10.4 Collisions and Explosions

Throughout this chapter, you have been learning how to apply the principles of Newtonian mechanics to realistic cases. You have learned how to treat objects with finite dimensions instead of point masses. You have been working with more than one mass when they were exerting forces on each other through strings, ropes, and cables. In this section, you will consider another type of interaction between masses — collisions and explosions in more than one dimension.

Figure 10.19 It would be extremely difficult to determine the forces on each fragment of this exploding firecracker. What can you say about the explosion in general?

Thus far in this chapter, you have had success by finding the net force on an object or system and then applying Newton's second law and the kinematic equations to describe the motion of the system. However, when analyzing collisions and explosions, it is difficult to determine the forces that objects are exerting on each other such as the fragments flying off in all directions in the photograph in Figure 10.19. Collision and explosion events will be easier to analyze if you determine the motion of the objects before and after they exert forces on each other and use the motion data to perform analyses of the interaction.

You might recall, from Chapter 5, that a rearrangement of Newton's second law led to the concepts of momentum and impulse. Momentum is defined as the product of an object's mass and velocity as described in the following box. In Chapter 6, you learned that momentum is conserved. The box that shows the mathematical relationship that describes conservation of momentum

SECTION OUTCOMES

- Use vector analysis in two dimensions for systems involving two or more masses.
- Apply the laws of conservation of momentum to two dimensional collisions and explosions.
- Determine in which real-life situations involving elastic and inelastic interactions the laws of conservation of momentum and energy are best used.

KEY TERMS

- elastic collision
- inelastic collision

is also repeated here. In Chapters 5 and 6, you applied these relationships to motion in one dimension only. Now you will extend your skills to include two dimensions.

DEFINITION OF MOMENTUM

Momentum is the product of an object's mass and its velocity.

$$\vec{p} = m\vec{v}$$

Quantity	Symbol	SI unit
momentum	$\vec{p}$	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$ (kilogram metres per second)
mass	m	kg (kilograms)
velocity	$\vec{v}$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$(\text{mass})(\text{velocity}) = \text{kg} \cdot \frac{\text{m}}{\text{s}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

Note: Momentum does not have a unique unit of its own.

PHYSICS FILE

When working with collisions, instead of using subscripts such as "2," physicists often use a superscript symbol called a "prime," which looks like an apostrophe, to represent the variables *after* a collision. The variable is said to be "primed." Look for this notation in the box on the right.

LAW OF CONSERVATION OF MOMENTUM

The sum of the momenta of two objects before collision is equal to the sum of their momenta after they collide.

$$\vec{P}_{\text{A}} + \vec{P}_{\text{B}} = \vec{P}'_{\text{A}} + \vec{P}'_{\text{B}}$$

$$m_{\text{A}}\vec{v}_{\text{A}} + m_{\text{B}}\vec{v}_{\text{B}} = m_{\text{A}}\vec{v}'_{\text{A}} + m_{\text{B}}\vec{v}'_{\text{B}}$$

Quantity	Symbol	SI unit
mass of object A	m_{A}	kg (kilograms)
mass of object B	m_{B}	kg (kilograms)
velocity of object A before the collision	$\vec{v}_{\text{A}}$	$\frac{\text{m}}{\text{s}}$ (metres per second)
velocity of object B before the collision	$\vec{v}_{\text{B}}$	$\frac{\text{m}}{\text{s}}$ (metres per second)
velocity of object A after the collision	$\vec{v}'_{\text{A}}$	$\frac{\text{m}}{\text{s}}$ (metres per second)
velocity of object B after the collision	$\vec{v}'_{\text{B}}$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Collisions in Two Dimensions

Very few collisions are confined to one dimension, as anyone who has played billiards knows. Nevertheless, you can work in one dimension at a time, because momentum is conserved in each dimension independently. For example, consider the car crash illustrated in Figure 10.20. Car A is heading north and car B is heading east when they collide at the intersection. The cars lock together and move off at an angle. You can find the total momentum of the entangled cars because the component of the momentum to the north must be the same as car A's original momentum. The eastward component of the momentum must be the same as car B's original momentum. You can use the Pythagorean theorem to find the resultant momentum, as shown in the following problems.

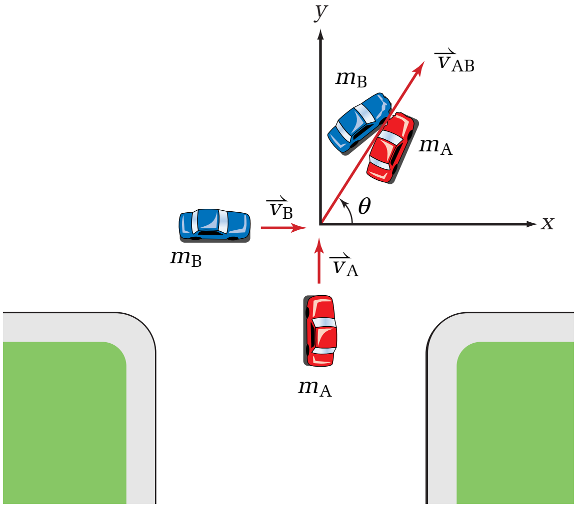
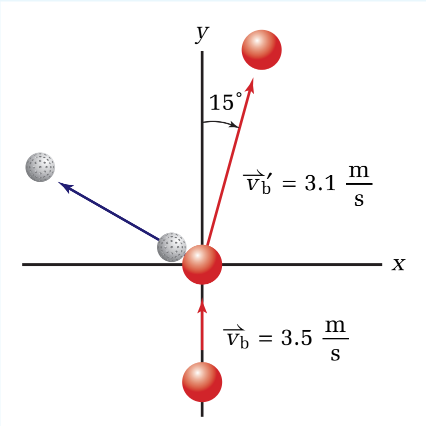


Figure 10.20 Momentum is conserved independently in both the north-south dimension and the east-west dimension.

MODEL PROBLEM

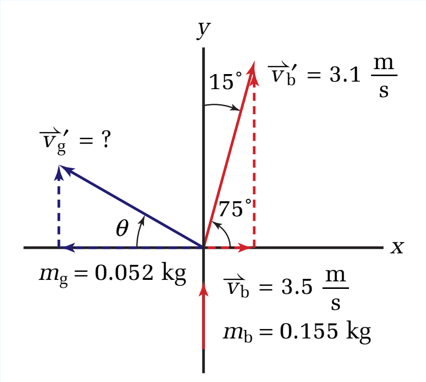
Applying Conservation of Momentum in Two Dimensions

A billiard ball of mass 0.155 kg is rolling directly away from you at 3.5 m/s. It collides with a stationary golf ball of mass 0.052 kg. The billiard ball rolls off at an angle of 15˚ clockwise from its original direction with a velocity of 3.1 m/s. What is the velocity of the golf ball?



Frame the Problem

- Sketch the vectors representing the momentum of the billiard ball and the golf ball immediately before and just after the collision. It is always helpful to superimpose an *x*–*y*-coordinate system on the vectors so that the origin is at the point of the contact of the two balls. For calculations, use the angles that the vectors make with the *x*-axis.
- Momentum is *conserved* in the *x* and *y* directions *independently*.
- The *total momentum* of the system (billiard ball and golf ball) *before* the collision is carried by the *billiard ball* and is all in the positive *y* direction.
- *After* the collision, both balls have *momentum* in both the *y* direction and the *x* direction.



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- Since the *momentum* in the *x direction* was *zero* before the collision, it must be *zero* after the collision. Therefore, the *x-components of the momentum* of the two balls after the collision must be *equal in magnitude* and *opposite in direction*.
- The *sum* of the *y*-components of the two balls *after* the collision must equal the *momentum* of the billiard ball *before* the collision.
- Use subscript "b" for the billiard ball and subscript "g" for the golf ball.
- Solve the problem in two ways — first using a scale diagram of the momentum vectors to visualize the solution and second using the method of components to obtain a precise answer.

