## Multiple Masses

## SECTION

## OUTCOMES

- Use vector analysis in two dimensions for systems involving two or more masses.
- Analyze systems of two or more masses including inclined planes.
- Use Newton's laws of motion and the concepts of frictional forces and normal forces.


## K E Y

TERMS

- tension
- counterweight
- system
- internal forces
- external forces

Until now, you have been considering only one object and the forces acting directly on that object alone. In everyday life, most forces are exerted by machines and most machines have many moving parts. Also, machines, such as the crane in Figure 10.9 use cables and pulleys to manipulate large loads. In this section, you will learn about tension in cables and how to analyze forces that affect multiple masses.

## Tension in Ropes and Cables

When a crane exerts a force on one end of a cable, each particle in the cable exerts an equal force on the next particle in the cable, creating tension throughout the cable. Every segment in a cable is pulling on the adjacent segment with a force equal in magnitude and opposite in direction to the force with which the adjacent segment is pulling on it. The cable then exerts a force on its load. Tension is the magnitude of the force exerted on and by a cable, rope, or string. How do engineers determine the amount of tension that these cables must be able to withstand? They apply Newton's laws of motion.

To avoid using complex mathematical analyses, you need to make several assumptions about cables and ropes that support loads. Your results will be quite close to the values calculated by computers that are programmed to take into account all of the non-ideal conditions. The simplifying assumptions are as follows.

- The mass of the rope or cable is so much smaller than the mass of the load that it does not significantly affect the motion or forces involved.
- The tension is the same at every point in the rope or cable.
- If a rope or cable passes over a pulley, the direction of the tension forces changes, but the magnitude remains the same. This statement is the same as saying that the pulley is frictionless and its mass is negligible.

Figure 10.9 Mobile construction cranes can withstand the tension necessary to lift loads of up to 1000 t .

## MODEL PROBLEM

## Tension in a Cable

An elevator filled with people has a total mass of 2245 kg . As the elevator begins to rise, the acceleration is $0.55 \mathrm{~m} / \mathrm{s}^{2}$. What is the tension in the cable that is lifting the elevator?

## Frame the Problem

- Draw a free-body diagram.
- The tension in the cable has the same magnitude as the force it exerts on the elevator.
- Two forces are acting on the elevator: the cable ( $\vec{F}_{\mathrm{T}}$ ) and gravity ( $\vec{F}_{\mathrm{g}}$ ).
- The elevator is rising and speeding up, so the acceleration is upward.
- Newton's second law applies to the problem.
- The motion is in one dimension, so let positive and negative signs indicate direction. Let "up" be positive and "down" be negative.


## Identify the Goal

The tension, $\left|\vec{F}_{\mathrm{T}}\right|$, in the rope

## Variables and Constants

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=2245 \mathrm{~kg}$ | $g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ | $\vec{F}_{\mathrm{T}}$ |
| $\vec{a}=0.55 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}[$ up $]$ |  | $\vec{F}_{\mathrm{g}}$ |

## Strategy

Apply Newton's second law and insert all of the forces acting on the elevator. Then solve for the tension.

Calculations

$$
\begin{aligned}
& \vec{F}=m \vec{a} \\
& \vec{F}_{\mathrm{T}}+\vec{F}_{\mathrm{g}}=m \vec{a} \\
& \vec{F}_{\mathrm{T}}=-\vec{F}_{\mathrm{g}}+m \vec{a} \\
& \vec{F}_{\mathrm{T}}=-(-m g)+m \vec{a}
\end{aligned}
$$

Substitute values and solve.

$$
\begin{aligned}
& \vec{F}_{\mathrm{T}}=(2245 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)+(2245 \mathrm{~kg})\left(0.55 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \\
& \vec{F}_{\mathrm{T}}=23258.2 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}} \\
& \vec{F}_{\mathrm{T}} \cong 2.3 \times 10^{4} \mathrm{~N}[\mathrm{up}]
\end{aligned}
$$

The tension in the cable is $2.3 \times 10^{4} \mathrm{~N}$.

