\text{h}}$		

continued ►

Strategy

Write the equation that relates angle of banking, speed, and radius of curvature, and solve for speed, v.

Calculations

$$\tan\theta = \frac{v^2}{rg}$$

$$v^2 = rg\tan\theta$$

$$v = \sqrt{rg\tan\theta}$$

Substitute the numerical values and solve.

$$v = \sqrt{(382\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(\tan 18.0^\circ)}$$

$$v = \sqrt{1217.61\ \frac{\text{m}^2}{\text{s}^2}}$$

$$v = 34.894\ \frac{\text{m}}{\text{s}}$$

$$v \cong 34.9\ \frac{\text{m}}{\text{s}}$$

(a) A vehicle driving at 34.9 m/s could round the curve without needing any friction for centripetal force.

Convert the velocity in m/s into km/h.

$$v = \left(34.894\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{3600\ \cancel{\text{s}}}{\text{h}}\right)\left(\frac{1\ \text{km}}{1000\ \cancel{\text{m}}}\right)$$

$$v = 125.619\ \frac{\text{km}}{\text{h}}$$

$$v \cong 126\ \frac{\text{km}}{\text{h}}$$

(b) Tracy was driving three times as fast as the speed of 126 km/h at which the normal force provides the needed centripetal force. Paul had to rely on friction for a large part of the needed centripetal force.

Validate

An angle of banking of 18° is very large compared to the banking on normal highway curves. You would expect that it was designed for speeds much higher than the highway speed limit. A speed of 126 km/h is higher than highway speed limits.

PRACTICE PROBLEMS

20. An engineer designed a turn on a road so that a 1225 kg car would need 4825 N of centripetal force when travelling around the curve at 72.5 km/h. What is the radius of curvature of the road?

21. A car exits a highway on a ramp that is banked at 15° to the horizontal. The exit ramp has a radius of curvature of 65 m. If the conditions are extremely icy and the driver cannot depend on any friction to help make the turn, at what speed should the driver travel so that the car will not skid off the ramp?

22. An icy curve with a radius of curvature of 175 m is banked at 12°. At what speed must a car travel to ensure that it does not leave the road?

23. An engineer must design a highway curve with a radius of curvature of 155 m that can accommodate cars travelling at 85 km/h. At what angle should the curve be banked?

You have studied just a few examples of circular motion that you observe or experience nearly every day. Although you rarely think about it, you have been experiencing several forms of circular motion every minute of your life. Simply existing on Earth's surface places you in uniform circular motion as Earth rotates. In addition, Earth is revolving around the Sun. In the next chapter, you will apply many of the concepts you have just learned about force and motion to the motion of planets, moons, and stars, as well as to artificial satellites.

11.2 Section Review

1. K/U Define uniform circular motion and describe the type of acceleration that is associated with it.
2. K/U Study the diagram in Figure 11.6 on page 552. Explain what approximation was made in the derivation that requires you to imagine what occurs as the angle becomes smaller and smaller.
3. C What are the benefits of using the concept of centripetal acceleration rather than working on a traditional Cartesian coordinate system?
4. K/U Explain how centripetal force differs from common forces, such as the forces of friction and gravity.
5. K/U If you were swinging a ball on a string around in a circle in a vertical plane, at what point in the path would the string be the most likely to break? Explain why. In what direction would the ball fly when the string broke?
6. C Explain why gravity does *not* affect circular motion in a horizontal plane, and why it *does* affect a similar motion in a vertical plane.
7. C Describe three examples in which different forces are contributing the centripetal force that is causing an object to follow a circular path.
8. MC When airplane pilots make very sharp turns, they are subjected to very large g forces. Based on your knowledge of centripetal force, explain why this occurs.
9. C A centrifugal force, if it existed, would be directed radially outward from the centre of a circle during circular motion. Explain why it feels as though you are being thrown outward when you are riding on an amusement park ride that causes you to spin in a circle.
10. K/U On a highway, why are sharp turns banked more steeply than gentle turns? Use vector diagrams to clarify your answer.
11. I Imagine that you are in a car on a major highway. When going around a curve, the car starts to slide sideways down the banking of the curve. Describe conditions that could cause this to happen.

UNIT PROJECT PREP

Parts of your catapult launch mechanism will move in part of a circle. The payload, once launched, will be a projectile.

- How will your launch mechanism apply enough centripetal force to the payload to move it in a circle, while still allowing the payload to be released?
- How will you ensure that the payload is launched at the optimum angle for maximum range?
- What data will you need to gather from a launch to produce the most complete possible analysis of the payload's actual path and flight parameters?

Physics Goes to the Fair!

There's no backing down. You've paid for your ticket, and you're in your seat with the restraint bar in place. Your heart is pounding as you look at the track in front of you. You're almost convinced you have nothing to worry about, but in the back of your mind, a worry flickers. Is this roller coaster safe? Well, rest easy. You'll be back on the fairground in no time, thanks to the physics of roller coaster design and the vigilance of Canada's provincial public safety inspectors.

Roller coasters have not always been safe. Early track designs employed circular loops. When coaster cars entered these loops at high speeds, they encountered excessive normal forces, putting riders at risk of whiplash and broken bones. When designers tried to correct the problem by decreasing the speed at which the cars entered the loops, the cars become projectiles, unable to make it through the loop without falling off the track.

Designers solved these problems with the clothoid loop. Clothoid loops are shaped like tear drops and have a constantly changing radius, where the radius at the bottom of the loop is larger than the radius at the top. The larger radius at the bottom allows the cars to enter the loop at high speeds. As the cars climb the loop, they are affected by gravity, but are still able to make it through the loop and maintain contact with the track because of the smaller radius at the top.

Once designers are confident they have ironed out all the kinks in amusements rides, it is the safety inspector's turn to make sure the rides are safe for the public. Alfred Byram is the Chief Public Safety Inspector with the New Brunswick government, and has worked for the past twenty-two years as an inspector, testing and inspecting elevators, and amusement rides at parks like Crystal Palace in Moncton.

Byram studied construction electricity at the New Brunswick Community College in Saint John, and then became a trained elevator mechanic before joining the government as a safety inspector. Before he could inspect amusement rides, Byram had to visit theme parks to gain onsite experience. Safety inspectors must be trained in mechanics, especially the mechanics of hydraulics. In order to make sure rides pass Canada's safety standards, Byram inspects ride tracks to make sure they are not worn, carts to make sure they are attached and intact, and hydraulic hoses to make sure they are in good working order.

Standards also require that nondestructive testing be done on all amusement rides. Nondestructive testing is testing that does not destroy the part or material being tested. An important test done on amusement rides is magnetic particle inspection, which uses magnetic fields and small magnetic particles, such as iron filings, to detect flaws in ferromagnetic components.

According to Byram, safety inspectors not only have to be good with parts, they have to be good with people too. Being able to interact diplomatically with amusement park owners and operators makes his job a lot easier. As for any requirement that inspectors be ride fanatics, Byram says there isn't one. Byram will only ride the Ferris wheel. He says, "It's all the thrill I need!"

Going Further

1. Most roller coaster tracks end with a series of parabolic hills that culminate with a sharp, steep drop. When riders descend the sharp drop they briefly undergo free fall. What is free fall? What kinds of forces are involved in free fall?