# Static Equilibrium and Torque 

SECTION
OUTCOMES

- Use vector analysis in two dimensions for systems involving static equilibrium and torques.
- Apply static torques to structures such as seesaws and bridges.
- Identify questions to investigate that arise from practical problems.


## K E Y

## TERMS

- translational motion
- rotational motion
- pivot point
- torque
- lever arm
- centre of mass

In Section 10.1, you performed experiments and did calculations to determine when an object was in equilibrium. If the net force on a stationary object is zero, then the object should remain motionless. To extend the concept of equilibrium, study Figure 10.14. If the ruler is lying motionless on your desk and you exert forces on it that are equal in magnitude and opposite in direction at the locations shown in the figure, will it remain motionless? The net force on the ruler is zero yet it will begin to move.


Figure 10.14 When you push on opposite sides of the ruler at points that are not directly across from each other, the ruler will rotate.

At first glance, this motion appears to contradict Newton's first and second laws. Before jumping to such a drastic conclusion, however, analyze the type of motion that will occur. The ruler will rotate around some point but the entire ruler will not move off in one direction. Until now, you have been considering objects as point masses and observing the motion of the point. Real objects, however, have dimensions and can rotate.

When working with large objects as opposed to point masses, you can divide their motion into two parts - translational motion which is the motion of the object as a whole and rotational motion - angular motion around one point. The point around which the rotation occurs is the pivot point. Figure 10.15 illustrates both types of motion. A wrench is rotating as it is sliding across a nearly frictionless surface. The methods you have previously used to determine if an object was in equilibrium apply to its translational motion. In this section, you will learn the concepts you need in order to determine whether an object is in rotational equilibrium.


Figure 10.15 The wrench is rotating around the mark on the wrench while the mark is moving in a straight line.

## Torque

Since an object is in translational equilibrium when the sum of all of the forces acting on the object is zero, you might expect that there is another quantity that must be zero to create rotational equilibrium. That quantity is called torque and is symbolized by the Greek letter tau ( $\tau$ ). Torque causes a change in the rotational or angular motion of an object.

You probably know more about torque than you realize. As a child, you might have played on a seesaw. You knew that if two children of different weights wanted to play, you would have to adjust the seesaw so that the larger child was closer to the pivot point of the seesaw to balance it. A childs weight exerts a torque on the seesaw. Also, in the Multi Lab on page 453, you knew that you would have to support the ruler at a point very close to the centre or it would fall off of your finger. The ruler's own weight was exerting a torque on the ruler.


## PHYSICS FILE

You will discover, if you continue to study physics, that many symbols for rotational motion are Greek letters. For example theta $(\theta)$ is often used to symbolize an angle. The symbol for angular velocity (the rate of change of an angle $\theta$ ) is omega ( $\omega$ ) and the symbol for angular acceleration is alpha $(\alpha)$.

Figure 10.17 To find the lever arm, extend the force vector and drop a perpendicular line from the pivot point to the line of the force.

From the examples above, you can assume that force is involved in torque. To develop a complete expression for torque, think back to the Multi Lab again and think about the force that was necessary in order to open the door. You probably noticed that when you were pushing at a point closer to the hinge, the force required to cause the door to rotate around the hinge - or pivot point - was greater. This observation tells you that the distance between the point where the force is exerted and the pivot point must be involved in torque.

Torque is often defined as the lever arm times the magnitude of the force where the lever arm is the perpendicular distance between the line along which the force is acting and the pivot point. Torque is positive if, alone, it would turn the object counterclockwise and negative if it would turn the object clockwise. Figure 10.17 illustrates positive and negative torque.


## TOROUE

Torque is the product of the lever arm and the magnitude of the force. Lever arm is the perpendicular distance between the line along which the force is acting and the pivot point. Torque is positive if it will rotate the object counterclockwise and negative if it will rotate the object clockwise.

$$
\tau=r_{\perp} F
$$

Quantity
torque
magnitude of the force
lever arm

## Unit Analysis

(force)(lever arm) $=\mathrm{N} \cdot \mathrm{m}$
Note: The unit of torque, the newton•metre, is not a joule. A newton-metre is a joule only when the force and the displacement of an object are in the same direction and the force is therefore doing work on the object. In the case of torque, the force and the lever arm are perpendicular. No object is moving a distance equal to the lever arm.