## Validate

The weight of the elevator is $(2245 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \cong 2.2 \times 10^{4} \mathrm{~N}$. The tension in the cable must support the weight of the elevator and exert an additional force to accelerate the elevator. Therefore, you would expect the tension to be a little larger than the weight of the elevator, which it is.

## PRACTICE PROBLEMS

14. A 32 kg child is practising climbing skills on a climbing wall, while being belayed (secured at the end of a rope) by a parent. The child loses her grip and dangles from the belay rope. When the parent starts lowering the child, the tension in the rope is 253 N . Find the acceleration of the child when she is first being lowered.
15. A 92 kg mountain climber rappels down a rope, applying friction with a figure eight (a piece of climbing equipment) to reduce his downward acceleration. The rope, which is damaged, can withstand a tension of only 675 N. Can the climber limit his descent to a constant speed without breaking the rope? If not, to what value can he limit his downward acceleration?
16. A 10.0 kg mass is hooked on a spring scale fastened to a hoist rope. As the hoist starts moving the mass, the scale momentarily reads 87 N. Find
(a) the direction of motion
(b) the acceleration of the mass
(c) the tension in the hoist rope
17. Pulling on the strap of a 15 kg backpack, a student accelerates it upward at $1.3 \mathrm{~m} / \mathrm{s}^{2}$. How hard is the student pulling on the strap?
18. A 485 kg elevator is rated to hold 15 people of average mass ( 75 kg ). The elevator cable can withstand a maximum tension of $3.74 \times 10^{4} \mathrm{~N}$, which is twice the maximum force that the load will create (a $200 \%$ safety factor). What is the greatest acceleration that the elevator can have with the maximum load?

## Connected Objects

Imagine how much energy it would require to lift an elevator carrying 20 people to the main deck of the CN Tower in Toronto, 346 m high. A rough calculation using the equation for gravitational potential energy ( $E_{g}=m g \Delta h$ ), which you learned in Chapter 7, would yield a value of about 10 million joules of energy. Is there a way to avoid using so much energy?

Elevators are not usually simply suspended from cables. Instead, the supporting cable passes up over a pulley and then back down to a heavy, movable counterweight, as shown in Figure 10.10. Gravitational forces acting downward on the counterweight create tension in the cable. The cable then exerts an upward force on the elevator cage. Most of the weight of the elevator and passengers is balanced by the counterweight. Only relatively small additional forces from the elevator motors are needed to raise and lower the
elevator and its counterweight. Although the elevator and counterweight move in different directions, they are connected by a cable, so they accelerate at the same rate.


Figure 10.10 Most elevators are connected by a cable to a counterweight that moves in the opposite direction to the elevator. A typical counterweight has a mass that is the same as the mass of the empty elevator plus about half the mass of a full load of passengers.

Elevators are only one of many examples of machines that have large masses connected by a cable that runs over a pulley. In fact, in 1784, mathematician George Atwood (1745-1807) built a machine similar to the simplified illustration in Figure 10.11. He used his machine to test and demonstrate the laws of uniformly accelerated motion and to determine the value of $g$, the acceleration due to gravity. The acceleration of Atwood's machine depended on $g$, but was small enough to measure accurately. In the following investigation, you will use an Atwood machine to measure $g$.


Figure 10.11 An Atwood machine uses a counterweight to reduce acceleration due to gravity.

George Atwood designed his machine to demonstrate the laws of motion. In this investigation, you will demonstrate those laws and determine the value of $g$.

## Problem

How can you determine the value of $g$, the acceleration due to gravity, by using an Atwood machine?

## Prediction

- Predict how changes in the difference between the two masses will affect the acceleration of the Atwood machine if the sum of the masses is held constant.
- When the difference between the two masses in an Atwood machine is held constant, predict how increasing the total mass (sum of the two masses) will affect their acceleration.