PROBLEM TIP

When you are working with many bits of data in one problem, it is often helpful to organize the data in a table such as the one shown here.

Object		P_x	P_y
before	A		
	B		
	total		
after	A		
	B		
	total		

Identify the Goal

The velocity, $\vec{v}'_g$, of the golf ball after the collision

Variables and Constants

Known

$m_b = 0.155$ kg $\vec{v}_b = 3.5 \, \frac{m}{s}$ [forward]

$m_g = 0.052$ kg $\vec{v}'_b = 3.1 \, \frac{m}{s}$ [15° clockwise from original]

Implied

$\vec{v}_g = 0.00 \, \frac{m}{s}$

Unknown

$\vec{v}'_g$

Method I

Strategy

Use the equation for the conservation of momentum and solve for the unknown momentum of the golf ball after the collision.

Calculate the magnitudes of momentum vectors of the billiard ball before and after the collision.

Calculations

$$\vec{p}_b + \vec{p}_g = \vec{p}'_b + \vec{p}'_g$$
$$\vec{p}'_g = \vec{p}_b + \vec{p}_g - \vec{p}'_b$$
$$\vec{p}_g = 0$$
$$\vec{p}'_g = \vec{p}_b - \vec{p}'_b$$

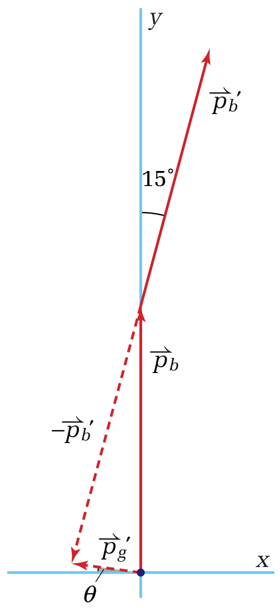
$$\vec{p} = m\vec{v}$$
$$|\vec{p}_b| = (0.155 \text{ kg})\left(3.5 \, \frac{m}{s}\right)$$
$$|\vec{p}_b| = 0.5425 \, \frac{kg \cdot m}{s}$$
$$|\vec{p}'_b| = (0.155 \text{ kg})\left(3.1 \, \frac{m}{s}\right)$$
$$|\vec{p}'_b| = 0.4805 \, \frac{kg \cdot m}{s}$$

On a coordinate system, make a scale diagram with the tail of $\vec{p}_b$ at the origin.

Draw $\vec{p}_b{}'$ with the tail at the tip of $\vec{p}_b$.

Draw $-\vec{p}_b{}'$ with the tail at the tip of $\vec{p}_b$.

Add $\vec{p}_b$ and $-\vec{p}_b{}'$ by drawing the resultant vector from the tail of $\vec{p}_b$ to the tip of $-\vec{p}_b{}'$. This is vector $\vec{p}_g{}'$.



scale: 1.0 cm = 0.20 $\frac{\text{kg}\cdot\text{m}}{\text{s}}$

Measure the length of $\vec{p}_g{}'$.

$\vec{p}_g{}'$ is 0.67 cm long

Measure angle of $\vec{p}_g{}'$.

$\theta = 34°$

Use the scale factor to determine magnitude of $\vec{p}_g{}'$.

$$\vec{p}_g{}' = (0.67\ \cancel{\text{cm}})\left(0.20\ \frac{\frac{\text{kg}\cdot\text{m}}{\text{s}}}{\cancel{\text{cm}}}\right)$$

$$\vec{p}_g{}' = 0.134\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Use the definition of momentum to solve for the velocity of the golf ball.

$$\vec{p}_g{}' = m_g \vec{v}_g{}'$$

$$\vec{v}_g{}' = \frac{\vec{p}_g{}'}{m_g}$$

$$|\vec{v}_g{}'| = \frac{0.134\ \frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}}}{0.052\ \cancel{\text{kg}}}$$

$$|\vec{v}_g{}'| = 2.577\ \frac{\text{m}}{\text{s}}$$

$$|\vec{v}_g{}'| \cong 2.6\ \frac{\text{m}}{\text{s}}$$

The velocity of the golf ball is 2.6 m/s at an angle of 34° clockwise from the negative x axis.

Method II

Strategy

Calculations

Write the expression for the conservation of momentum in the x direction.

$$m_b v_{bx} + m_g v_{gx} = m_b v'_{bx} + m_g v'_{gx}$$

Note that the x-component of the momentum of both balls was zero before the collision. Then solve for the x-component of the velocity of the golf ball after the collision.

$$0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = m_b v'_{bx} + m_g v'_{gx}$$

$$m_g v'_{gx} = -m_b v'_{bx}$$

$$v'_{gx} = -\frac{m_b v'_{bx}}{m_g}$$

Substitute values and solve.

$$v'_{gx} = -\frac{(0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}} \cos 75°)}{0.052\ \text{kg}}$$

$$v'_{gx} = -2.3916\ \frac{\text{m}}{\text{s}}$$

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Strategy

Carry out the same procedure for the *y*-components.

Calculations

$$m_b v_{by} + m_g v_{gy} = m_b v'_{by} + m_g v'_{gy}$$

$$m_b v_{by} + 0.0\ \frac{kg \cdot m}{s} = m_b v'_{by} + m_g v'_{gy}$$

$$m_g v'_{gy} = m_b v_{by} - m_b v'_{by}$$

$$v'_{gy} = \frac{m_b v_{by} - m_b v'_{by}}{m_g}$$

$$v'_{gy} = \frac{(0.155\ \text{kg})(3.5\ \frac{\text{m}}{\text{s}}) - (0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}} \sin 75°)}{0.052\ \text{kg}}$$

$$v'_{gy} = 1.507\ \frac{\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of the golf ball.

$$|\vec{v}'_g|^2 = v'^2_{gx} + v'^2_{gy}$$

$$|\vec{v}'_g|^2 = \left(-2.3916\ \frac{\text{m}}{\text{s}}\right)^2 + \left(1.507\ \frac{\text{m}}{\text{s}}\right)^2$$

$$|\vec{v}'_g|^2 = 5.7198\ \frac{\text{m}^2}{\text{s}^2} + 2.271\ \frac{\text{m}^2}{\text{s}^2}$$

$$|\vec{v}'_g|^2 = 7.9908\ \frac{\text{m}^2}{\text{s}^2}$$

$$|\vec{v}'_g| = 2.8268\ \frac{\text{m}}{\text{s}}$$

$$|\vec{v}'_g| \cong 2.8\ \frac{\text{m}}{\text{s}}$$

Use the tangent function to find the direction of the velocity vector.

Since the *x*-component is negative and the *y*-component is positive, the vector is in the second quadrant. Use positive values to find the magnitude of the reference angle.

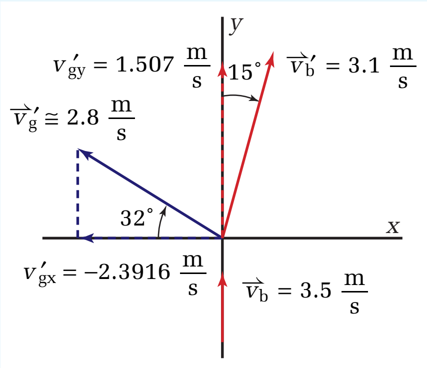
$$\tan\theta = \frac{v'_{gy}}{v'_{gx}}$$

$$\tan\theta = \frac{1.507\ \frac{\text{m}}{\text{s}}}{2.3916\ \frac{\text{m}}{\text{s}}}$$

$$\theta = \tan^{-1} 0.6301$$

$$\theta = 32.22°$$

$$\theta \cong 32°$$



The velocity of the golf ball after the collision is 2.8 m/s at 32° clockwise from the negative *x*-axis. (At more advanced levels, you will be expected to report angles counterclockwise from the positive *x*-axis. In this case, the angle would be 180° − 32° = 148° counterclockwise from the *x*-axis.)