Before applying the equation for torque, you need to answer one more question. At what point does the force of gravity act on an object? Think back to the observation you made when balancing a ruler with your finger. When your finger was not at the centre of the ruler, the ruler's own weight exerted a torque on the ruler and caused it to rotate and fall. When you found the balance point, gravity was still acting on the ruler but it did not rotate. Therefore, the lever arm must have been zero. Gravity was acting down on the ruler at the same point at which your finger was exerting an upward force. Gravity acts on the centre of mass of an object the point at which an object can be balanced.

When an object is not symmetrical, such as the piece of cardboard that you cut for the Multi Lab, it still has a centre of mass. It is just a little harder to predict where the centre of mass is located. Imagine that you divided an object into a very large number of very small (point) identical masses as shown in Figure 10.18. You can then think of the coordinates ( $x, y$, and $z$ ) of the centre of mass as the average of the coordinates of each tiny mass. In any problem, you will be given enough information to know where the centre of mass of an object is located.


Figure 10.18 Divide the object into many masses of identical size. Determine the $x$ coordinate of each mass and calculate the average. The result is the $x$ coordinate of the centre of mass.

Math Link
The mathematical equations for determining the coordinates of the centre of mass of an object are given below.

$$
\begin{aligned}
& x_{\mathrm{c}}=\frac{\sum m_{\mathrm{i}} x_{\mathrm{i}}}{M} \\
& y_{\mathrm{c}}=\frac{\sum m_{\mathrm{i}} y_{\mathrm{i}}}{M} \\
& z_{\mathrm{c}}=\frac{\sum m_{\mathrm{i}} z_{\mathrm{i}}}{M}
\end{aligned}
$$

The Greek letter $\sum$, capital sigma, means sum. The expressions means that you must multiply the mass of each small segment of mass by its distance in a given direction then add them together. Finally, you divide by the total mass, $M$.

## MODEL PROBLEM

(a) A cable is attached to the shaft of a 395 kg crane at a point 4.5 m from the hinged (pivot) point. When the crane is horizontal, the cable makes a $55^{\circ}$ angle with the crane and pulls on the crane with a force of 1250 N . What torque is the cable exerting on the crane?
(b) The crane is 5.0 m long and its centre of mass is at the centre of the crane. What torque is the cranes weight exerting on the crane?


## Frame the Problem

- Make separate sketches of force of the cable acting on the crane and the weight of the crane acting on the crane. Draw a perpendicular line from the force line to the pivot point.
- Use trigonometric functions to find the lengths of the lever arms.
- Use the equation for torque to determine the magnitude of the torque.

- Visualize the direction in which the crane would turn if each force acted in isolation to determine the sign of the torque.


## Identify the Goal

(a) torque, $\tau_{\mathrm{c}}$, exerted by cable
(b) torque, $\tau_{\mathrm{w}}$, exerted by weight of crane

## Variables and Constants

## Known

$$
\begin{array}{ll}
\left|\vec{F}_{\mathrm{c}}\right|=1250 \mathrm{~N} & \mathrm{l}_{\text {total }}=5.0 \mathrm{~m} \\
\theta=55^{\circ} & \mathrm{l}_{\text {to cable }}=4.5 \mathrm{~m} \\
m=395 \mathrm{~kg} &
\end{array}
$$

## Implied

$g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$

## Unknown

$\tau_{\mathrm{c}}$
$\tau_{\mathrm{w}}$

## Strategy

Analyze the triangle formed by the crane, cable, and lever arm and, using the sine function, determine the length of the lever arm.