## Equipment

- retort stand
- clamps
- masses: $100 \mathrm{~g}(2), 20 \mathrm{~g}(1), 10 \mathrm{~g}(10)$, or similar identical masses, such as 1 inch plate washers
- 2 plastic cups to hold masses
- light string

Traditional instrumentation
lab pulley
lab timer
metre stick

## Probeware

Smart Pulley® or photogates or ultrasonic range finder
motion analysis software
computer


## Procedure

## Constant Mass Difference

1. Set up a data table to record $m_{1}, m_{2}$, total mass, $\Delta d$ and $\Delta t$ (if you use traditional equipment), and $a$.
2. Set up an Atwood machine at the edge of a table, so that $m_{1}=120 \mathrm{~g}$ and $m_{2}=100 \mathrm{~g}$.
3. Lift the heavier mass as close as possible to the pulley. Release the mass and make the measurements necessary for finding its downward acceleration. Catch the mass before it hits the floor.

- Using traditional equipment, find displacement $(\Delta d)$ and the time interval $(\Delta t)$ while the mass descends smoothly.
- Using probeware, measure velocity ( $v$ ) and graph velocity versus time. Find acceleration from the slope of the line during an interval when velocity was increasing steadily.

4. Increase each mass by 10 g and repeat the observations. Continue increasing mass and finding acceleration until you have a total of five mass-acceleration data pairs.
5. Graph acceleration versus total mass. Draw a best-fit line through your data points.

## Constant Total Mass

6. Set up a data table to record $m_{1}, m_{2}$, mass difference ( $\Delta m$ ), $\Delta d$ and $\Delta t$ (if you use traditional equipment), and $a$.
7. Make $m_{1}=150 \mathrm{~g}$ and $m_{2}=160 \mathrm{~g}$. Make observations to find the downward acceleration, using the same method as in step 3.
8. Transfer one 10 g mass from $m_{1}$ to $m_{2}$. The mass difference will now be 30 g , but the total mass will not have changed. Repeat your measurements.
9. Repeat step 8 until you have data for five mass difference-acceleration pairs.
10. Graph acceleration versus mass difference. Draw a best-fit line or curve through your data points.

## Analyze and Conclude

1. Based on your graphs for step 5 , what type of relationship exists between total mass and acceleration in an Atwood machine? Use appropriate curve-straightening techniques to support your answer (see Skill Set 4, Mathematical Modelling and Curve Straightening). Write the relationship symbolically.
2. Based on your graphs for step 10, what type of relationship exists between mass difference and acceleration in an Atwood machine? Write the relationship symbolically.
3. How well do your results support your prediction?
4. String that is equal in length to the string connecting the masses over the pulley is sometimes tied to the bottoms of the two masses, where it hangs suspended between them. Explain why this would reduce
experimental errors. Hint: Consider the mass of the string as the apparatus moves and how that affects $m_{1}$ and $m_{2}$.
5. Mathematical analysis shows that the acceleration of an ideal (frictionless) Atwood machine is given by $a=g \frac{m_{1}-m_{2}}{m_{1}+m_{2}}$. Use this relationship and your experimental results to find an experimental result for $g$.
6. Calculate experimental error in your value of $g$. Suggest the most likely causes of experimental error in your apparatus and procedure.

## Apply and Extend

7. Start with Newton's second law in the form $\vec{a}=\frac{\vec{F}}{m}$ and derive the equation for $a$ in question 5 above. Hint: Write $\vec{F}$ and $m$ in terms of the forces and masses in the Atwood machine.
8. Using the formula $a=g \frac{m_{1}-m_{2}}{m_{1}+m_{2}}$ for an Atwood machine, find the acceleration when $m_{1}=2 m_{2}$.
9. Under what circumstances would the acceleration of the Atwood machine be zero?
10. What combination of masses would make the acceleration of an Atwood machine equal to $\frac{1}{2} g$ ?

## Web Link

www.mcgrawhill.ca/links/atlphysics
For some interactive activities involving the Atwood machine, go to the above Internet site and click on Web Links.