Validate

Since all of the momentum before the collision was in the positive *y* direction, the *y*-component of momentum after the collision had to be in the positive *y* direction, which it was. Since there was no momentum in the *x* direction before the collision, the *x*-components of the momentum after the collision had to be in opposite directions, which they were. The scale diagram method and the method of components were in good agreement.

PRACTICE PROBLEMS

Use both the scale diagram method and the method of components to solve each problem.

35. A 0.150 kg billiard ball (A) is rolling toward a stationary billiard ball (B) at 10.0 m/s. After the collision, ball A rolls off at 7.7 m/s at an angle of 40.0° clockwise from its original direction. What is the speed and direction of ball B after the collision?

36. A bowling ball with a mass of 6.00 kg rolls with a velocity of 1.20 m/s toward a single standing bowling pin that has a mass of 0.220 kg. When the ball strikes the bowling pin, the pin flies off at an angle of 70.0° counterclockwise from the original direction of the ball, with a velocity of 3.60 m/s. What was the velocity of the bowling ball after it hit the pin?

37. Car A (1750 kg) is travelling due south and car B (1450 kg) is travelling due east. They reach the same intersection at the same time and collide. The cars lock together and move off at 35.8 km/h[E31.6°S]. What was the velocity of each car before they collided?

Angular Momentum

Why is a bicycle easy to balance when you are riding, but falls over when you come to a stop? Why does a toy gyroscope, like the one in Figure 10.21, balance on a pointed pedestal when it is spinning, but falls off the pedestal when it stops spinning? The answer lies in the conservation of angular momentum.

Figure 10.21 When a spinning object begins to fall, its angular momentum resists the direction of the fall.

When an object is moving on a curved path or rotating, it has angular momentum. Angular momentum and linear (or translational) momentum are similar in that they are both dependent on an object's mass and velocity. Analyze Figure 10.22 to find the third quantity that affects angular momentum.

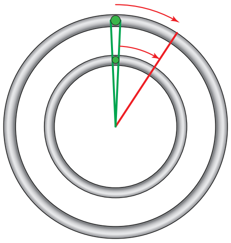


Figure 10.22 As the distance from the centre of rotation increases, a unit of mass must move faster in order to maintain a constant rate of rotation.

Web Link

www.mcgrawhill.ca/links/atlphysics

For information on current accident-investigation research topics and technological developments related to vehicle safety, go to the above Internet site and click on **Web Links**.

PHYSICS FILE

Although Kepler knew nothing about angular momentum, his second law, the law of areas which you will study in Chapter 12, is an excellent example of the conservation of angular momentum. With somewhat complex mathematics, it is possible to write the law of conservation of angular momentum for a planet in orbit and show that it is equivalent to Kepler's second law.

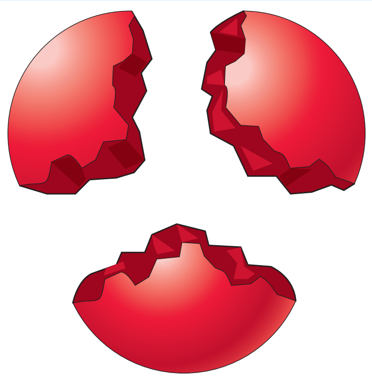
Picture the movement of a unit of mass in each of the two wheels illustrated in Figure 10.22. If the two wheels are rotating at the same rate, each unit of mass in the large wheel is moving faster than a unit of mass in the small wheel. Thus, r, the distance of a mass from the centre of rotation, affects the angular momentum. The magnitude of the angular momentum, L, of a particle that is moving in a circle is equal to the product of its mass, velocity, and distance from the centre of rotation, or $L = mvr$. You will not pursue a quantitative study of angular momentum any further in this course, but it is essential to be aware of the law of conservation of angular momentum in order to have a complete picture of the important conservation laws of physics. Similar to conservation of linear momentum, the angular momentum of an isolated system is conserved.

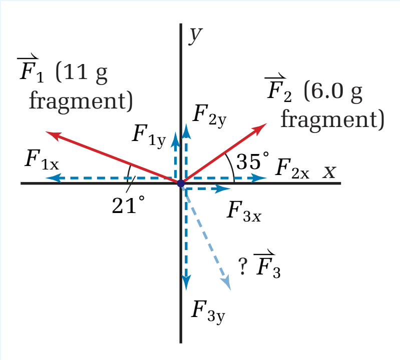
Explosions

Take another look at the photograph in Figure 10.19. How can you analyze something as complex as an explosion? Start with conditions before the explosion. Since there was no motion at all, the momentum was zero. Therefore, the total momentum of all of the parts after the explosion must also be zero according to conservation of momentum. The situation is very much like the concept of recoil that you studied in chapter 7. You can even think of recoil as an explosion in which two fragments are produced and they move in one dimension. Since momentum is conserved independently in each dimension, you can treat the fragments from an explosion in exactly the same way that you treated the two objects in recoil. The sum of the components of the momentum of all of the fragments in each dimension after the explosion must be zero. The following model problem shows you how to work with the fragments in two dimensions. Real explosions, of course, occur in three dimensions which does not require any new concepts. You would simply work with the x-, y-, and z-components of the momentum instead of just the x- and y-components.

MODEL PROBLEM

A 25 g spherical fire cracker explodes into three parts. You were able to get a photograph taken under a strobe light of two dimensions of the explosion. However, one of the fragments was out of the range of the photograph. After the explosion, you measured the mass of the two fragments and calculated the velocity of the fragments from the photograph. By superimposing a coordinate system on the photograph, you measured the angles at which the fragments moved. A 6.0 g fragment moved off at an angle of 35° with the positive *x* axis at a velocity of 42 m/s. An 11 g fragment moved off at an angle of 21° cw with the negative *x* axis. What was the velocity of the third fragment?





Frame the Problem

- Make a sketch of the momentum vectors after the explosion.
- After an explosion, the *vector sum* of the *momentum* of all of the fragments must be *zero*.

Identify the Goal

the velocity, $\vec{v}_3$, of the third fragment after the explosion

Variables and Constants

Known		Unknown
$m_1 = 6.0$ g	$m_2 = 11$ g	m_3
$\vec{v}_1 = 42\ \frac{\text{m}}{\text{s}}$	$\vec{v}_2 = 33\ \frac{\text{m}}{\text{s}}$	$\vec{v}_3$
$\theta_1 = 35°$ccw from x axis	$\theta_2 = 21°$cw from $-x$ axis	θ_3

Strategy

Find the magnitude of the momentum of fragments 1 and 2.

