10.1 Using Vector Components to Analyze Motion

SECTION OUTCOMES

- Use vector analysis in two dimensions for systems involving relative motion.
- Add and subtract vectors using the method of components.
- Use free body diagrams to analyze force problems.

KEY TERMS

- resolve (vectors)
- components

Figure 10.1 Rounding a sharp curve, dropping down a steep slope before levelling off makes down-hill skiing exhilarating.

Skiing would not be much fun if all ski runs were straight. You develop skills when you have to judge how fast you can take a curve or come to a quick stop. In fact, nearly every sport involves sudden turns, stops, and starts. Motion is rarely in a straight line. Nevertheless, nearly all of the problem solving skills that you developed in Chapter 3 involved straight line motion. How can you describe and analyze forces and motion that follows curved lines or motion that changes direction abruptly?

Equations of Motion

When you developed and applied the equations of motion listed below, you read that you would be using them almost entirely in one dimension at a time so vector notations were not included in the equations. Before developing new skills that allow you to work in two or three dimensions, review the kinematic equations themselves. Be sure that you know the conditions under which each of the equations applies. Some equations apply only to uniform motion (motion with a constant velocity) and others apply only to uniformly accelerated motion (motion with a constant acceleration).

Uniform motion

- definition of velocity $v = \frac{\Delta d}{\Delta t}$
- displacement in terms of velocity and time $\Delta d = v\Delta t$

Uniformly accelerated motion

- definition of acceleration $a = \frac{\Delta v}{\Delta t}$ or $a = \frac{v_2 - v_1}{\Delta t}$
- final velocity in terms of initial velocity, acceleration, and time interval $v_2 = v_1 + a\Delta t$
- displacement in terms of initial velocity, final velocity, and time interval $\Delta d = \frac{(v_1 + v_2)}{2}\Delta t$
- displacement in terms of initial velocity, acceleration, and time interval $\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$
- final velocity in terms of initial velocity, acceleration, and displacement $v_2^2 = v_1^2 + 2a\Delta d$

PHYSICS FILE

To use the vector form of some of the equations of motion would require mathematical techniques and notations that are beyond the scope of this textbook. Therefore, it is best to apply the equations to one dimension at a time and not to attempt to use vector notations.

Vector Components

In Chapter 3, when you solved vector addition and subtraction problems graphically, you probably noticed that the method is very imprecise. Measurements with rulers and protractors create a large uncertainty. So you will not be surprised to learn that there is a more precise method.

An important clue to the more precise method lies in investigations and problems in which you added vectors that were at an angle of 90°, or a right angle, relative to each other. For example, when you determine the vector sum of a canoe's velocity and the river's velocity (Figure 10.2) you can use the Pythagorean theorem to calculate the sum precisely. You do not have to measure with a ruler. The precise method for adding and subtracting vectors is based on right triangles and the rules of trigonometry.

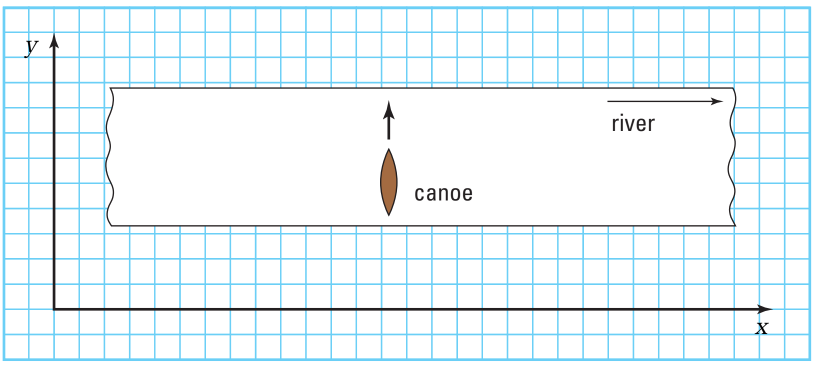


Figure 10.2 Use the Pythagorean theorem to calculate vectors precisely.

ELECTRONIC LEARNING PARTNER

To learn more about vector components, go to your Electronic Learning Partner.

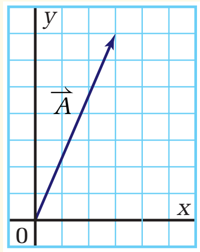
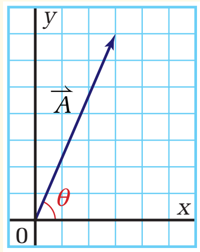
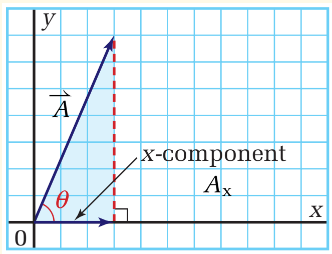
The vectors that you want to add or subtract are rarely at right angles to each other; however, any vector can be separated, or **resolved**, into components that are at right angles to each other. **Components** are parts of a vector that lie on the axes of a coordinate system. Since components are confined to one dimension, they can be added algebraically.

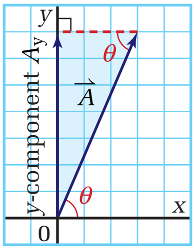
When working with vector components, the *x*–*y*-coordinate system is much more convenient to use than a system based on compass directions. Follow the steps in Table 10.1 to learn how to resolve a vector into components. If you need to review the definitions of the trigonometric functions, sine, cosine, and tangent, turn to Skill Set 5.

PHYSICS FILE

When working in an *x*–*y*-coordinate system, mathematicians and physicists report the direction of a vector by giving the angle that the vector makes with the positive *x*-axis. You find the angle by starting at the positive *x*-axis and rotating counterclockwise until you reach the location of the vector. If a vector has an angle greater than 90°, it lies in a quadrant other than the first. Vectors in the second, third, or fourth quadrants have at least one component that is negative. Figure 10.3 summarizes the signs of the *x*- and *y*-components in the four quadrants. When you use an angle to calculate the magnitude of the components, you would use the angle the vector makes with the nearest *x*-axis. The model problems show you how to find the components of a vector.

Table 10.1 Resolving Vectors into *x*- and *y*-Components

Procedural Step	Graph
Draw the vector with its tail at the origin of the coordinate system.	
Identify the angle that the vector makes with the *x*-axis and label it "θ."	
Draw a vertical line from the tip of the vector to the *x*-axis. The line from the origin to the base of this vertical line is the *x*-component of the vector.	
Write the equation that defines $\cos\theta$.	$\cos\theta = \frac{A_x}{\lvert\vec{A}\rvert}$
Solve for the *x*-component, A_x.	$A_x = \lvert\vec{A}\rvert\cos\theta$

Procedural Step	Graph
Draw a horizontal line from the tip of the vector to the *y*-axis. The line from the origin to the base of this line is the *y*-component.	
Write the equation that defines sin θ.	$\sin\theta = \frac{A_y}{\lvert\vec{A}\rvert}$
Solve for the *y*-component, A_y	$A_y = \lvert\vec{A}\rvert \sin\theta$

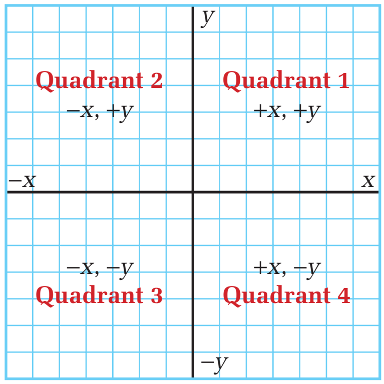


Figure 10.3 The signs of vector components are summarized on this coordinate system.