## Assigning Direction to the Motion of Connected Objects

When two objects are connected by a flexible cable or rope that runs over a pulley, such as the masses in an Atwood machine, they are moving in different directions. However, connected objects move as a unit. For some calculations, you need to work with the forces acting on the combined objects and the acceleration of the combined objects. How can you treat the pair of objects as a unit when two objects are moving in different directions?

Since the connecting cable or rope changes only the direction of the forces acting on the objects and has no effect on the magnitude of the forces, you can assign the direction of the motion as being from one end of the cable or rope to the other. You can call one end "negative" and the other end "positive," as shown in Figure 10.12.

After you have assigned the direction to two or more objects, you can work with all of them as a unit or with each one alone. When you choose to work with one or a group of objects as a unit, the selected objects are called a system. The forces exerted through a rope or cable, between any two objects within the system, are called internal forces because they represent one part of the system acting on another part of the system. Internal forces do not affect the motion of the system as a whole. When you apply Newton's laws to a system of two or more objects, internal forces do not contribute to the net force on the system. Only external forces, such as gravity or friction, affect the motion of the system. When you apply Newton's laws to one mass alone, it becomes your system, and the force exerted by the rope on that object is now an external force and does affect the motion of the system. The following model problem will show you how to choose systems and apply Newton's laws after you have defined a system.


## MODEL PROBLEM

## Motion of Connected Objects

An Atwood machine is made of two objects connected by a rope that runs over a pulley. The object on the left ( $m_{1}$ ) has a mass of 8.5 kg and the object on the right ( $m_{2}$ ) has a mass of $17 \mathbf{k g}$.
(a) What is the acceleration of the masses?


## Frame the Problem

- Draw free-body diagrams. Draw one diagram of the system moving as a system and diagrams of each of the two objects as individual systems.
- Let the negative direction point from the centre to the 8.5 kg mass and the positive direction point from the centre to the 17 kg mass.
- Both objects move with the same acceleration.
- The force of gravity acts on both objects.
- The tension is constant throughout the rope.
- The rope exerts a force of equal magnitude and opposite direction on each object.
- When you isolate the individual objects, the tension in the rope is an external force acting on the object.
- Newton's second law applies to the combination of the two objects and to each individual object.


## Identify the Goal

(a) The acceleration, $\vec{a}$, of the two objects
(b) The tension, $\left|\vec{F}_{\mathrm{T}}\right|$, in the rope

## Variables and Constants

## Known

$m_{1}=8.5 \mathrm{~kg}$
$m_{2}=17 \mathrm{~kg}$

Implied
$g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$

Unknown
$\vec{F}_{\mathrm{g} 1} \quad \vec{F}_{\mathrm{T}}$
$\vec{F}_{\mathrm{g} 2}$

## Strategy

Apply Newton's second law to the system of two masses to find the acceleration.
The mass of the combination is the sum of the individual masses.

## Calculations

$$
\begin{aligned}
\vec{F} & =m \vec{a} \\
\vec{F}_{\mathrm{g} 1}+\vec{F}_{\mathrm{g} 2} & =\left(m_{1}+m_{2}\right) \vec{a} \\
-m_{1} g+m_{2} g & =\left(m_{1}+m_{2}\right) \vec{a} \\
\vec{a} & =\frac{\left(m_{2}-m_{1}\right) g}{m_{1}+m_{2}} \\
\vec{a} & =\frac{(17 \mathrm{~kg}-8.5 \mathrm{~kg}) 9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}{8.5 \mathrm{~kg}+17 \mathrm{~kg}} \\
\vec{a} & =3.27 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \\
\vec{a} & \cong 3.3 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}[\text { to the right }]
\end{aligned}
$$

(a) The acceleration of the combination of objects is $3.3 \mathrm{~m} / \mathrm{s}^{2}$ to the right.