Calculations

$$\vec{p} = m\vec{v}$$
$$\vec{p}_1 = (0.006\ \text{kg})\left(42\ \frac{\text{m}}{\text{s}}\right)$$
$$\vec{p}_1 = 0.252\ \frac{\text{kg·m}}{\text{s}}$$

$$\vec{p} = m\vec{v}$$
$$\vec{p}_2 = (0.011\ \text{kg})\left(33\ \frac{\text{m}}{\text{s}}\right)$$
$$\vec{p}_2 = 0.363\ \frac{\text{kg·m}}{\text{s}}$$

Find the x- and y-components of the momentum of fragments 1 and 2.

$$p_{1x} = |\vec{p}_1| \cos 35°$$
$$p_{1x} = \left(0.252\ \frac{\text{kg·m}}{\text{s}}\right)(0.81915)$$
$$p_{1x} = 0.206426\ \frac{\text{kg·m}}{\text{s}}$$

$$p_{2x} = -|\vec{p}_2| \cos 21°$$
$$p_{2x} = -\left(0.363\ \frac{\text{kg·m}}{\text{s}}\right)(0.93358)$$
$$p_{2x} = -0.33889\ \frac{\text{kg·m}}{\text{s}}$$

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Strategy	Calculations
Since fragment 2 is in the second quadrant, the *x*-component of the momentum is negative.	$p_{1y} = \lvert\vec{p}_1\rvert \sin 35^\circ$ $p_{1y} = \left(0.252\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)(0.573576)$ $p_{1y} = 0.14454\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$ $p_{2y} = \lvert\vec{p}_2\rvert \sin 21^\circ$ $p_{2y} = \left(0.363\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)(0.358368)$ $p_{2y} = 0.130088\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$
Make a table of the components of all three fragments and the total.	(see table below)

x-components	*y*-components
$0.206426\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$	$0.14454\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$
$-0.33889\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$	$0.130088\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$
p_{3x}	p_{3y}
0.0	0.0

Solve for the components of the momentum of the third fragment.

$$p_{3x} + 0.206426\ \frac{\text{kg}\cdot\text{m}}{\text{s}} - 0.33889\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = 0.0$$
$$p_{3x} = -0.206426\ \frac{\text{kg}\cdot\text{m}}{\text{s}} + 0.33889\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$
$$p_{3x} = 0.132464\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$p_{3y} + 0.14454\ \frac{\text{kg}\cdot\text{m}}{\text{s}} + 0.130088\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = 0.0$$
$$p_{3y} = -0.14454\ \frac{\text{kg}\cdot\text{m}}{\text{s}} - 0.130088\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$
$$p_{3y} = -0.274628\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the momentum of fragment 3.

$$\lvert\vec{p}_3\rvert^2 = p_{3x}^2 + p_{3y}^2$$
$$\lvert\vec{p}_3\rvert^2 = \left(0.132464\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2 + \left(-0.274628\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2$$
$$\lvert\vec{p}_3\rvert^2 = 0.017547\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2 + 0.0754205\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2$$
$$\lvert\vec{p}_3\rvert^2 = 0.0929673\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2$$
$$\lvert\vec{p}_3\rvert = 0.30491\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)$$
$$\lvert\vec{p}_3\rvert \cong 0.30\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)$$

Use the tangent function to find the direction of the momentum of fragment 3.

$$\tan\theta = \frac{p_{3y}}{p_{3x}}$$
$$\theta = \tan^{-1}\frac{p_{3y}}{p_{3x}}$$

Since the *x*-component of the momentum is positive and the *y*-component is negative, the vector lies in the fourth quadrant.

$$\theta = \tan^{-1}\frac{0.274628\ \cancel{\frac{\text{kg}\cdot\text{m}}{\text{s}}}}{0.132464\ \cancel{\frac{\text{kg}\cdot\text{m}}{\text{s}}}}$$
$$\theta = 64.250^\circ$$
$$\theta \cong 64^\circ$$

Find the mass of fragment 3 from the total original mass and the mass of fragments 1 and 2.

$$6.0\text{ g} + 11\text{ g} + m_3 = 25\text{ g}$$
$$m_3 = 25\text{ g} - 6.0\text{ g} - 11\text{ g}$$
$$m_3 = 8.0\text{ g}$$

Find the velocity of fragment 3 from the definition of momentum.

$$|\vec{p}| = m|\vec{v}|$$
$$|\vec{v}| = \frac{|\vec{p}|}{m}$$
$$|\vec{v}| = \frac{0.30491\ \frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}}}{0.0080\ \cancel{\text{kg}}}$$
$$|\vec{v}| = 38.11375\ \frac{\text{m}}{\text{s}}$$
$$|\vec{v}| \cong 38\ \frac{\text{m}}{\text{s}}$$

The velocity of fragment 3 was 38 m/s at an angle of 64˚ cw from the positive x axis.

Validate

All of the units cancelled properly to give m/s which is correct for velocity. The magnitude of the velocity of fragment 3 is of the same range as the velocities of fragments 1 and 2 which is to be expected. The angle is reasonable based on the original sketch.

PRACTICE PROBLEMS

38. You accidentally dropped a 3.5 kg glass platter. Before it hit the floor, the motion was entirely in the vertical direction. When it hit the floor, it broke into three pieces and they all moved out in the plane of the floor. Imagine a coordinate system on the floor. Piece 1 had a mass of 1.3 kg and it moved off with a velocity of 1.8 m/s at an angle of 52˚ counterclockwise from the positive x axis. Piece 2 with a mass of 1.2 kg moved off with a velocity of 2.5 m/s at an angle of 61˚ clockwise from the negative x axis. Find the mass and the velocity of piece 3.

Energy and Momentum in Collisions

Momentum is conserved in the two collisions pictured in Figure 10.23, but the two cases are quite different. When the metal spheres in the Newton's cradle collided, both momentum and kinetic energy were conserved. When the cars in the photograph crashed, kinetic energy was *not* conserved. This feature divides all collisions into two classes. Collisions in which kinetic energy is conserved are said to be **elastic.** When kinetic energy is *not* conserved, the collisions are **inelastic.**

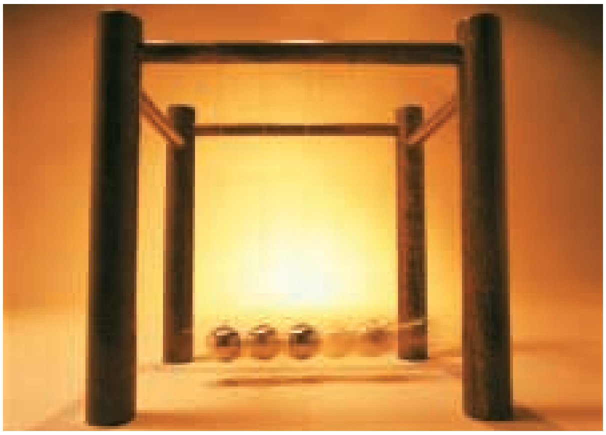


Figure 10.23 How do the collisions pictured here differ from each other?

You can determine whether a collision is elastic or inelastic by calculating both the momentum and the kinetic energy before and after the collision. Since momentum is always conserved at the instant of the collision, you can use the law of conservation of momentum to find unknown values for velocity. Then, use the known and calculated values for velocity to calculate the total kinetic energy before and after the collision. You will probably recall that the equation for kinetic energy is $E = \frac{1}{2}mv^2$.

MODEL PROBLEM

Classifying a Collision

A 0.0520 kg golf ball is moving east with a velocity of 2.10 m/s when it collides, head on, with a 0.155 kg billiard ball. If the golf ball rolls directly backward with a velocity of –1.04 m/s, was the collision elastic?

Frame the Problem

- *Momentum* is always conserved in a collision.
- If the collision is *elastic*, kinetic energy must also be conserved.
- The motion is in one dimension, so only positive and negative signs are necessary to indicate directions.

Identify the Goal

Is the total kinetic energy of the system before the collision, E_{kg}, equal to the total kinetic of the system after the collision, $E'_{kg} + E'_{kb}$?

Variables and Constants

Known		Implied	Unknown	
$m_g = 0.0520\ \text{kg}$	$v_g = +2.10\ \frac{\text{m}}{\text{s}}$	$v_b = 0.0\ \frac{\text{m}}{\text{s}}$	v'_b	E'_{kg}
$m_b = 0.155\ \text{kg}$	$v'_g = -1.04\ \frac{\text{m}}{\text{s}}$		E_{kg}	E'_{kb}

Strategy

Since momentum is always conserved, use the law of conservation of momentum to find the velocity of the billiard ball after the collision.