MODEL PROBLEM

Resolving Vectors

Find the *x*- and *y*-components of vector $\Delta\vec{d}$, which has a magnitude of 64 m at an angle of 120°.

Frame the Problem

- The angle is between 90° and 180°, so it is in the *second quadrant*. Therefore, the *x-component* is *negative* and the *y-component* is *positive*.
- Use *trigonometric functions* to find the components of the vector.

Identify the Goal

The components, Δd_x and Δd_y, of vector $\Delta\vec{d}$

Variables and Constants

Known	Unknown
$\Delta\vec{d} = 64$ m	Δd_x
$\theta = 120°$	Δd_y

PROBLEM TIP

To avoid confusion, always choose to use the angle that the vector makes with the *x*-axis. Regardless of the quadrant in which the vector is located, the *x*-component will always be the cosine of the angle times the magnitude of the vector. The *y*-component will always be the sine of the angle times the magnitude of the vector. Mathematicians call the angle that the vector makes with the closest *x*-axis the "reference angle."

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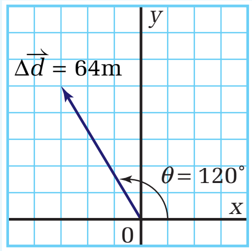
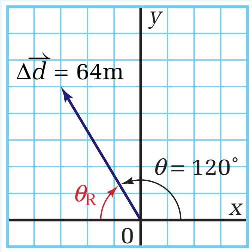
Strategy

Calculations

Draw the vector with its tail at the origin of an x–y-coordinate system.

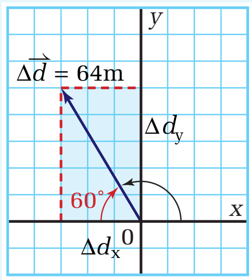


Identify the angle with the closest x-axis. Label it "θ_R" or "reference angle."



$\theta_R = 180° - 120°$
$\theta_R = 60°$

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.



Calculate the components according to the directions in Table 10.1.

$\Delta d_x = \lvert\Delta\vec{d}\rvert \cos\theta$	$\Delta d_y = \lvert\Delta\vec{d}\rvert \sin\theta$
$\Delta d_x = 64\text{ m } \cos 60°$	$\Delta d_y = 64\text{ m } \sin 60°$
$\Delta d_x = 64\text{ m } (0.5000)$	$\Delta d_y = 64\text{ m } (0.8660)$
$\Delta d_x = 32\text{ m}$	$\Delta d_y = 55.4\text{ m}$

Determine signs of the components.

The x-component lies on the negative x-axis so it is negative. The y-component lies on the positive y-axis so it is positive.

The x-component of the vector is –32 m and the y-component is +55 m.

Validate

Use the Pythagorean theorem to check your answers.

$$|\Delta\vec{d}|^2 = \Delta d_x{}^2 + \Delta d_y{}^2$$
$$|\Delta\vec{d}|^2 = (32\text{ m})^2 + (55.4\text{ m})^2$$
$$|\Delta\vec{d}|^2 = 1024\text{ m}^2 + 3069.2\text{ m}^2$$
$$|\Delta\vec{d}|^2 = 4093.2\text{ m}^2$$
$$|\Delta\vec{d}| = 64\text{ m}$$

The value agrees with the original vector.

PRACTICE PROBLEMS

1. Resolve the following vectors into their components.

(a) a position of 16 m at an angle of 75°

(b) an acceleration of 8.1 m/s^2 at an angle of 145°

(c) a velocity of 16.0 m/s at an angle of 225°

2. Resolve the following vectors into their components.

(a) a displacement of 20.0 km[N20.0°E]

(b) a velocity of 3.0 m/s[E30.0°S]

(c) a velocity of 6.8 m/s[W70.0°N]

3. A hot-air balloon has drifted 60.0 km [E60.0°N] from its launch point. It lands in a field beside a road that runs in a north-south direction. The balloonists radio back to their ground crew to come and pick them up. The ground crew can travel only on roads that run north-south or east-west. The roads are laid out in a grid pattern, with intersections every 2.0 km. How far east and then how far north will the pickup van need to travel in order to reach the balloon?

Vector Addition and Subtraction by Using Components

Examine Figure 10.4 to begin to see how resolving vectors into their components will allow you to add or subtract vectors in a precise yet uncomplicated way. In the figure, $\vec{R}$ is the resultant vector for the addition of $\vec{A}$, $\vec{B}$, and $\vec{C}$. You can also see that R_x is equal in length to $A_x + B_x + C_x$. The same is true for the y-components.

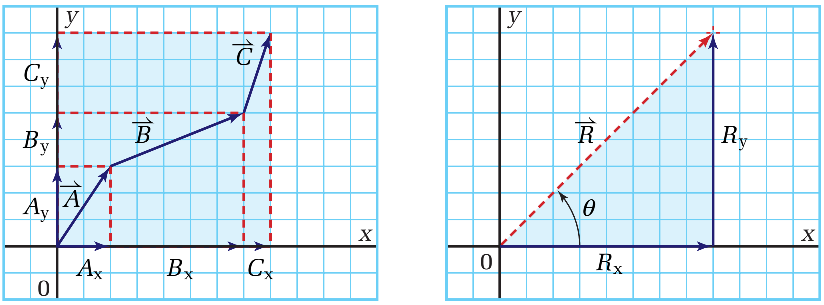


Figure 10.4 The projections of each vector on the x- and y-axes are the components of the vector.

When you want to add or subtract vectors, you can separate the vectors into their components, add or subtract the components, and then find the resultant vector by using the Pythagorean theorem.

$$R_x = A_x + B_x + C_x + \cdots \quad R_y = A_y + B_y + C_y + \cdots \quad |\vec{R}|^2 = R_x{}^2 + R_y{}^2$$

You can find the angle, θ, from the components of the resultant vector, because they make the sides of a right triangle. Find the ratio of R_y to R_x, then use your calculator to find the angle for which the tangent is the ratio.

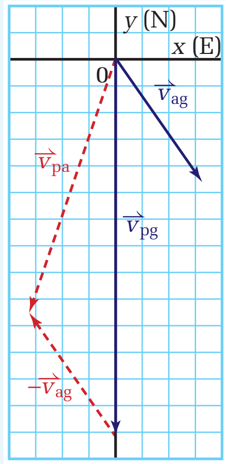
$$\tan \theta = \frac{R_y}{R_x}$$

MODEL PROBLEM

Using Vector Components

You are the pilot of a small plane and want to reach an airport, 5.0×10^2 km due south, in 4.0 h. A wind is blowing at 5.0×10^1 km/h[S35°E]. With what heading and airspeed should you fly to reach the airport on time?