## Calculations

$$
\begin{aligned}
\sin \theta & =\frac{\text { opposite side }}{\text { hypotenuse }} \\
\sin \theta & =\frac{r_{\perp}}{l_{\text {to cable }}} \\
r_{\perp} & =l_{\text {to cable }} \sin \theta \\
r_{\perp} & =(4.5 \mathrm{~m}) \sin 55^{\circ} \\
r_{\perp} & =3.6862 \mathrm{~m}
\end{aligned}
$$

Use the equation for torque.

$$
\begin{aligned}
& \tau_{\mathrm{C}}=r_{\perp} F \\
& \tau_{\mathrm{C}}=r_{\perp \mathrm{C}}\left|\vec{F}_{\mathrm{c}}\right| \\
& \tau_{\mathrm{C}}=(3.6862 \mathrm{~m})(1250 \mathrm{~N}) \\
& \tau_{\mathrm{C}}=4607.73 \mathrm{~N} \cdot \mathrm{~m} \\
& \tau_{\mathrm{c}} \cong 4.6 \times 10^{3} \mathrm{~N} \cdot \mathrm{~m}
\end{aligned}
$$

Determine direction that the crane would rotate around its hinge (pivot point).
counterclockwise
$\tau_{\mathrm{C}} \cong+4.6 \times 10^{3} \mathrm{~N} \cdot \mathrm{~m}$
(a) The torque due to the force of the cable on the crane is $+4.6 \times 10^{3} \mathrm{~N} \cdot \mathrm{~m}$.

Since the force of the crane's weight is perpendicular to the crane, the lever arm is the exact length from the hinge to the centre of the crane. Simply apply the equation for torque.

Determine direction that the crane would rotate around its hinge (pivot point).
$\tau=r_{\perp} F$
$\tau_{\mathrm{w}}=r_{\perp w}\left|\vec{F}_{\mathrm{g}}\right|$
$\tau_{\mathrm{w}}=r_{\perp w} m g$
$\tau_{\mathrm{w}}=\left[\left(\frac{1}{2}\right)(5.0 \mathrm{~m})\right](395 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)$
$\tau_{\mathrm{w}}=9687.375 \mathrm{~N} \cdot \mathrm{~m}$
$\tau_{\mathrm{w}} \cong 9.7 \times 10^{3} \mathrm{~N} \cdot \mathrm{~m}$
clockwise
$\tau_{\mathrm{w}} \cong-9.7 \times 10^{3} \mathrm{~N} \cdot \mathrm{~m}$
(b) The torque due to the weight of the crane is $-9.7 \times 10^{3} \mathrm{~N} \cdot \mathrm{~m}$

## Validate

The units are newton metres, the correct unit for torque. The cable and the weight of the crane are pulling in opposite directions so you would expect the signs of the torques to be opposite which they are.

## PRACTICE PROBLEMS

29. Consider your forearm a rigid rod and your elbow joint a pivot point. If your forearm is horizontal and your biceps muscle exerts a force of 1250 N directly upward at its point of attachment 0.052 m from your elbow, what torque does it exert on your forearm?

30. A 64 kg painter is standing three fourths of the distance up a ladder that is 3.0 m long. If the ladder makes an angle of $69^{\circ}$ with the ground, what torque does the painter's weight exert on the ladder?


## TRY THIS...

Cut an irregular shape from cardboard or use the piece you used in the Multi-Lab, Balancing Act. Push a pin through the cardboard very near to the edge and be sure that the cardboard rotates freely around the pin. Hang a string with a weight on the bottom from the string for a plumb line. Draw a line on the cardboard where the string hangs. Repeat the process for several points near the edge of the cardboard. The point where the lines cross is the centre of mass.


## Static Equilibrium

Having developed the concept of torque, you now have the skills necessary to determine whether an object is in static equilibrium, that is, the object will not move in any way. Conditions for both translational and rotational equilibrium must be met.

## CONDITIONS FOR STATIC EOUILIBRIUM

An object is in static equilibrium if the vector sum of all of the forces acting on it is zero and if the sum of all of the torques acting on it is zero.

$$
\sum \stackrel{\rightharpoonup}{F}=0 \quad \sum \tau=0
$$

Note: The capital sigma can be read "the sum of all."

A useful feature of torque is that if an object is in static equilibrium then the sum of the torques around any pivot point is zero. When solving a problem, you can choose any point on the object as the pivot point. You will find in the model and practice problems below, that the appropriate choice of a pivot point can simplify the calculations considerably because you can reduce the number of unknown quantities.