Calculations

$$m_g v_g + m_b v_b = m_g v'_g + m_b v'_b$$
$$m_g v_g + 0.0 - m_g v'_g = m_b v'_b$$
$$v'_b = \frac{m_g v_g - m_g v'_g}{m_b}$$
$$v'_b = \frac{(0.0520\ \text{kg})(2.10\ \frac{\text{m}}{\text{s}}) - (0.0520\ \text{kg})(-1.04\ \frac{\text{m}}{\text{s}})}{0.155\ \text{kg}}$$
$$v'_b = 1.0534\ \frac{\text{m}}{\text{s}}$$

Calculate the kinetic energy of the golf ball before the collision.

$$E_{kg} = \frac{1}{2} m_g v_g^2$$
$$E_{kg} = \frac{1}{2}(0.0520\ \text{kg})\left(2.10\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E_{kg} = 0.114\ 66\ \text{J}$$

Calculate the sum of the kinetic energies of the balls after the collision.

$$E'_{kg} = \frac{1}{2} m_g v'^2_g$$
$$E'_{kg} = \frac{1}{2}(0.0520\ \text{kg})\left(-1.04\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E'_{kg} = 0.028\ 12\ \text{J}$$

$$E'_{kb} = \frac{1}{2} m_b v'^2_b$$
$$E'_{kb} = \frac{1}{2}(0.155\ \text{kg})\left(1.0534\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E'_{kb} = 0.086\ 00\ \text{J}$$

$$E'_{kg} + E'_{kb} = 0.028\ 12\ \text{J} + 0.085\ 99\ \text{J}$$
$$E'_{kg} + E'_{kb} = 0.114\ 12\ \text{J}$$

The kinetic energies before and after the collision are the same to the third decimal place. Therefore, the collision was probably elastic.

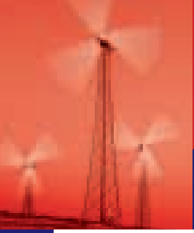
Validate

Although the kinetic energies before and after the collision differ in the fourth decimal place, the difference is less than 1%. Since the data contained only three significant digits, this difference could easily be due to the precision of the measurement. Therefore, it is fair to say that the collision was elastic.

PRACTICE PROBLEMS

39. A billiard ball of mass 0.155 kg moves with a velocity of 12.5 m/s toward a stationary billiard ball of identical mass and strikes it with a glancing blow. The first billiard ball moves off at an angle of 29.7° clockwise from its original direction, with a velocity of 9.56 m/s. Determine whether the collision was elastic.

40. Car A, with a mass of 1735 kg, was travelling north at 45.5 km/h and Car B, with a mass of 2540 kg, was travelling west at 37.7 km/h when they collided at an intersection. If the cars stuck together after the collision, what was their combined momentum? Was the collision elastic or inelastic?



INVESTIGATION 10-B

Collisions in Two Dimensions

TARGET SKILLS
- Initiating and planning
- Performing and recording
- Analyzing and interpreting

You can investigate the conservation of momentum and energy in two dimensional collisions in several ways. If you have access to an air table, strobe light, and Polaroid™ camera, you can use the procedure below. Alternatively, your teacher might provide you with simulations of the air table. If this is your option, use the simulations and follow the directions under "Analyze and Conclude." A final option is to find a simulation of two dimensional collisions on the Internet. If you choose this option, follow the directions that accompany the simulation.

Problem

Verify the law of conservation of momentum in two dimensions.

Equipment

- air table
- 2 pucks
- strobe light
- Polaroid™ camera
- ruler
- protractor
- laboratory balance

Procedure

1. Using the laboratory balance, determine the mass of each of two pucks. If the pucks have nearly the same mass, add some mass to one of the pucks with modelling clay.
2. With the air pressure on, place one of the pucks near the centre of the table. If it will not remain still, one partner should very gently hold it in place. Push a second puck toward the stationary puck so it will make a glancing collision with the central puck. Observe the motion of the pucks before and after the collision. Practice making the collision until you are confident you can do it every time.
3. Place two markers (masking tape) at two points on the edge of the table to use to determine the scale of the photographs.
4. Set up the strobe light so it is aimed toward the centre of the air table.
5. Set up the Polaroid™ camera so the centre of the image will be near the centre of the table. Set the shutter speed so that it will get all of the motion of the pucks before and after the collision.
6. Turn off the room lights and take a picture of a collision. Take enough photographs so that each pair of partners will have one to analyze.

Analyze and Conclude

1. Determine the scale of the photograph by measuring the distance between your markers on the table and the distance between the markers in the photograph.
2. On the photograph, measure the distances that the pucks moved before and after the collision. Calculate the actual distances using the scale that you determined in Step 7.
3. Measure the angles that the pucks took after the collision relative to the direction of the puck that was moving before the collision.
4. Using the rate at which the strobe light was flashing, determine the time between flashes.
5. Calculate the velocity, momentum, and kinetic energy of each puck before and after the collision.
6. Compare the total momentum of the pucks before and after the collision. Comment on how well the motion of the pucks obeyed the law of conservation of momentum.
7. Compare the total kinetic energy of the pucks before and after the collision. Comment on whether the collision was elastic or inelastic.

Elastic Collisions

By now, you have probably concluded that when objects collide, become deformed, and stick together, the collision is inelastic. Physicists say that such a collision is *completely inelastic*. Conversely, when hard objects such as billiard balls collide, bounce off each other, and return to their original shape, they have undergone elastic collisions. Very few collisions are perfectly elastic, but in many cases, the loss of kinetic energy is so small that it can be neglected.

Since both kinetic energy and momentum are conserved in perfectly elastic collisions, as many as four independent equations can be used to solve problems. Since you have four equations, you can solve for up to four unknown quantities. When combining these equations, however, the math becomes quite complex for all cases except head-on collisions, for which all motion is in one dimension.

An analysis of head-on collisions yields some very informative results, however. For example, if you know the velocities of the two masses before a collision, you can determine what the velocities will be after the collision. The following derivation applies to a mass, m_1, that is moving toward a stationary mass, m_2. Follow the steps to find the velocities of the two objects after the collision in terms of their masses and the velocity of the first mass before the collision. Since the motion in head-on collisions is in one dimension, vector notations will not be used.

- Write the equations for the conservation of momentum and kinetic energy for a perfectly elastic collision, inserting zero for the velocity of the second mass before the collision.

$$m_1v_1 + 0 = m_1v_1' + m_2v_2'$$
$$\frac{1}{2}m_1v_1^2 + 0 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

- Multiply by 2 both sides of the equation for conservation of kinetic energy.

$$m_1v_1^2 = m_1v_1'^2 + m_2v_2'^2$$

- Algebraically rearrange both equations so that terms describing mass 1 are on the left-hand side of the equations and terms describing mass 2 are on the right-hand side.

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2$$

- Factor m_1 out of the left-hand side of both equations.

$$m_1(v_1 - v_1') = m_2v_2'$$
$$m_1(v_1^2 - v_1'^2) = m_2v_2'^2$$

Language Link

In science, the word "elastic" does not mean "easily stretched." In fact, it can mean exactly the opposite. For example, glass is very elastic, up to its breaking point. Also, "elastic" is the opposite of "plastic." Find the correct meanings of the words "elastic" and "plastic" and then explain why "elastic" is an appropriate term to apply to collisions in which kinetic energy is conserved.

- Divide the first equation by the second equation.

$$\frac{\cancel{m_1}(v_1 - v_1')}{\cancel{m_1}(v_1^2 - v_1'^2)} = \frac{\cancel{m_2}v_2'}{\cancel{m_2}v_2'^2}$$

- Notice that the masses cancel. Expand the expression in the denominator on the left. Notice that it is the difference of perfect squares.

$$\frac{\cancel{(v_1 - v_1')}}{\cancel{(v_1 - v_1')}(v_1 + v_1')} = \frac{\cancel{v_2'}}{v_2'^{\cancel{2}}}$$

- Simplify. Solve the equation for v_2' by inverting. Also, solve the equation for v_1'.