Frame the Problem



- Make a sketch of the problem.
- Your destination is directly *south*.
- A strong wind is blowing *east of south*. The *heading* of the plane will have to account for the wind.
- The *vector sum* of the *velocity* of the plane in relation to the air and the *velocity* of the air in relation to the ground, must be the same as the needed *total velocity* of the plane in relation to the ground.
- You must use *vector addition*.

Identify the Goal

The velocity, $\vec{v}_{pa}$, of the plane relative to the air
(**Note:** A velocity has not been reported until both the magnitude and direction are given.)

Variables and Constants

Known

$\vec{v}_{ag} = 5.0 \times 10^1 \dfrac{\text{km}}{\text{h}}[\text{S}35°\text{E}]$

$\Delta\vec{d}_{pg} = 5.0 \times 10^2\ \text{km[S]}$

$\Delta t = 4.0\ \text{h}$

Unknown

$\vec{v}_{pa}$ $v_{pa\,x}$ $v_{pa\,y}$

θ_{ag} $v_{ag\,x}$ $v_{ag\,y}$

$\vec{v}_{pg}$ $v_{pg\,x}$ $v_{pg\,y}$

θ_{pg}

Strategy

You can find the ground speed and direction that the plane must attain to arrive on time by using the mathematical definition for velocity.

Calculations

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$$

$$\vec{v}_{pg} = \frac{\Delta\vec{d}_{pg}}{\Delta t_{pg}}$$

$$\vec{v}_{pg} = \frac{5.0 \times 10^2\ \text{km[S]}}{4.0\ \text{h}}$$

$$\vec{v}_{pg} = 125\ \frac{\text{km}}{\text{h}}[\text{S}]$$

Since you now know the wind velocity and the necessary velocity of the plane in relation to the ground, you can use the expression for the vector sum of the velocities to find the velocity of the plane in relation to the air. This quantity *is* the heading and airspeed of the plane. First, solve for $\vec{v}_{pa}$.

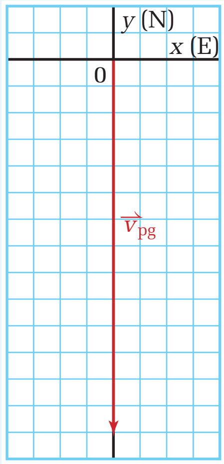
$$\vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$$
$$\vec{v}_{pg} - \vec{v}_{ag} = \vec{v}_{pa} + \vec{v}_{ag} - \vec{v}_{ag}$$
$$\vec{v}_{pg} - \vec{v}_{ag} = \vec{v}_{pa}$$
$$\vec{v}_{pa} = \vec{v}_{pg} - \vec{v}_{ag}$$

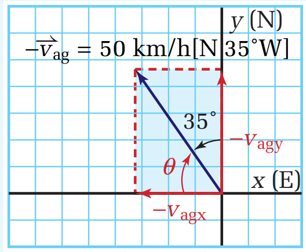
PROBLEM TIP

Recall from Chapter 3 that the negative of a vector has the same magnitude but is opposite in direction to the original vector

Draw the two known vectors on an x–y-coordinate system ($+y$ coincides with north), with their tails at the origin.

Identify the angles they make with the x-axis.





Define and draw the vector $-\vec{v}_{ag}$.

$$\theta_{ag} = 90° - 35° = 55°$$

Find the x- and y-components of the vectors $\vec{v}_{pg}$ *and* $-\vec{v}_{ag}$.

$\vec{v}_{pg}$ is pointed directly south, or along the negative y-axis. Therefore, it has no x-component. Its y-component is the same as the vector.

$$v_{pg\,y} = |\vec{v}_{pg}|$$
$$v_{pg\,y} = -125\ \frac{\text{km}}{\text{h}}$$

$$-v_{ag\ x} = |\vec{v}_{ag}| \cos\theta_{ag}$$
$$-v_{ag\ x} = 50\ \frac{\text{km}}{\text{h}} \cos 55°$$
$$-v_{ag\ x} = 50\ \frac{\text{km}}{\text{h}}(0.5736)$$
$$-v_{ag\ x} = 28.68\ \frac{\text{km}}{\text{h}}$$
$$-v_{ag\ y} = |\vec{v}_{ag}| \sin\theta_{ag}$$
$$-v_{ag\ y} = 50\ \frac{\text{km}}{\text{h}} \sin 55°$$
$$-v_{ag\ y} = 50\ \frac{\text{km}}{\text{h}}(0.8191)$$
$$-v_{ag\ y} = 40.96\ \frac{\text{km}}{\text{h}}$$

Determine the signs of the components.

$\vec{v}_{pg}$ has no x-component and the y-component is negative.

$-\vec{v}_{ag}$ is in the second quadrant, so the x-component is negative and the y-component is positive.

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Strategy

Make a table in which to list the x- and y-components of the vectors.

Add the components of $\vec{v}_{pg}$ and $-\vec{v}_{ag}$ to obtain the components of $\vec{v}_{pa}$.

Calculations

Vector	x-component	y-component
$\vec{v}_{pg}$	$0.0\ \frac{km}{h}$	$-125\ \frac{km}{h}$
$-\vec{v}_{ag}$	$-28.68\ \frac{km}{h}$	$40.96\ \frac{km}{h}$
$\vec{v}_{pa}$	$-28.68\ \frac{km}{h}$	$-84.04\ \frac{km}{h}$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{pa}$.

$$|\vec{v}_{pa}|^2 = (v_{pa\ x})^2 + (v_{pa\ y})^2$$

$$|\vec{v}_{pa}|^2 = \left(-28.68\ \frac{km}{h}\right)^2 + \left(-84.04\ \frac{km}{h}\right)^2$$

$$|\vec{v}_{pa}|^2 = 822.54\left(\frac{km}{h}\right)^2 + 7062.7\left(\frac{km}{h}\right)^2$$

$$|\vec{v}_{pa}|^2 = 7885.3\left(\frac{km}{h}\right)^2$$

$$|\vec{v}_{pa}| = 88.8\ \frac{km}{h}$$

Find the angle the resultant makes with the x-axis.

$$\tan\theta_{pa} = \frac{84.04\ \frac{km}{h}}{28.68\ \frac{km}{h}} = 2.93$$

Since the components are both negative, the vector lies in the third quadrant. However, use positive values to find the reference angle. The result will give the angle from the negative x-axis into the fourth quadrant.

$$\theta_{pa} = \tan^{-1} 2.93$$

$$\theta_{pa} = 71.1^\circ$$

The plane's airspeed must be 89 km/h and it must fly at a heading of [W71°S].

Validate

The wind is blowing toward the southeast and the pilot wants to fly directly south. The component of the wind blowing south will help the plane to get there faster, but the component of the wind blowing east will blow the plane off course if the pilot does not compensate. The pilot must head slightly west to make up for the wind blowing east, so you would expect that the pilot would have to fly slightly west of south. (Note that [W71°S] is the same as [S19°W].) This is in perfect agreement with the calculations.