## MODEL PROBLEMS

1. A firefighter stands two thirds of the distance up a 4.0 m ladder with a mass of 35 kg that is leaning on a burning house. Assume that friction between the ladder and the side of the house is negligible. The total mass of the firefighter and gear is 95 kg . The ladder makes an angle of $58^{\circ}$ with the ground. What must be the coefficient of static friction between the ladder and the ground to prevent the ladder from sliding backwards?


## Frame the Problem

- Sketch and label the forces acting on the ladder. Make a separate sketch with lengths, angles, and lever arms labelled.
- Assign symbols to all of the forces.
$\vec{F}_{\mathrm{Nh}}$ - normal force of house on top of ladder
$\vec{F}_{\mathrm{f}}$ - frictional force of ground on base of ladder
$\vec{F}_{\mathrm{gl}}$ - weight of ladder
$\vec{F}_{\mathrm{gFF}}$ - weight of firefighter

$\vec{F}_{\text {NG }}$ - normal force of ground on base of ladder

Analyze what you need to reach your goal.

- To find the coefficient of static friction, you need to know the frictional force of the ground on the base of the ladder and the normal force of the ground on the base of the ladder.
- Setting the sum of the forces in the $y$ direction to zero will lead to the value of the normal force of the ground on the ladder.
- Since the frictional force of the ground on the base of the ladder and the normal force of the house on the top of the ladder are the only forces in the $x$ direction, they must add to zero. If you can find the normal force of the house on the ladder, you can find the frictional force. Neither of these forces are known.
- By properly choosing a pivot point, you can use torque to find the normal force of the house on the ladder.
- In this case, the best pivot point would be the point of contact of the ladder with the ground.
- Set up equations for translational equilibrium in the $x$ and $y$ directions and an equation for rotational equilibrium.


## PROBLEM TIP

Choose a pivot point that will make the lever arm zero for the maximum number of forces. This reduces the number of terms in the equation for rotational equilibrium.

## Identify the Goal

coefficient of static friction, $\mu_{\mathrm{s}}$, between the ground and the base of the ladder

## Variables and Constants

## Known

$l_{\text {ladder }}=4.0 \mathrm{~m} \quad m_{\mathrm{FF}}=95 \mathrm{~kg}$
$m_{\text {ladder }}=35 \mathrm{~kg} \quad \theta=58^{\circ}$

## Implied

$g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$

Unknown
$\vec{F}_{\mathrm{Nh}} \quad \vec{F}_{\mathrm{gFF}}$
$\vec{F}_{\mathrm{f}} \quad \vec{F}_{\mathrm{NG}}$
$\vec{F}_{\mathrm{gl}}$
Calculations
$\sum F_{\mathrm{y}}=0$
$F_{\mathrm{NG}}+F_{\mathrm{gl}}+F_{\mathrm{gFF}}=0$
$F_{\mathrm{NG}}-m_{\text {ladder }} g-m_{\mathrm{FF}} g=0$
$F_{\mathrm{NG}}=m_{\text {ladder }} g+m_{\mathrm{FF}} g$
$F_{\mathrm{NG}}=(35 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)+(95 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)$
$F_{\mathrm{NG}}=343.35 \mathrm{~N}+931.95 \mathrm{~N}$
$F_{\mathrm{NG}}=1275.3 \mathrm{~N}$

## Strategy

## Calculations

Set the sum of the torques around the point of contact with the ground equal to zero.