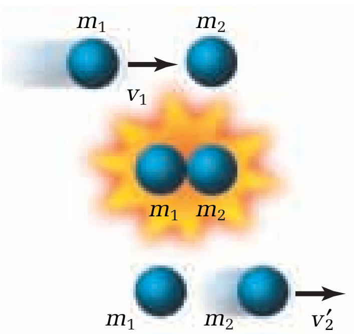
$$\frac{1}{(v_1 + v_1')} = \frac{1}{v_2'}$$
$$v_2' = v_1 + v_1'$$
$$v_1' = v_2' - v_1$$

- Develop two separate equations by substituting the values for v_1' and v_2' above into the equation for conservation of momentum, $m_1v_1 - m_1v_1' = m_2v_2'$. Expand and rearrange the equations and then solve for v_1' (left) and v_2' (right).

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1 - m_1v_1' = m_2(v_1 + v_1')$$
$$m_1v_1 - m_1v_1' = m_2v_1 + m_2v_1'$$
$$m_1v_1' + m_2v_1' = m_1v_1 - m_2v_1$$
$$v_1'(m_1 + m_2) = v_1(m_1 - m_2)$$
$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$$

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1 - m_1(v_2' - v_1) = m_2v_2'$$
$$m_1v_1 - m_1v_2' + m_1v_1 = m_2v_2'$$
$$2m_1v_1 = m_1v_2' + m_2v_2'$$
$$2m_1v\ = (m_1 + m_2)v_2'$$
$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

The two equations derived above allow you to find the velocities of two masses after a head-on collision in which a moving mass collides with a stationary mass. Without doing any calculations, however, you can draw some general conclusions. First, consider the case in which the two masses are identical.



When one moving mass collides head on with an identical stationary mass, the first mass stops. The second mass then moves with a velocity identical to the original velocity of the first mass.

Case 1: $m_1 = m_2$

Since the masses are equal, call them both "m." Substitute m into the two equations for the velocities of the two masses after the collision. Then, mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$$
$$v_1' = \left(\frac{m - m}{m + m}\right)v_1$$
$$v_1' = \left(\frac{0}{m + m}\right)v_1$$
$$v_1' = 0$$

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$
$$v_2' = \left(\frac{2m}{m + m}\right)v_1$$
$$v_2' = \left(\frac{\cancel{2m}}{\cancel{2m}}\right)v_1$$
$$v_2' = v_1$$

Case 2: $m_1 >>> m_2$

Since mass 1 is much larger than mass 2, you can almost ignore the mass of the second object in your calculations. You can therefore make the following approximations.

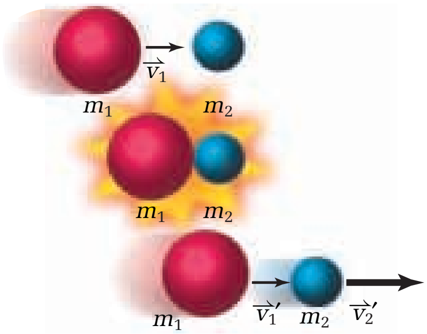
$$m_1 - m_2 \cong m_1 \quad \text{and} \quad m_1 + m_2 \cong m_1$$

Substitute these approximations into the two equations for the velocities of the two masses after the collision. Then, mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' \cong \left(\frac{\cancel{m_1}}{\cancel{m_1}}\right) v_1 \qquad v_2' \cong \left(\frac{2\cancel{m_1}}{\cancel{m_1}}\right) v_1$$

$$v_1' \cong v_1 \qquad v_2' \cong 2v_1$$



When one moving mass collides head on with a much smaller stationary mass, the first mass continues at nearly the same speed. The second mass then moves with a velocity that is approximately twice the original velocity of the first mass.

Case 3: $m_1 <<< m_2$

Since mass 1 is much smaller than mass 2, you can ignore the mass of the first object in your calculations. You can therefore make the following approximations.

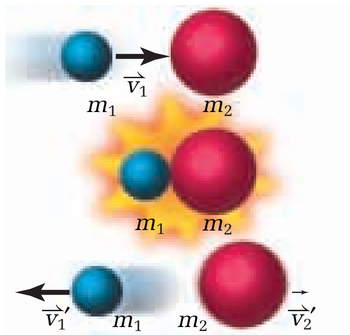
$$m_1 - m_2 \cong -m_2 \quad \text{and} \quad m_1 + m_2 \cong m_2 \quad \text{and} \quad m_1 \cong 0$$

Substitute these approximations into the two equations for the velocities of the two masses after the collision. Then mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' \cong \left(\frac{-\cancel{m_2}}{\cancel{m_2}}\right) v_1 \qquad v_2' \cong \left(\frac{0}{m_2}\right) v_1$$

$$v_1' \cong -v_1 \qquad v_2' \cong 0$$



When one moving mass collides head on with a much larger stationary mass, the first mass bounces backward with a velocity opposite in direction and almost the same in magnitude as its original velocity. The motion of the second mass is almost imperceptible.

Conceptual Problems

- Using the special cases of elastic collisions, qualitatively explain what would happen in each of the following situations.
  - **(a)** A bowling ball collides head on with a single bowling pin.
  - **(b)** A golf ball hits a tree.
  - **(c)** A marble collides head on with another marble that is not moving.

- Cars, trucks, and motorcycles do not undergo elastic collisions, but the general trend of the motion is similar to the motion of objects involved in elastic collisions. Describe, in very general terms, what would happen in each of the following cases. In each case, assume that the vehicles did not become attached to each other.
  - **(a)** A very small car runs into the back of a parked tractor-trailer.
  - **(b)** A mid-sized car runs into the back of another mid-sized car that has stopped at a traffic light.
  - **(c)** A pickup truck runs into a parked motorcycle.

Inelastic Collisions

When you are working with inelastic collisions, you can apply only the law of conservation of momentum to the motion of the objects at the instant of the collision. Depending on the situation, however, you might be able to apply the laws of conservation of energy to motion just before or just after the collision. For example, a ballistic pendulum can be used to measure the velocity of a projectile such as a bullet, as illustrated in Figure 10.24. When the bullet collides with the wooden block of the ballistic pendulum, it becomes embedded in the wood, making the collision completely inelastic.

After the collision, however, you can apply the law of conservation of mechanical energy to the motion of the pendulum. The kinetic energy of the pendulum at the instant after the collision is converted into potential energy of the pendulum bob. By measuring the height to which the pendulum rises, you can calculate the velocity of the bullet just before it hit the pendulum, as shown in the following model problem.

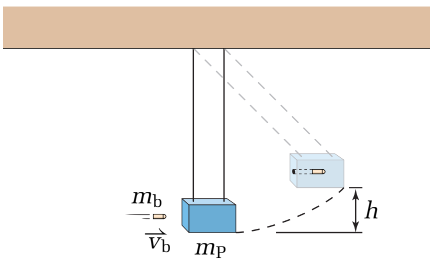


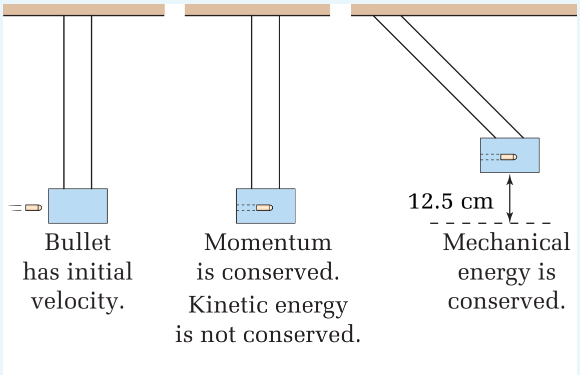
Figure 10.24 A ballistic pendulum is designed to have as little friction as possible. Therefore, you can assume that, at the top of its swing, the gravitational potential energy of the pendulum bob is equal to the kinetic energy of the pendulum bob at the lowest point of its motion.