PRACTICE PROBLEMS

4. A pleasure boat heads out of a marina for sightseeing. It travels 2.7 km due south to a small island. Then it travels 3.4 km[S26°E] to another island. Finally, it turns and heads [E12°N] for 1.9 km to a third island.
   (a) Determine the boat's displacement for the entire journey.
   (b) In what direction should the boat be pointed to head straight home?
5. A jet-ski driver wants to head to an island in the St. Lawrence River that is 5.0 km [W20.0°S] away. If he is travelling at a speed of 40.0 km/h relative to the water and the St. Lawrence is flowing 6.0 km/h[E]
   (a) in what direction should he head the jet-ski?
   (b) how long will it take him to reach the island?
6. A space shuttle is approaching the International Space Station at a velocity of 12 m/s relative to the space station. A landing cable is fired toward the space station with a velocity of 3.0 m/s, at an angle of 25° relative to the direction of the shuttle. What velocity will the cable appear to have to an observer looking out of a window in the space station?

Vector Components and Dynamics

One of the most useful applications of vector addition and subtraction using components is in dynamics. You will recall from Chapter 5 that Newton's first and second laws are:

- *An object at rest or in uniform motion will remain at rest or in uniform motion unless acted upon by an external force.*
- *When a force $\vec{F}$, acts on a mass m, the resulting acceleration $\vec{a}$, is proportional to the force and inversely proportional to the mass.*

$$\vec{a} = \frac{\vec{F}}{m} \qquad \text{or} \qquad \vec{F} = m\vec{a}$$

The force, $\vec{F}$, in both laws always represents the net force or the vector sum of all of the forces acting on the mass. In some cases, you do not know the direction of the net force. You must first do vector addition to find it. In other cases, you want an object to be in equilibrium, that is, to be stationary. A point mass is in equilibrium if the vector sum of all of the forces acting on it is zero. You might know two or more forces and want to find the equilibrating force — the force that will cause to vector sum to be zero. For example, if $\vec{F}_1$ = 425 N[E63°N] and $\vec{F}_2$ = 385 N[W15°N], what force would create equilibrium? Figure 10.5 shows you how to approach such a problem. The two forces are shown in part A, $\vec{F}_1$ is resolved into components in part B, and $\vec{F}_2$ is resolved into components in part C.

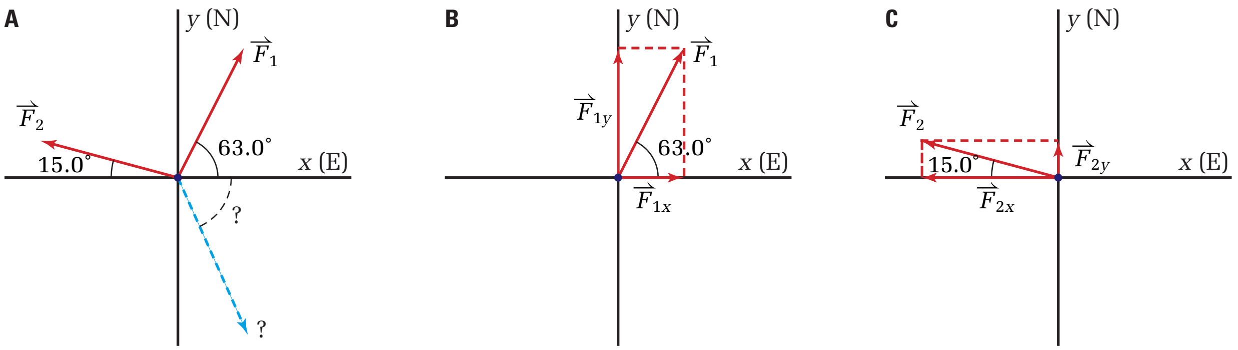


Figure 10.5 Find the *x*- and *y*-components of the known forces then find an *x* and a *y* component that will make all of the components add to zero.

Make a table such as Table 10.2 to record the components of the known forces. Include a row for the equilibrating force, $\vec{F}_3$. Set the sum of the *x*-components to zero and the sum of the *y*-components to zero and solve for the components of the third force. Use the components to find the magnitude and direction of the equilibrating force, $\vec{F}_3$.

Table 10.2

Vector	*x*-component	*y*-component
$\vec{F}_1$	192.94 N	378.67 N
$\vec{F}_2$	−371.88 N	99.645 N
$\vec{F}_3$	$\vec{F}_{3x}$	$\vec{F}_{3y}$
$\vec{F}_{net}$	0.0 N	0.0 N

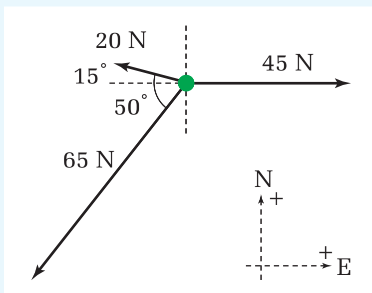
MODEL PROBLEM

Three children are each pulling on their older sibling, who has a mass of 65 kg. The forces exerted by each child are listed below. Use vector components to find the acceleration of the older sibling.

$\vec{F}_1 = 45$ N[E] $\quad$ $\vec{F}_2 = 65$ N[S40°W] $\quad$ $\vec{F}_3 = 20$ N[N75°W]

Frame the Problem

- The force of gravity on the older sibling is balanced by the normal force of the ground. Therefore, you can neglect vertical forces because there is no motion in the vertical plane.
- Draw a free body diagram representing horizontal forces on the older sibling.



- The *net force* in the horizontal plane will determine the magnitude and direction of the *acceleration* of the older sibling,
- *Newton's second law* applies to this problem.

Identify the Goal

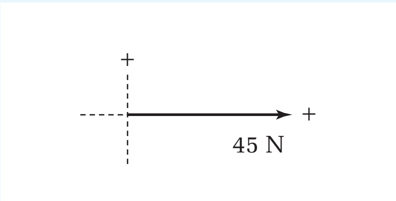
The acceleration, $\vec{a}$, of the older sibling

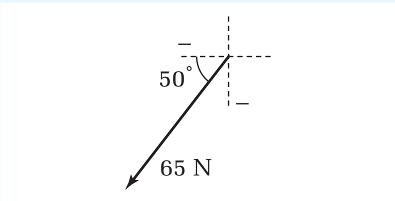
Variables and Constants

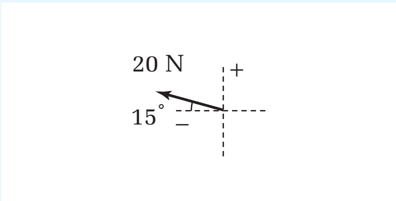
Known	Unknown
$\vec{F}_1 = 45\ \text{N[E]}$	$\vec{a}$
$\vec{F}_2 = 65\ \text{N[S40°W]}$	θ
$\vec{F}_3 = 20\ \text{N[N75°W]}$	

Strategy and Calculations

Draw each vector with its tail at the origin of an x–y-coordinate system where $+y$ coincides with north and $+x$ coincides with east.