Apply Newton's second law to $m_{1}$ and solve for tension.

$$
\begin{aligned}
\vec{F} & =m_{\vec{a}} \\
\vec{F}_{\mathrm{g} 1}+\vec{F}_{\mathrm{T}} & =m_{1} \vec{a} \\
-m_{1} g+\vec{F}_{\mathrm{T}} & =m_{1} \vec{a} \\
\vec{F}_{\mathrm{T}} & =m_{1} g+m_{1} \vec{a} \\
\vec{F}_{\mathrm{T}} & =(8.5 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)+(8.5 \mathrm{~kg})\left(3.27 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \\
\vec{F}_{\mathrm{T}} & =111.18 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}} \\
\vec{F}_{\mathrm{T}} & \cong 1.1 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

(b) The tension in the rope is $1.1 \times 10^{2} \mathrm{~N}$.

## Validate

You can test your solution by applying Newton's second law to the second mass.

$$
\begin{aligned}
\vec{F}_{\mathrm{g} 2}+\vec{F}_{\mathrm{T}} & =m_{2} \vec{a} \\
m_{2} g+\vec{F}_{\mathrm{T}} & =m_{2} \vec{a} \\
\vec{F}_{\mathrm{T}} & =m_{2} \vec{a}-m_{2} g \\
\vec{F}_{\mathrm{T}} & =(17 \mathrm{~kg})\left(3.27 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)-(17 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \\
\vec{F}_{\mathrm{T}} & =-111.18 \mathrm{~N} \\
\vec{F}_{\mathrm{T}} & =-1.1 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

The magnitudes of the tensions calculated from the two masses independently agree. Also, notice that the application of Newton's second law correctly gave the direction of the force on the second mass.
19. An Atwood machine consists of masses of 3.8 kg and 4.2 kg . What is the acceleration of the masses? What is the tension in the rope?
20. The smaller mass on an Atwood machine is 5.2 kg . If the masses accelerate at $4.6 \mathrm{~m} / \mathrm{s}^{2}$, what is the mass of the second object? What is the tension in the rope?
21. The smaller mass on an Atwood machine is 45 kg . If the tension in the rope is 512 N , what is the mass of the second object? What is the acceleration of the objects?
22. A 3.0 kg counterweight is connected to a 4.5 kg window that freely slides vertically in its frame. How much force must you exert to start the window opening with an acceleration of $0.25 \mathrm{~m} / \mathrm{s}^{2}$ ?
23. Two gymnasts of identical 37 kg mass dangle from opposite sides of a rope that passes over a frictionless, weightless pulley. If one of the gymnasts starts to pull herself up the rope with an acceleration of $1.0 \mathrm{~m} / \mathrm{s}^{2}$, what happens to her? What happens to the other gymnast?

## Objects Connected at an Angle

In the lab, a falling weight is often used to provide a constant force to accelerate dynamics carts. Gravitational forces acting downward on the weight create tension in the connecting string. The pulley changes the direction of the forces, so the string exerts a horizontal force on the cart. Both masses experience the same acceleration because they are connected, but the cart and weight move at right angles to each other.

You can approach problems with connected objects such as the lab cart and weight in the same way that you solved problems involving the Atwood machine. A mass can even be on an inclined plane. Even if a block is sliding, with friction, over a surface, the mathematical treatment is much the same. Study Figure 10.13 and follow the directions below to learn how to treat connected objects that are moving both horizontally and vertically.

- Analyze the forces on each individual object, then label the diagram with the forces.
- Assign a direction to the motion.
- Draw the connecting string or rope as though it was a straight line. Be sure that the force vectors are in the same direction relative to each mass.
- Draw a free-body diagram of the combination and of each individual mass.
- Apply Newton's second law to each free-body diagram.

Figure 10.13 When you visualize the string "straightened," the force of gravity appears to pull down on mass 1, but to the side on mass 2. Although it might look strange, be assured that these directions are correct regarding the way in which the forces affect the motion of the objects.


## MODEL PROBLEM

## Connected Objects

A block ( $m_{1}$ ) with a mass of 615 g is resting on a plane inclined at an angle of $33^{\circ}$. The coefficient of kinetic friction between the block and the plane is 0.19 . The block is attached to a string that runs over a pulley. A freely hanging object ( $\mathrm{m}_{2}$ ) with a mass of 525 g is attached to the other end of the string as shown in the diagram.