MODEL PROBLEMS

Energy Conservation Before and After a Collision

1. **A forensic expert needed to find the velocity of a bullet fired from a gun in order to predict the trajectory of a bullet. He fired a 5.50 g bullet into a ballistic pendulum with a bob that had of mass 1.75 kg. The pendulum swung to a height of 12.5 cm above its rest position before dropping back down. What was the velocity of the bullet just before it hit and became embedded in the pendulum bob?**

Frame the Problem



- Sketch the positions of the bullet and pendulum bob just before the collision, just after the collision, and with the pendulum at its highest point.
- When the bullet hit the pendulum, *momentum* was *conserved*.
- If you can find the *velocity* of the combined bullet and pendulum bob after the collision, you can use conservation of momentum to find the *velocity* of the bullet before the collision.
- The collision was completely inelastic so *kinetic energy* was *not* conserved.
- However, you can assume that the friction of the pendulum is negligible, so *mechanical energy* of the pendulum was *conserved*.
- The *gravitational potential energy* of the combined masses at the highest point of the pendulum is equal to the *kinetic energy* of the combined masses at the lowest point of the pendulum.
- If you know the kinetic energy of the combined masses just after the collision, you can find the *velocity* of the masses just *after* the collision.
- Use the subscripts "b" for the bullet and "p" for the pendulum.

Identify the Goal

The velocity, v_b, of the bullet just before it hit the ballistic pendulum

Variables and Constants

Known		Implied	Unknown	
$m_b = 5.50$ g	$\Delta h = 12.5$ cm	$g = 9.81\ \frac{m}{s^2}$	$\vec{v}_b$	E_g
$m_p = 1.75$ kg			$\vec{v}_p$	E_k

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Strategy

To find the velocity of the combined masses of the bullet and pendulum bob just after the collision, use the relationship that describes the conservation of mechanical energy of the pendulum.

Substitute the expressions for kinetic energy and gravitational potential energy that you learned in Chapter 7. Solve for velocity. Convert all units to SI units.

Define the direction of the bullet as positive during and immediately after the collision.

Apply the conservation of momentum to find the velocity of the bullet before the collision. Convert all units to SI units.

Calculations

$$E_{k(\text{bottom})} = E_{g(\text{top})}$$

$$\frac{1}{2}\cancel{m}v^2_{\text{bottom}} = \cancel{m}g\Delta h$$

$$v^2_{\text{bottom}} = 2g\Delta h$$

$$v_{\text{bottom}} = \sqrt{2g\Delta h}$$

$$v_{\text{bottom}} = \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(12.5\ \cancel{\text{cm}})\left(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)}$$

$$v_{\text{bottom}} = \sqrt{2.4525\ \frac{\text{m}^2}{\text{s}^2}}$$

$$v_{\text{bottom}} = \pm 1.566\ \frac{\text{m}}{\text{s}}$$

$$m_b\vec{v}_b + m_p\vec{v}_p = m_b\vec{v}'_b + m_p\vec{v}'_p$$

$$m_b\vec{v}_b + 0 = (m_b + m_p)\vec{v}'_{b/p}$$

$$\vec{v}_b = \frac{(m_b + m_p)v'_{b/p}}{m_b}$$

$$\vec{v}_b = \frac{\left[5.50\ \cancel{\text{g}}\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right) + 1.75\ \text{kg}\right]1.566\ \frac{\text{m}}{\text{s}}}{5.50\ \cancel{\text{g}}\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)}$$

$$\vec{v}_b = \frac{(1.7555\ \cancel{\text{kg}})1.566\ \frac{\text{m}}{\text{s}}}{0.005\ 50\ \cancel{\text{kg}}}$$

$$\vec{v}_b = 499.8387\ \frac{\text{m}}{\text{s}}$$

$$\vec{v}_b \cong 5.00 \times 10^2\ \frac{\text{m}}{\text{s}}[\text{in positive direction}]$$

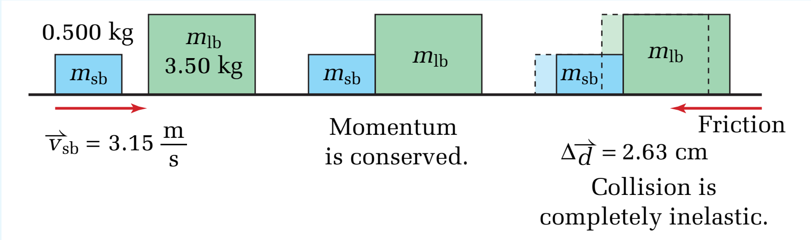
The velocity of the bullet just before the collision was about 500 m/s in the positive direction.

Validate

In both calculations, the units cancelled to give metres per second, which is correct for velocity. The velocity of 500 m/s is a reasonable velocity for a bullet.

2. A block of wood with a mass of 0.500 kg slides across the floor toward a 3.50 kg block of wood. Just before the collision, the small block is travelling at 3.15 m/s. Because some nails are sticking out of the blocks, the blocks stick together when they collide. Scratch marks on the floor show that they slid 2.63 cm before coming to a stop. What is the coefficient of friction between the wooden blocks and the floor?

Frame the Problem



- Sketch the blocks just before, at the moment of, and after the collision, when they came to a stop.
- *Momentum* is conserved during the collision.
- Since the blocks stuck together, the collision was *completely inelastic*, so *kinetic energy* was *not* conserved. Some kinetic energy was lost to sound, heat, and deformation of the wood during the collision.
- Some *kinetic energy* remained after the collision.
- The force of *friction* did *work* on the moving blocks, converting the remaining kinetic energy into heat.
- Due to the *law of conservation of energy*, you know that the *work* done by the force of *friction* was equal to the *kinetic energy* of the blocks at the instant after the collision.
- Since the motion is in one direction, use a plus sign to symbolize direction.
- Use the subscripts "sb" for the small block, "lb" for the large block, and "cb" for connected blocks.

Identify the Goal

The coefficient of friction, μ, between the wooden blocks and the floor

Variables and Constants

Known		Implied	Unknown		
$m_{sb} = 0.500$ kg	$\vec{v}_{sb} = 3.15\ \frac{m}{s}$	$g = 9.81\ \frac{m}{s^2}$	μ	W	$\vec{F}_N$
$m_{lb} = 3.50$ kg	$\Delta\vec{d} = 2.63$ cm	$\vec{v}_{lb} = 0.00\ \frac{m}{s}$	$\vec{F}_f$	E_k	$\vec{v}'_{cb}$

Strategy

Apply the law of conservation of momentum to find the velocity of the connected blocks of wood after the collision.

Calculations

$$m_{sb}\vec{v}_{sb} + m_{lb}\vec{v}_{lb} = m_{sb}\vec{v}'_{sb} + m_{lb}\vec{v}'_{lb}$$

$$m_{sb}\vec{v}_{sb} + 0 = (m_{sb} + m_{lb})\vec{v}'_{cb}$$

$$\vec{v}'_{cb} = \frac{m_{sb}\vec{v}_{sb}}{m_{sb} + m_{lb}}$$

$$\vec{v}'_{cb} = \frac{(0.500\ \text{kg})(3.15\ \frac{m}{s})}{0.500\ \text{kg} + 3.50\ \text{kg}}$$

$$\vec{v}'_{cb} = \frac{1.575\ \cancel{\text{kg}}\frac{m}{s}}{4.00\ \cancel{\text{kg}}}$$

$$\vec{v}'_{cb} = 0.393\ 75\ \frac{m}{s}\,[\text{to the right}]$$

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Strategy

Due to the law of conservation of energy, the work done on the blocks by the force of friction is equal to the kinetic energy of the connected blocks after the collision.