Find the angle with the nearest x-axis.

East coincides with the x-axis so the angle is 0°

In the angle [S40°W], the 40° angle is with the $-y$-axis. The angle with the $-x$-axis is $90° - 40° = 50°$

In the angle [N75°W] the 75° angle is with the $+y$-axis. The angle with the $-x$-axis is $90° - 75° = 15°$

Find the x-component of each force vectors.

$F_{1x} = |\vec{F}_1| \cos 0°$

$F_{1x} = (45\ \text{N})(1.000)$

$F_{1x} = 45\ \text{N}$

$F_{2x} = -|\vec{F}_2| \cos 50°$

$F_{2x} = -(65\ \text{N})(0.6428)$

$F_{2x} = -41.78\ \text{N}$

The angle is in the third quadrant so x is negative.

$F_{3x} = -|\vec{F}_3| \cos 15°$

$F_{3x} = -(20\ \text{N})(0.9659)$

$F_{3x} = -19.32\ \text{N}$

The angle is in the second quadrant so x is negative.

Find the y-components of each force vector.

$F_{1y} = |\vec{F}_1| \sin 0°$

$F_{1y} = (45\ \text{N})(0.0)$

$F_{1y} = 0.0\ \text{N}$

$F_{2y} = -|\vec{F}_2| \sin 50°$

$F_{2y} = -(65\ \text{N})(0.7660)$

$F_{2y} = -49.79\ \text{N}$

The angle is in the third quadrant so y is negative.

$F_{3y} = |\vec{F}_3| \sin 15°$

$F_{3y} = (20\ \text{N})(0.2588)$

$F_{3y} = 5.176\ \text{N}$

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Make a table in which to list the *x*- and *y*-components. Add them to find the components of the resultant vector.

Vector	*x*-component	*y*-component
$\vec{F}_1$	45 N	0.0 N
$\vec{F}_2$	–41.78 N	–49.79 N
$\vec{F}_3$	–19.32 N	5.176 N
$\vec{F}_{net}$	–16.1 N	–44.614 N

Use the Pythagorean Theorem to find the magnitude of the net force.

$$|\vec{F}_{net}|^2 = (F_{x\ net})^2 + (F_{y\ net})^2$$
$$|\vec{F}_{net}|^2 = (-16.1\ \mathrm{N})^2 + (-44.614\ \mathrm{N})^2$$
$$|\vec{F}_{net}|^2 = 259.21\ \mathrm{N}^2 + 1990.41\ \mathrm{N}^2$$
$$|\vec{F}_{net}|^2 = 2249.62\ \mathrm{N}^2$$
$$|\vec{F}_{net}| = 47.430\ \mathrm{N}$$

Use trigonometry to find the angle θ.

Since both the *x*- and *y*-components are negative, the angle is in the third quadrant.

$$\tan\theta = \frac{-44.614\ \mathrm{N}}{-16.1\ \mathrm{N}}$$
$$\tan\theta = 2.7711$$
$$\theta = \tan^{-1} 2.7711$$
$$\theta = 70.16^\circ$$

The net force on the older sibling is 46 N at an angle of 70˚ from the *x*-axis in the third quadrant. This result is equivalent to 47 N[W70˚S] or 47 N[S20˚W].

Apply Newton's second law in terms of acceleration to find the older sibling's acceleration.

$$\vec{a} = \frac{\vec{F}}{m}$$
$$\vec{a} = \frac{47.43\ \mathrm{N[S20^\circ W]}}{65\ \mathrm{kg}}$$
$$\vec{a} = 0.72969\ \frac{\frac{\cancel{\mathrm{kg}}\cdot\mathrm{m}}{\mathrm{s}^2}}{\cancel{\mathrm{kg}}}[\mathrm{S20^\circ W}]$$
$$\vec{a} = 0.79269\ \frac{\mathrm{m}}{\mathrm{s}^2}[\mathrm{S20^\circ W}]$$

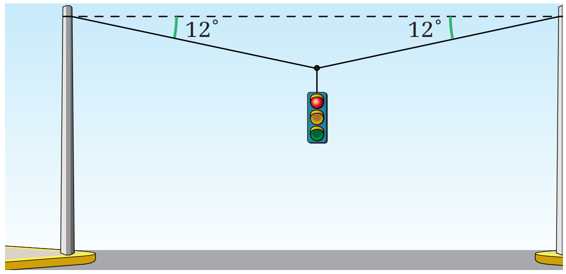
The acceleration of the older sibling is 0.73 $\frac{\mathrm{m}}{\mathrm{s}^2}$[S20˚W].

Validate

Using components gives nearly the same answer as the scale diagram method. You would expect the method of components to yield more accurate results. Also, the units cancelled to give m/s^2 which is correct for acceleration.

PRACTICE PROBLEMS

7. For each of the following combinations of forces, find the equilibrating force — the force that will make the vector sum equal to zero.
   (a) $\vec{F}_1 = 154$ N[E22°S], $\vec{F}_2 = 203$ N[W74°N] What is $\vec{F}_3$?
   (b) $\vec{F}_1 = 782$ N[E12°N], $\vec{F}_2 = 629$ N[W24°S] What is $\vec{F}_3$?
   (c) $\vec{F}_1 = 48$ N[W81°N], $\vec{F}_2 = 61$ N[E63°N], $\vec{F}_3 = 78$ N[E15°S] What is $\vec{F}_4$?
8. Three young children are pulling on a stuffed animal toy. Amy is pulling with a force of 15 N[N58°E] and Buffy is pulling with a force of 18 N[S23°E]. With what force must Caitlin pull to prevent the toy from moving?
9. A traffic light hangs in the centre of the road from cables as shown in the figure. If the mass of the traffic light is 65 kg, what force must the cables exert on the light to prevent it from falling? (Hint: Since the angles that the cables make with the horizontal are the same (12°) they both exert forces of the same magnitude.)



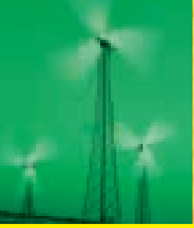
Working with Common Forces

A few common forces influence nearly all forms of motion that occur on or near Earth's surface. These common forces are the gravitational force, frictional forces, a normal force, and in many cases an applied force. Although you are familiar with these forces from your previous study of Chapters 4 and 5, some features of these forces warrant a more detailed examination.

A *gravitational force* acting on a mass is the weight of that mass thus weight is a force. Since Newton's second law applies to any net force and if the force of gravity is the only force acting on an object, you can find an object's weight by applying Newton's second law.

- State Newton's second law — $\vec{F} = m\vec{a}$
- If gravity is the only force acting on a mass, near Earth's surface, the acceleration of the mass will have a magnitude equal to g. The direction of the gravitational force near Earth's surface is toward Earth's centre. — $\vec{F}_g = mg$[down]
- Since the direction is always the same, you will often see the equation written without vector notations. Therefore, an object's weight is often written as shown. — $F_g = mg$

The critical point to remember when using the equation for weight is that the value of g varies slightly from place to place on Earth's surface because Earth is not a perfect sphere. So, although you will typically use the value 9.81 m/s^2, be aware that it can vary depending on location.