$$
\begin{aligned}
& \sum_{\tau_{\mathrm{NG}}} \tau=0 \\
& r_{\perp}+\tau_{\mathrm{gl}}+\tau_{\mathrm{FF}}+\tau_{\mathrm{Nh}}=0 \\
& F_{\mathrm{NG}}+r_{\perp} F_{\mathrm{f}}-r_{\perp} F_{\mathrm{gl}}-r_{\perp} F_{\mathrm{FF}}+r_{\perp} F_{\mathrm{Nh}}=0 \\
& F_{\mathrm{Nh}}=\frac{-r_{\perp} F_{\mathrm{NG}}-r_{\perp} F_{\mathrm{f}}+r_{\perp} F_{\mathrm{gl}}+r_{\perp} F_{\mathrm{FF}}}{r_{\perp}} \\
& F_{\mathrm{Nh}}=\frac{-(0 \mathrm{~m})\left(F_{\mathrm{NG}}\right)-(0 \mathrm{~m})\left(F_{\mathrm{f}}\right)+\left(2.0 \mathrm{~m} \cos 58^{\circ}\right)(35 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)+\left(2.66 \mathrm{~m} \cos 58^{\circ}\right)(95 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}{4.0 \mathrm{~m} \cos 58^{\circ}} \\
& F_{\mathrm{Nh}}=\frac{363.896 \mathrm{~N} \cdot \text { har }+1313.66 \mathrm{~N} \cdot \text { mI }}{2.1196 \mathrm{mI}} \\
& F_{\mathrm{Nh}}=791.48 \mathrm{~N}
\end{aligned}
$$

Set the sum of the forces in $\quad \sum F_{\mathrm{x}}=0$
the $x$ direction equal to zero.

$$
F_{\mathrm{f}}+F_{\mathrm{Nh}}=0
$$

Note that, in the last calculation, $F_{\mathrm{Nh}}$ was positive because it would cause a positive torque. However, the force
$F_{\mathrm{f}}=-F_{\mathrm{Nh}}$
$F_{\mathrm{f}}=-(-791.48 \mathrm{~N})$
$F_{\mathrm{f}}=791.48 \mathrm{~N}$ itself points in the negative direction so it must be considered negative.

Use the forces calculated

$$
F_{\mathrm{f}}=\mu F_{\mathrm{N}}
$$

above and the equation for sliding friction to find the frictional coefficient.

$$
\begin{aligned}
\mu & =\frac{F_{\mathrm{f}}}{F_{\mathrm{N}}} \\
\mu & =\frac{791.48 \mathrm{~A}}{1275.3 \mathrm{~A}} \\
\mu & =0.6206 \\
\mu & \cong 0.62
\end{aligned}
$$

The coefficient of static friction between the base of the ladder and the ground must be at least 0.62

## Validate

All of the units cancelled making the coefficient of friction unitless which is correct. The value is in the range in which you would expect a coefficient of friction to be.
2. An 18 kg sign hangs symmetrically from a 7.8 kg uniform rod that is 1.3 m long and is attached to a wall. A cable that makes a $35^{\circ}$ angle with the rod helps to support the rod at its end as shown.
(a) What is the tension in the cable?
(b) What force is the wall exerting on the rod?


## Frame the Problem

- Make a labelled sketch of the rod with the forces acting on it and a sketch of the rod with lengths, angles, lever arms, and pivot point labelled.
- You cannot assume that the force the wall exerts on the rod is perpendicular to the wall. It could have an $x$ - and a $y$-component.

- Assign symbols to all of the forces.
$\vec{F}_{\text {Tc }}$ - tension in cable
$\vec{F}_{\mathrm{gr}}$ - weight of rod
$\vec{F}_{\text {gs }}$ - weight of sign
$\vec{F}_{\text {wx }}$ - x-component of force of wall on rod
$\vec{F}_{\text {wy }}$ - $y$-component of force of wall on rod
- Since the rod is uniform, you can assume that the centre of mass
 is at the centre of the rod. Similarly, since the sign in hung symmetrically, you can assume that the weight of the sign acts at the centre of the sign and thus the centre of the rod.
- Choose the point of contact between the wall and the rod as the pivot point to eliminate two unknowns ( $F_{\mathrm{wx}}$ and $F_{\mathrm{wy}}$ ) from the equation for rotational equilibrium.
- Use the equation for rotational equilibrium to find the tension in the cable.
- Use the equations for translational equilibrium in the $x$ and $y$ directions to find the $x$ - and a $y$-components of the force of the wall on the rod.