Substitute the expressions for work and kinetic energy into the equations.

Friction is the force doing the work, and it is always parallel to the direction of motion. Substitute the formula for the force of friction.

Since the blocks are moving horizontally, the normal force is the weight of the blocks. Substitute the weight into the expression and solve for the coefficient of friction.

Calculations

W (to stop blocks) $= E_k$ (after collision)

$$F_{||}\Delta d = \frac{1}{2}mv^2$$

$$F_f\Delta d = \frac{1}{2}mv^2$$

$$\mu F_N \Delta d = \frac{1}{2}mv^2$$

$$\mu mg\Delta d = \frac{1}{2}mv^2$$

$$\mu = \frac{\frac{1}{2}\cancel{m}v^2}{\cancel{m}g\Delta d}$$

$$\mu = \frac{v^2}{2g\Delta d}$$

$$\mu = \frac{(0.393\ 75\ \frac{\text{m}}{\text{s}})^2}{2(9.81\ \frac{\text{m}}{\text{s}^2})(2.63\ \cancel{\text{cm}})(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}})}$$

$$\mu = \frac{0.15\ 504\ \cancel{\frac{\text{m}^2}{\text{s}^2}}}{0.5160\ \cancel{\frac{\text{m}^2}{\text{s}^2}}}$$

$$\mu = 0.300\ 46$$

$$\mu \cong 0.300$$

The coefficient of friction between the blocks and the floor is 0.300.

Validate

All of the units cancel, which is correct because the coefficient of friction is unitless. The value of 0.300 is quite reasonable for a coefficient of friction between wood and another similar surface.

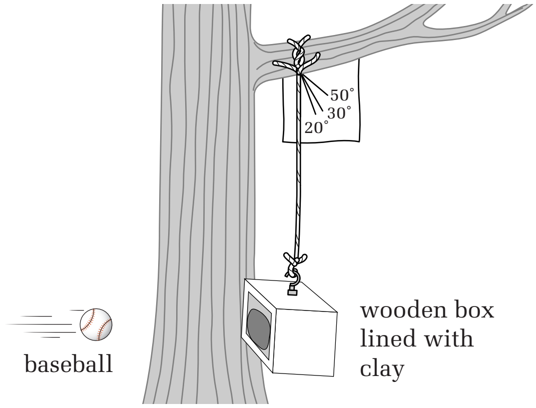
PRACTICE PROBLEMS

41. A 12.5 g bullet is shot into a ballistic pendulum that has a mass of 2.37 kg. The pendulum rises a distance of 9.55 cm above its resting position. What was the speed of the bullet?

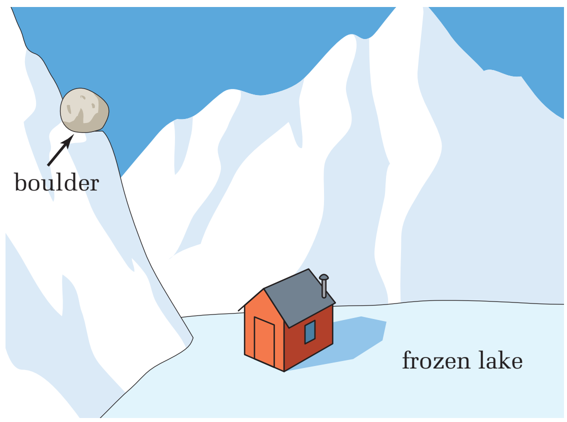
42. A student flings a 23 g ball of putty at a 225 g cart sitting on a slanted air track that is 1.5 m long. The track is slanted at an angle of 25° with the horizontal. If the putty is travelling at 4.2 m/s parallel to the track when it hits the cart, will the cart reach the end of the track before it stops and slides back down? Support your answer with calculations.

43. A car with a mass of 1875 kg is travelling along a country road when the driver sees a deer dart out onto the road. The driver slams on the brakes and manages to stop before hitting the deer. The driver of a second car (mass of 2135 kg) is driving too close and does not see the deer. When the driver realizes that the car ahead is stopping, he hits the brakes but is unable to stop. The cars lock together and skid another 4.58 m. All of the motion is along a straight line. If the coefficient of friction between the dry concrete and rubber tires is 0.750, what was the speed of the second car when it hit the stopped car?

44. You and some classmates read that the record for the speed of a pitched baseball is 46.0 m/s. You wanted to know how fast your school's star baseball pitcher could throw. Not having a radar gun, you used the concepts you learned in physics class. You made a pendulum with a rope and a small box lined with a thick layer of soft clay, so that the baseball would stick to the inside of the box. You drew a large protractor on a piece of paper and placed it at the top, so that one student could read the maximum angle of the rope when the pendulum swung up. The rope was 0.955 m long, the box with clay had a mass of 5.64 kg, and the baseball had a mass of 0.350 kg. Your star pitcher pitched a fastball into the box and the student reading the angle recorded a value of 20.0° from the resting, vertical position. How fast did your star pitcher pitch the ball?

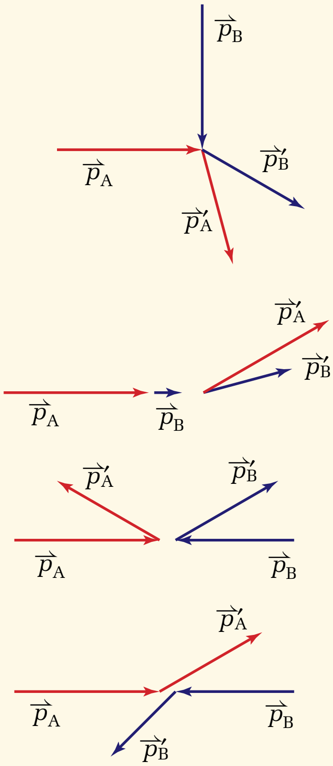


45. A 55.6 kg boulder sat on the side of a mountain beside a lake. The boulder was 14.6 m above the surface of the lake. One winter night, the boulder rolled down the mountain, directly into a 204 kg ice-fishing shack that was sitting on the frozen lake. What was the velocity of the boulder and shack at the instant that they began to slide across the ice? If the coefficient of friction between the shack and the rough ice was 0.392, how far did the shack and boulder slide?



10.4 Section Review

1. **I** The vectors in the following diagrams represent the momentum of objects before and after a collision. Which of the diagrams (there might be more than one) does *not* represent real collisions? Explain your reasoning.



2. **C** Some collision problems have two unknown variables, such as the velocities of two cars before a collision. Explain how it is possible to find two unknowns by using only the law of conservation of momentum.

3. **MC** Two cars of identical mass are approaching the same intersection, one from the south and one from the west. They reach the intersection at the same time and collide. The cars lock together and move away at an angle of 22° counter-clockwise from the road, heading east. Which car was travelling faster than the other before the collision? Explain your reasoning.

4. **K/U** What is the difference between an elastic collision and an inelastic collision?

5. **C** Describe an example of an elastic collision and an example of an inelastic collision that were not discussed in the text.

6. **C** Given a set of data for a collision, describe a step-by-step procedure that you could use to determine whether the collision was elastic.

7. **I** The results of the head-on collision in which the moving mass was much larger than the stationary mass ($m_1 >>> m_2$) showed that (a) that the velocity of mass 1 after the collision was almost the same as it had been before the collision and (b) that mass 2, which was stationary before the collision, attained a velocity nearly double that of mass 1 after the collision. Explain how it is possible for kinetic energy $\left(\frac{1}{2}mv^2\right)$ to be conserved in such a collision, when there was a negligible change in the velocity of mass 1 and a large increase in the velocity of mass 2.

8. **MC** Imagine that you have a very powerful water pistol. Describe in detail an experiment that you could perform, including the measurements that you would make, to determine the velocity of the water as it leaves the pistol.