QUICK LAB

Maintaining Equilibrium

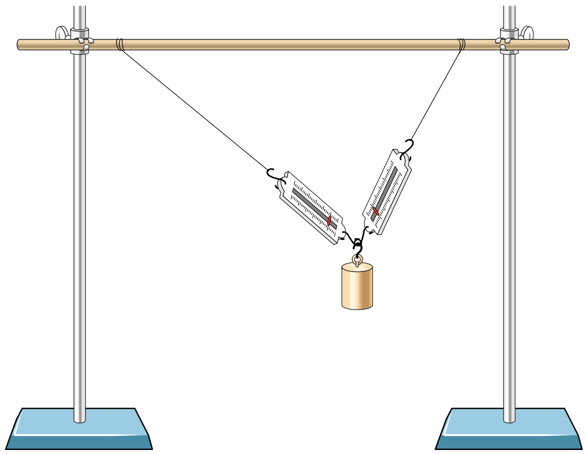
TARGET SKILLS

- **Analyzing and interpreting**
- **Performing and recording**

When the vector sum of all of the forces acting on a point mass is zero, the mass is said to be in equilibrium. You have just been performing calculations to create an equilibrium. How close do your calculations agree with experimental observation? You will answer that question in this activity.

Set up a force table similar to the one shown in the photograph. Adjust three of the pulleys so that their strings pass over the 30˚, 105˚, and 185˚ marks. Place masses totalling 150 g (at 30˚), 200 g (at 105˚), and 175 g (at 185˚) on the holders hanging from the strings over the pulleys. Calculate the force which will bring the forces on the strings to equilibrium holding the central ring over the central point on the force table. Test your calculation by placing masses on the string over the fourth pulley and adjusting the pulley to the calculated angle.

Choose another combination of masses and experiment with the force table until you have established another equilibrium condition. Observe the angles and determine the forces on the strings. Calculate the net force pulling on the central ring on the force table to find out how well your calculation agrees with your observation. Calculate the percent difference between the two values.



Assemble a holder which consists of a rod or dowel supported horizontally by clamps on two retort stands as shown in the diagram. Attach strings of different lengths near the ends of the rod or dowel. Attach Newton spring scales to the end of each string. Hang an object of unknown mass on the scales. From the readings on the scales, calculate the mass of the object. With a single Newton spring scale, determine the mass of the object.

Analyze and Conclude

1. Discuss the degree to which your calculations agree with your observations with the force table.
2. Describe any possible sources of error in your observations.
3. What is the percent difference between your calculated value for the object of unknown mass and your observed value?
4. Comment on the significance of such calculations in the design of structures such as buildings or bridges.

Although objects on Earth's surface are always subject to gravity, many objects are not moving at all. To make the net force equal to zero, something must be exerting a force that is equal in magnitude and opposite in direction to the force of gravity. Recall that Newton's third law states,

For every action force on object B due to object A, there is a reaction force, equal in magnitude but opposite in direction, due to object B acting back on object A.

In the case of stationary objects, the force that is "equal and opposite" to the gravitational force is the *normal* force exerted by the ground or other surface on which the object is resting. In cases such as this — an object resting on the ground — the normal force is equal, in magnitude, to the weight of the object.

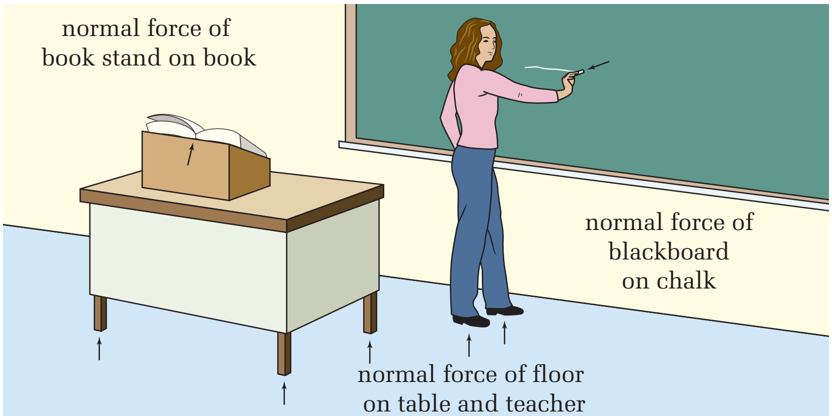


Figure 10.6 Normal forces are perpendicular to the plane of the surface that is exerting the normal force.

When objects are falling due to the gravitational force, they rarely fall with the acceleration due to gravity or 9.81 m/s^2. The acceleration of falling objects is smaller than g because the force of air *friction* is acting in the direction opposite to their motion, thus reducing the downward acceleration. Virtually every moving object that you encounter is subject to some form of friction. However, use caution when solving problems involving friction. Falling objects experience air friction which is a type of fluid friction. Fluid friction is very complex and increases as the speed of the object increases. You will not do any detailed analyzes of fluid friction.

The familiar equation for the force of friction that you have used in the past applies only to sliding friction. When two surfaces are sliding past each other, the magnitude of the force of friction which each is exerting on the other is proportional to the magnitude of the *normal* force. Note that the equation is written without vector notations because it is only the magnitudes of the forces that are related.

$$F_{\mathrm{f}} = \mu F_{\mathrm{N}}$$

MISCONCEPTION

The Normal Force is Not Always the Weight

Students often equate a normal force of a surface on an object to the weight of the object. Although this is occasionally true, it is not always the case. The normal force is the force pressing two surfaces together and is always perpendicular to the plane of the surfaces. In some situations, the normal force is unrelated to the weight of an object.

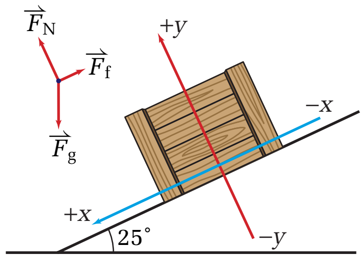


Figure 10.7 Assign a coordinate system to the diagram with the *x* axis parallel to the inclined plane. Let the positive *x* direction be down the plane.

The proportionality constant, μ, is the coefficient of friction and must be experimentally determined for any two surfaces that slide on each other. Also, you probably recall from Chapter 4, for every combination of surfaces, there are two frictional coefficients. The coefficient of kinetic friction, μ_k, applies when one object is moving relative to the other while the coefficient of static friction, μ_s, applies to an object at rest that is just starting to move.

Both the frictional force and the normal force have features that are critical to remember when analyzing problems. One example will illustrate both of these features. Consider the crate on the inclined plane shown in Figure 10.7. Given that the mass of the crate is 38 kg, the coefficient of friction is 0.48, and the angle of the incline is 25°, what is the acceleration of the crate?