## Identify the Goal

(a) the tension, $\left|\vec{F}_{\text {Tc }}\right|$, in the cable
(b) the force, $\vec{F}_{\mathrm{w}}$, that the wall exerts on the rod

## Variables and Constants

## Known

$$
\begin{array}{ll}
m_{\mathrm{s}}=18 \mathrm{~kg} & \mathrm{l}_{\mathrm{r}}=1.3 \mathrm{~m} \\
m_{\mathrm{r}}=7.8 \mathrm{~kg} & \theta=35^{\circ}
\end{array}
$$

## Implied <br> $g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$

Unknown
$\vec{F}_{\mathrm{Tc}} \quad \vec{F}_{\mathrm{gs}}$
$\vec{F}_{\mathrm{gr}} \quad \vec{F}_{\mathrm{w}}$

## Strategy

Apply the equation for rotational equilibrium around the pivot point at the point where the rod contacts the wall.

## Calculations

$\sum \tau=0$
$\tau_{\mathrm{wx}}+\tau_{\mathrm{wy}}+\tau_{\mathrm{r}}+\tau_{\mathrm{s}}+\tau_{\mathrm{c}}=0$
$r_{\perp} F_{\mathrm{wx}}+r_{\perp} F_{\mathrm{wy}}-r_{\perp} F_{\mathrm{r}}-r_{\perp} F_{\mathrm{s}}+r_{\perp} F_{\mathrm{c}}=0$
$F_{\mathrm{C}}=\frac{-r_{\perp} F_{\mathrm{wx}}-r_{\perp} F_{\mathrm{wy}}+r_{\perp} F_{\mathrm{r}}+r_{\perp} F_{\mathrm{s}}}{r_{\perp \mathrm{c}}}$
$F_{\mathrm{C}}=\frac{-(0 \mathrm{~m})\left(F_{\mathrm{wx}}\right)-(0 \mathrm{~m})\left(F_{\mathrm{wy}}\right)+(0.65 \mathrm{~m})(7.8 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)+(0.65 \mathrm{~m})(18 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}{(1.3 \mathrm{~m}) \sin 35^{\circ}}$
$F_{\mathrm{C}}=\frac{49.7367 \mathrm{~N} \cdot \text { br }+114.777 \mathrm{~N} \cdot \text { pr }}{0.745649 \mathrm{mr}}$
$F_{\mathrm{C}}=\frac{164.5137 \mathrm{~N}}{0.745649}$
$F_{\mathrm{C}}=220.631 \mathrm{~N}$
$F_{\mathrm{C}} \cong 2.2 \times 10^{2} \mathrm{~N}$
(a) The tension in the cable is $2.2 \times 10^{2} \mathrm{~N}$.

Apply the equations for translational equilibrium

$$
\begin{aligned}
& \sum_{\mathrm{x}}=0 \\
& F_{\mathrm{wx}}+F_{\mathrm{Tx}}=0 \\
& F_{\mathrm{wx}}=-F_{\mathrm{Tx}} \\
& F_{\mathrm{wx}}=-(-220.631 \mathrm{~N}) \cos 35^{\circ} \\
& F_{\mathrm{wx}}=180.73 \mathrm{~N} \\
& \sum_{\mathrm{y}}=0 \\
& F_{\mathrm{wy}}+F_{\mathrm{gr}}+F_{\mathrm{gs}}+F_{\mathrm{Ty}}=0 \\
& F_{\mathrm{wy}}=-F_{\mathrm{gr}}-F_{\mathrm{gs}}-F_{\mathrm{Ty}} \\
& F_{\mathrm{wy}}=-(7.8 \mathrm{~kg})\left(-9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)-(18 \mathrm{~kg})\left(-9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)-(220.631 \mathrm{~N}) \sin 35^{\circ} \\
& F_{\mathrm{wy}}=76.518 \mathrm{~N}+176.58 \mathrm{~N}-126.5487 \mathrm{~N} \\
& F_{\mathrm{wy}}=126.549 \mathrm{~N}
\end{aligned}
$$