(a) After the masses have started moving, what is their acceleration?
(b) What is the tension in the string?

## Frame the Problem

- Sketch the apparatus in its correct configuration with forces added. Assign positive and negative directions.
- Then sketch the apparatus with the string in a straight line. Be sure that the forces are all at the same angles relative to the masses as in the first sketch.
- Make free body diagrams of the two masses as a system and of each mass as its own individual system.

- Only the block $\left(m_{1}\right)$ is experiencing friction with the plane.
- Both a component of the gravitational force and a frictional force on the block are creating a force in the negative direction.
- The only force in the positive direction is the gravitational force on $m_{2}$.
- Treat the two masses as a system to find their acceleration.
- To find the tension in the string, select only one mass as a system. Choosing the mass that is subject to the fewest forces makes the calculations simpler.
- Apply Newton's second law to the system of two masses and then to $m_{2}$ as a system.


## Identify the Goal

(a) acceleration, $\vec{a}$, of the two masses
(b) the tension, $\left|\stackrel{\rightharpoonup}{F}_{\mathrm{T}}\right|$, in the string

## Variables and Constants

## Known

$$
\begin{array}{ll}
m_{1}=615 \mathrm{~g} & \mu=0.19 \\
m_{2}=525 \mathrm{~g} & \theta=33^{\circ}
\end{array}
$$

## Implied

$g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$

## Unknown

$\stackrel{\rightharpoonup}{a}$
$\left|\vec{F}_{\mathrm{T}}\right|$

## Strategy

- To find the frictional force, you need the normal force. To find the normal force, apply Newton's second law to the direction perpendicular to the inclined plane. Since there is no motion perpendicular to the plane, the acceleration is zero.


## Calculations

$$
\begin{aligned}
& F_{\perp}=m a_{\perp} \\
& F_{\mathrm{N}}+F_{\mathrm{g} 1 \perp}=m a_{\perp} \\
& F_{\mathrm{N}}=-F_{\mathrm{g} 1 \perp}+m a_{\perp} \\
& F_{\mathrm{N}}=-\left(-m_{1} g \cos \theta\right)+0 \\
& F_{\mathrm{N}}=(0.615 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \cos 33^{\circ} \\
& F_{\mathrm{N}}=5.0598 \mathrm{~N}
\end{aligned}
$$

## Strategy

- Since the two masses have the same acceleration, define the system as the combination of the masses. Find the acceleration from Newton's second law in the direction of the string when it is drawn horizontal. Call it the $x$ direction.

Calculations
$F_{\mathrm{x}}=m a_{\mathrm{x}}$
$F_{\mathrm{f}}+F_{\mathrm{g} 1 \mathrm{x}}+F_{\mathrm{g} 2 \mathrm{x}}=m a_{\mathrm{x}}$
$-\mu F_{\mathrm{N}}-m_{1} g \sin \theta+m_{2} g=\left(m_{1}+m_{2}\right) a_{\mathrm{x}}$
$a_{\mathrm{x}}=\frac{-\mu F_{\mathrm{N}}-m_{1} g \sin \theta+m_{2} g}{m_{1}+m_{2}}$
$a_{\mathrm{x}}=\frac{-(0.19)(5.0598 \mathrm{~N})-(0.615 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \sin 33^{\circ}+(0.525 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}{(0.615 \mathrm{~kg}+0.525 \mathrm{~kg})}$
$a_{\mathrm{x}}=\frac{-0.96136 \mathrm{~N}-3.2859 \mathrm{~N}+5.15025 \mathrm{~N}}{1.14 \mathrm{~kg}}$
$a_{\mathrm{x}}=\frac{0.90299 \mathrm{~N}}{1.14 \mathrm{~kg}}$
$a_{\mathrm{x}}=0.7921 \frac{\frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}}{\mathrm{~kg}}$
$a_{\mathrm{x}} \cong 0.79 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
(a) The acceleration of the two masses is $0.79 \mathrm{~m} / \mathrm{s}^{2}$.