- To find the force of friction, you need to know the normal force. Remember, the normal force is *not* the weight of the crate. To find the normal force, apply Newton's second law to the *y*-components of all of the forces. Since there is no motion in the *y* direction, the acceleration is zero.

$$F_y = ma_y$$
$$F_{gy} + F_N = ma_y$$
$$-(38\ \text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(\cos 25°) + F_N = (38\ \text{kg})\left(0.0\frac{\text{m}}{\text{s}^2}\right)$$
$$F_N = 337.85\ \text{N}$$

- To find the acceleration of the crate, apply Newton's second law to the components of the forces in the *x* direction. From this equation, you can solve for the acceleration of the crate.

$$F_x = ma_x$$
$$F_{gx} + F_f = ma_x$$
$$mg\sin\theta - \mu F_N = ma_x$$
$$(38\ \text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(\sin 25°) - (0.48)(337.85\ \text{N}) = (38\ \text{kg})a_x$$
$$a_x = \frac{157.54\ \text{N} - 162.17\ \text{N}}{38\ \text{kg}}$$
$$a_x = -0.12184\frac{\text{m}}{\text{s}^2}$$
$$a_x \cong -0.12\frac{\text{m}}{\text{s}^2}$$

What do you notice that immediately tells you that there is something wrong with this answer? The normal force was calculated correctly. The fact that it is not equal to the weight of the crate was taken into account. It appears that all of the values were substituted into the expression for Newton's second law in the *y* direction correctly. However, the sign of the acceleration is negative. This sign implies that the crate will spontaneously accelerate *up* the inclined plane in the absence of an applied force. Obviously this will not happen. Further inspection of the calculation of the acceleration reveals that the frictional force was greater than the component of the gravitational force along the incline. This calculation of the frictional force is the source of the problem. *The magnitude of the frictional force is not always* μF_N.

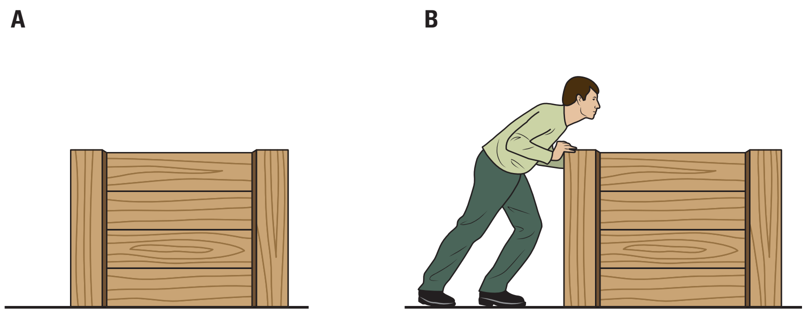


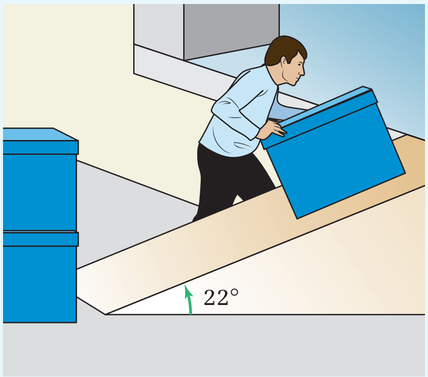
Figure 10.8 **(A)** When there is no applied force acting horizontally on the crate, there is *no* frictional force. **(B)** As someone applies a horizontal force, the frictional force increases with the applied force until the applied force reaches μF_N. When the applied force exceeds μF_N, the crate begins to move.

To understand why this is true, consider the crate in Figure 10.8 A. It is not moving so the horizontal acceleration must be zero indicating that the net horizontal force is zero. Since there is no applied force acting horizontally, the frictional force must be zero. In part B, someone is exerting an applied force but the crate is still stationary indicating that the acceleration and the net force on the crate must be zero. In this case, the frictional force must be equal in magnitude to the applied force regardless of its magnitude. Until the applied force is great enough to overcome the maximum possible static frictional force, the crate will not begin to move. This is the critical point to remember — the frictional force increases with the opposing force until it reaches its maximum and then the object begins to move. A frictional force can *never* be larger than its opposing force. For this reason, the equation for the force of friction is sometimes written,

$$F_{f(max)} = \mu F_N.$$

MODEL PROBLEM

A worker places a large plastic waste container with a mass of 84 kg on the ramp of a loading dock as shown in the figure. The ramp makes an angle of 22° with the horizontal. The workers turns to pick up another container before pushing the first one up the ramp.



(a) If the coefficient of static friction is 0.47, will the crate slide down the ramp?

(b) If the crate does slide down, what will be its acceleration?

(c) If the crate does not slide down, and the worker starts to push it up the ramp with an applied force parallel to the plane, with what force will he be pushing when the crate begins to move?

(d) If the worker continues to push with the same force after the crate starts moving, and the coefficient of kinetic friction is 0.25, at what rate will the container start to accelerate?

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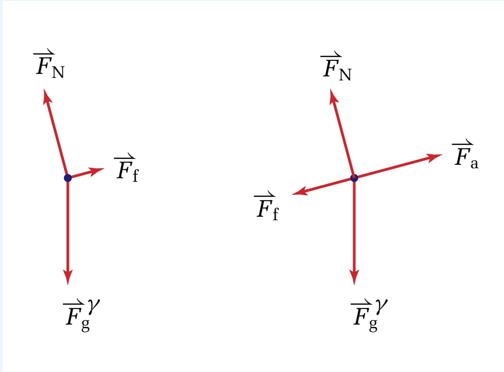
Frame the Problem

- Sketch free body diagrams of the container sitting on the ramp and the container with the applied force of the worker.
- If the *component* of the *force of gravity* that is *parallel* to the ramp is greater than the *maximum possible frictional force*, it will slide down the ramp. Otherwise, it will remain motionless.
- To start the container sliding up the ramp, the worker will have to *apply* a *force* that is equal in magnitude to the *maximum possible force of friction*. The *coefficient of static friction* applies to this situation.
- As soon as the container starts to slide, the *coefficient of kinetic friction* applies.
- The *acceleration* will depend on the *net force* and the *mass* of the container.



Identify the Goal

(a) whether the container will slide down the ramp

(b) acceleration, a_x, of the container down the ramp

(c) applied force, $\vec{F}_a$, needed to start the container sliding up the ramp

(d) acceleration, a_x, of the container up the ramp

Variables

Known		Implied	Unknown
$m = 84$ kg	$\mu_s = 0.47$	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	a_x (up)
$\theta = 22^\circ$	$\mu_k = 0.25$		a_x (down)

Strategy

Find the x component of the force of gravity.