Use the Pythagorean theorem

$$
\begin{aligned}
&\left|\stackrel{\rightharpoonup}{F}_{\mathrm{w}}\right|^{2}=F_{\mathrm{wx}}^{2}+F_{\mathrm{wy}}^{2} \\
&\left|\stackrel{\rightharpoonup}{F}_{\mathrm{w}}\right|^{2}=(180.73 \mathrm{~N})^{2}+(126.549 \mathrm{~N})^{2} \\
&\left|\stackrel{\rightharpoonup}{F}_{\mathrm{w}}\right|^{2}=48678.1 \mathrm{~N}^{2} \\
&\left|\vec{F}_{\mathrm{w}}\right|=220.63 \mathrm{~N} \\
&\left|\stackrel{\rightharpoonup}{F}_{\mathrm{w}}\right| \cong 2.2 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

Use the tangent function to find the angle that the force of the wall on the rod makes with the rod.
$\tan \theta=\frac{F_{\mathrm{wy}}}{F_{\mathrm{wx}}}$
$\theta=\tan ^{-1} \frac{F_{\mathrm{wy}}}{F_{\mathrm{wx}}}$
$\theta=\tan ^{-1} \frac{126.549 \mathrm{~A}}{180.73 \mathrm{~N}}$
$\theta=\tan ^{-1} 0.70021$
$\theta=35^{\circ}$
(b) The force of the wall on the rod is $2.2 \times 10^{2} \mathrm{~N}$ at an angle of $35^{\circ}$ with the horizontal rod.

## Validate

All of the units cancel properly to give newtons for force and unitless for the tangent ratio. The values are reasonable.

## PRACTICE PROBLEMS

31. An Olympic diver with a mass of 54 kg stands at the end of a uniform, 3.8 m diving board with a mass of 25 kg . The fulcrum supporting the diving board is 1.3 m from the bolted end. What force must the bolt at the end exert on the diving board to hold the board in place?

32. You are holding a 16 kg steel ball in your hand while keeping your forearm level. Your forearm has a mass of 2.3 kg with a centre of mass as shown in the diagram. Given the dimensions in the diagram, what force would your biceps muscle have to exert on the bone of your forearm to hold your forearm in place?

33. A 24 kg light fixture is hanging from a uniform, 3.5 kg horizontal beam that is 1.6 m long. A supporting cable makes an angle of $33^{\circ}$ with the beam.

(a) Find the tension in the cable.
(b) Find the force exerted on the beam by the pole.
34. You have a summer job as a house painter. You are standing three fourths of the way up a uniform, 2.6 m ladder that has a mass of 7.5 kg . The ladder makes an angle of $73^{\circ}$ with the ground. Assume that your mass is 55 kg . What must be the force of friction of the ground on the base of the ladder to prevent it from slipping? Assume that the friction between the top of the ladder and the house is negligible.


Use the concepts you have learned about static equilibrium to design and build a decorative mobile. For example, a mobile for an infants room might be made with tiny stuffed toys. You might choose to use foil figures such as angels or the moon and stars as shown in the illustration. Use your imagination to choose the items that will be hanging freely.


Use at least four dowels for the horizontal rods and light string from which to suspend the figures. Determine the masses of your figures and dowels. Design the mobile on paper. Calculate the positions where you will have to attach the supporting string to each dowel so that it will hang horizontally in equilibrium. Build and test your mobile. If it is not in equilibrium, make adjustments in the positions of the strings until it hangs properly.

## Analyze and Conclude

1. How well did your mobile balance when you assembled it from your design and calculations?
2. If you had to make significant adjustments to create equilibrium, describe possible reasons for errors.
3. Describe at least two types of structures for which proper calculations of static equilibrium would be critical for the function of the structure.

### 10.3 Section Review

1. C Explain how the vector sum of all of the forces acting on an object can be zero and yet the motion of the object can change.
2. © Describe the two different types of motion that can occur at the same time but you can analyze separately.
3. ©OC Describe three actions that you carry out every day that involve torque.
4. K/U How do you determine the sign (positive or negative) of torque?
5. K/U Define "lever arm."
6. K/U When is the unit "newton-metre" not equivalent to a joule?
7. © How can an object exert a torque on itself?
8. K/U What are the conditions for static equilibrium?