- To find the tension in the string, the tension must be an external force. Choose $m_{2}$ as the system because fewer forces are acting on it. Apply Newton's second law.

$$
\begin{aligned}
& F_{\mathrm{x}}=m a_{\mathrm{x}} \\
& F_{\mathrm{T}}+F_{\mathrm{g} 2}=m_{2} a_{\mathrm{x}} \\
& F_{\mathrm{T}}=-F_{\mathrm{g} 2}+m_{2} a_{\mathrm{x}} \\
& F_{\mathrm{T}}=-m_{2} g+m_{2} a_{\mathrm{x}} \\
& F_{\mathrm{T}}=m_{2}\left(-g+a_{\mathrm{x}}\right) \\
& F_{\mathrm{T}}=0.525 \mathrm{~kg}\left(-9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}+0.7921 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \\
& F_{\mathrm{T}}=-4.734 \mathrm{~N} \\
& F_{\mathrm{T}} \cong-4.7 \mathrm{~N}
\end{aligned}
$$

(b) The tension in the string is 4.7 N .

## Validate

The force on $m_{2}$ is negative which you would expect from the diagrams. If you choose $m_{1}$ as your system and applied Newton's second law, you would find that $F_{\mathrm{T}}=+4.734 \mathrm{~N}$. (Try it.)

## PRACTICE PROBLEMS

24. A Fletcher's trolley apparatus consists of a 1.90 kg cart on a level track attached to a light string passing over a pulley and holding a 0.500 kg mass suspended in the air.
Neglecting friction, calculate
(a) the tension in the string when the suspended mass is released
(b) the acceleration of the trolley
25. A 40.0 g glider on an air track is connected to a 25.0 g mass suspended by a string passing over a frictionless pulley. When the mass is released, how long will it take the glider to travel the 0.85 m to the other end of the track? (Assume the mass does not hit the floor, so there is constant acceleration during the experiment.)
26. The objects in the diagram have the following masses: $m_{1}=228 \mathrm{~g}, m_{2}=615 \mathrm{~g}$, and $m_{3}=455 \mathrm{~g}$. The coefficient of kinetic friction between the block and the table is 0.260 . Mass $m_{3}$ is 1.95 m above the floor. What will be the time interval between the instant that the masses start to move and the instant when mass $m_{3}$ hits the floor?

27. The block in the diagram has a mass of 145 g and the freely hanging object has a mass of 85 g . The coefficient of kinetic friction between the block and the ramp is 0.18 . The ramp makes an angle of $22^{\circ}$ with the horizontal.
(a) What will be the speed of the masses 2.5 s after they just start to move?
(b) What is the tension in the string while they are moving?

28. The block in the diagram has a mass of 725 g , and the hanging object has a mass of 595 g . The coefficient of static friction between the block and the inclined plane is 0.47 , and the coefficient of kinetic friction is 0.12 . The inclined plane makes an angle of $34^{\circ}$ with the horizontal.

(a) What force directed up the incline would you have to apply to the block, to make the objects start to move?
(b) After the objects start to move, what will be their acceleration?
(c) What will be the tension in the string when the objects are moving?

### 10.2 Section Review

1. C How does an Atwood machine make it easier to determine $g$ (the acceleration due to gravity), rather than by just measuring the acceleration of a free-falling object?
2. K/U List the simplifying assumptions usually made about supporting cables and ropes. Why are simplifying assumptions needed?
3. K/U Two objects are moving in different directions. Under what circumstances can you treat this as a one-dimensional problem?
4. ©O By the mid-1800s, steam-driven elevators with counterweights had been developed. However, they were not in common use until 1852, when Elisha Otis invented an elevator with a safety device that prevented the elevator from falling if the cable broke. How do you think that the invention of a safe elevator changed modern society?