Calculations

$$\sin\theta = \frac{F_{gx}}{|\vec{F}_g|}$$

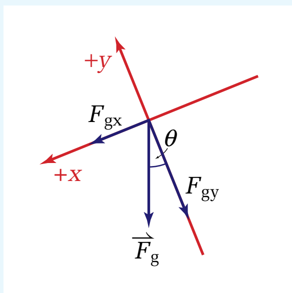
$$F_{gx} = |\vec{F}_g| \sin\theta$$

$$F_{gx} = mg\sin\theta$$

$$F_{gx} = (84 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)\sin 22^\circ$$

$$F_{gx} = (824.04 \text{ N})(0.374607)$$

$$F_{gx} = 308.69 \text{ N}$$



Find the maximum possible force of static friction.

$$F_{f(max)} = \mu_s F_N$$
$$F_{f(max)} = \mu_s mg\cos\theta$$
$$F_{f(max)} = (0.47)(84\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 22^\circ$$
$$F_{f(max)} = (387.29\ \text{N})(0.92718)$$
$$F_{f(max)} = 359.097\ \text{N}$$

(a) The magnitude of the maximum possible force of friction is larger than the component of the force of gravity along the ramp. Therefore, the container will not slide.

(b) The acceleration of the container down the ramp is zero.

Find the net force in the y direction by applying Newton's second law to the components of force in the y direction.

$$F_y = ma_y$$
$$F_{gy} + F_N = ma_y$$
$$-mg\cos\theta + F_N = ma_y$$

There is no motion in the y direction so the acceleration is zero.

$$-mg\cos\theta + F_N = 0$$
$$F_N = mg\cos\theta$$
$$F_N = (84\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 22^\circ$$
$$F_N = (824.04\ \text{N})(0.92718)$$
$$F_N = 764.04\ \text{N}$$

Find the applied force by applying Newton's second law to the components of force in the x direction. The force of friction will be positive (down the ramp) because the applied force is in the negative direction (up the ramp).

$$F_x = ma_x$$
$$F_a + F_{gx} + F_f = ma_x$$
$$F_a + mg\sin\theta + \mu_s F_N = ma_x$$

The acceleration in the x direction will be zero until the applied force has overcome the force of static friction.

$$F_a + mg\sin\theta + \mu_s F_N = 0$$
$$F_a = -mg\sin\theta - \mu_s F_N$$
$$F_a = -(84\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 22^\circ - (0.47)(764.04\ \text{N})$$
$$F_a = -(824.04\ \text{N})(0.37461) - (359.09\ \text{N})$$
$$F_a = -308.69\ \text{N} - 359.09\ \text{N}$$
$$F_a = -667.787\ \text{N}$$
$$F_a \cong -6.7 \times 10^2\ \text{N}$$

(c) The applied force necessary to overcome the component of gravity along the ramp and to overcome the force of static friction is 6.7×10^2 N up the plane in the negative direction.

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Strategy

As soon as the container begins to move, the coefficient of kinetic friction replaces the coefficient of static friction. Find the acceleration in the x direction by applying Newton's second law to the components of force in the x direction and using the coefficient of kinetic friction. Use the (unrounded) value for the applied force calculated above.

Calculations

$$F_a + mg\sin\theta + \mu_s F_N = ma_x$$

$$ma_x = F_a + mg\sin\theta + \mu_k F_N$$

$$a_x = \frac{F_a + mg\sin\theta + \mu_k F_N}{m}$$

$$a_x = \frac{-667.79\text{ N} + (84\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 22^\circ + (0.25)(764.04\text{ N})}{84\text{ kg}}$$

$$a_x = \frac{-667.79\text{ N} + (824.04\text{ N})(0.37461) + (191.01\text{ N})}{84\text{ kg}}$$

$$a_x = \frac{-667.79\text{ N} + 308.69\text{ N} + 191.01\text{ N}}{84\text{ kg}}$$

$$a_x = -2.0011\ \frac{\text{m}}{\text{s}^2}$$

$$a_x \cong -2.0\ \frac{\text{m}}{\text{s}^2}$$

(d) At the instant that the container starts to move, its acceleration up the ramp will be 2.0 m/s^2. This is quite fast so the worker will reduce the applied force immediately.

Validate

Recalling that the units $\text{kg}\cdot\text{m/s}^2$ are equivalent to a N, all of the units cancel to give the correct units. Personal experience tells you that it is reasonable that the container does not slide down the ramp. The applied force needed to start pushing the crate up the ramp is smaller than the weight of the container which is expected.

PRACTICE PROBLEMS

10. A cardboard box (of negligible mass) filled with 45 kg of paper sits on a ramp that makes an angle of 21° with the horizontal. The coefficient of static friction is 0.42.

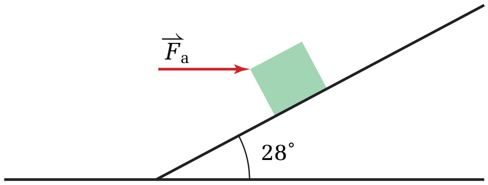
- **(a)** Will the box slide down the ramp? Explain.
- **(b)** If not, how much force would you have to apply directly down the ramp to start the box sliding?

11. A 61 kg plastic container sits on a ramp.

- **(a)** If the coefficient of static friction is 0.37, at what angle of the ramp would the container just start to slide?
- **(b)** If the coefficient of kinetic friction is 0.18, what would be the acceleration of the container just after it started to slide?

12. A plastic recycling container filled with paper, making a total of 55 kg, is sitting on a ramp that makes an angle of 33° with the horizontal. If the coefficient of kinetic friction is 0.23, with what applied force directly up the ramp would you have to push to keep the container sliding up the ramp at a constant velocity?

13. A new worker in a warehouse is pushing an 85 kg crate up a 28° ramp. The coefficient of static friction is 0.46. Instead of pushing directly up the ramp, the worker is pushing directly horizontally as shown in the diagram.



(a) How hard does the worker have to push to start the crate moving up the ramp? (Hint: A component of the applied force is perpendicular to the ramp thus increasing the normal force.)

(b) An experienced worker stops and tells the new worker to kneel down a little and push directly up the ramp. How hard does the worker have to push to start the crate moving up the ramp from this position?

10.1 Section Review

1. C Explain how to resolve a vector.

2. C What use is the Pythagorean theorem in resolving vectors?

3. K/U Draw examples of velocity vectors for the following cases:
   (a) the x- and y-components are both positive
   (b) the x-component is positive and the y-component is negative
   (c) the x-component is negative and the y-component is positive
   (d) the x-component is zero and the y-component is negative

4. K/U Consider a standard x–y-coordinate system. In which quadrant(s) does a vector have:
   (a) two positive components?
   (b) two negative components?
   (c) one positive and one negative component?

5. K/U When applying Newton's first and second laws, what is the most important fact to remember about the force in these laws?

6. C Under what conditions is it valid to use 9.81 m/s^2 in the equation $F_g = mg$ for weight?

7. K/U Why are there two coefficients of friction for every pair of surfaces that slide across each other?

8. C How does fluid friction differ from sliding friction?

9. MC Give two examples of normal forces that do not involve the weight of an object.

10. C What is the significance of writing the equation for sliding friction with (max) as a subscript? $F_{f(max)} = \mu F_N$

11. MC Do some research to learn how balancing forces is important to bridge building. How do engineers account for the weight of the cars or trains that will be using the bridge?