9.3 Interference of Waves and Related Properties

SECTION OUTCOMES

- Explain the phenomena of wave interference and diffraction.
- Conduct investigations on wave interference including Young's double-slit experiment.
- Compare and describe the properties of sound and electromagnetic radiation.

KEY TERMS

- noise
- music
- harmonics
- fundamental frequency
- sound spectrum
- closed air column
- resonance lengths
- open air column
- beat
- beat frequency
- coherent
- fringe

The property of a phenomenon that most clearly establishes that it has wave properties is interference. Interference can be demonstrated in a variety of ways. In Chapter 8, you learned that when a wave on a spring reflects from one end, the reflected wave interferes with the incident wave and can, under the right conditions, establish a standing wave. When you looked at water waves in two dimensions, you saw nodal and antinodal lines created by interference. You will see — or hear — the same phenomena in sound and light waves. One of the most pleasant ways to experience standing waves is in music.

Music, Noise, and Resonance in Air Columns

If you strike two stones together, you produce a sound that is immediately recognizable, but which has no specific pitch. A **noise**, such as this, is a mixture of many sound frequencies with no recognizable relationship to each other. **Music**, on the other hand, is a mixture dominated by sound frequencies known as **harmonics** that are whole-number multiples of the lowest frequency or **fundamental frequency**. By plotting the intensity of the various sound frequencies that make up a sound, you can see graphically the difference between music and noise. The **sound spectrum** of music consists of a number of discrete frequencies, while the sound spectrum for noise shows a continuous or nearly continuous range of frequencies.

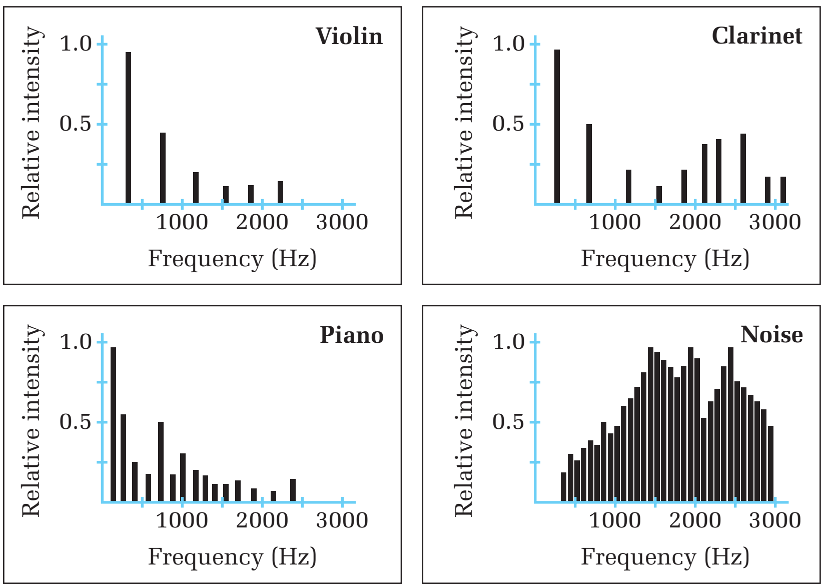


Figure 9.37 Sound spectra of notes played on musical instruments and of a noise

TRY THIS...

Borrow a trumpet, trombone, or other brass instrument. Take the mouthpiece out of the instrument and blow through it. The sound that your vibrating lips make is noise. Re-insert the mouthpiece into the instrument, and then blow again. (You might need a few practice tries to produce a note.) The air column of the instrument will resonate with only one of the frequencies your lips are producing. It is this frequency that you hear.

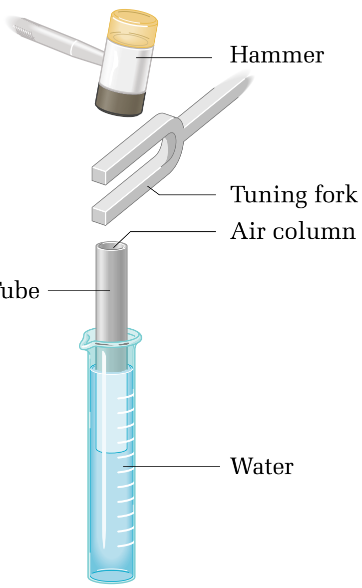
Musical instruments can produce sounds that are harmonious because the vibrations that are set up are essentially in one dimension only. In a guitar or violin, the vibrations are set up in a string. In a wind instrument, like a trombone or clarinet, vibrations are set up in a narrow column of air. Many musical sounds, including the human voice, are produced by resonance in air columns. Resonance in a linear air column occurs when a standing wave is established. The air column (the same as a spring) can sustain a standing wave only for frequencies of vibration that are whole-number multiples of a fundamental. Thus, the sound waves that are emitted from the air column are all whole-number multiples (or integral multiples) of a fundamental, and the total effect is perceived as musical.

QUICK LAB

Resonance Lengths of a Closed Air Column

TARGET SKILLS

- Performing and recording
- Identifying variables



Place a 50 cm long piece of plastic pipe inside a large graduated cylinder almost completely filled with water that is at room temperature. Sound a 512 Hz tuning fork and hold it over the open end of the plastic pipe. Raise the pipe slowly out of the water while keeping the tuning fork positioned over the open end. Measure the lengths of the air column for which resonance occurs. Repeat the procedure using a 1024 Hz tuning fork.

Analyze and Conclude

1. Use a thermometer to measure the room temperature. Calculate the speed of sound in air, and from that, the wavelength of the sound produced by the 512 Hz tuning fork.
2. By how much is one resonance length longer than the previous one? (If you were able to determine three or more resonance lengths, was this increase in length constant?) What fraction of a wavelength is this increase in resonance length?
3. Repeat questions 1 and 2 for the 1024 Hz tuning fork.

Resonance Lengths of a Closed Air Column

An air column that is closed at one end and open at the other is called a **closed air column**. If a tuning fork is held over the open end and the length of the column is increased, the loudness of the sound will increase very sharply for specific lengths of the tube, called **resonance lengths**. If a different tuning fork is used, there will still be distinct resonance lengths, but they will be different than those you found with the first fork.

Resonance in a Closed Air Column

Resonance occurs in an air column when the length of the air column meets the criteria for supporting a standing wave. The tuning fork produces a sound wave, which travels down the air column and is reflected off the closed end. This reflected wave interferes with the wave from the tuning fork, producing a standing wave.

It is easier to see how resonance occurs if sound waves are represented in the same manner as standing waves on a rope or spring, as presented in Chapter 8. In the case of sound waves, however, the amplitude of the wave does not represent the actual position of the molecules in the air as it does for the rope. The amplitude of the sound wave represents the extent of the longitudinal displacement of the molecules in the air at that specific point along the air column.

In Figure 9.38, the arrows on the right hand side show the distance that the air molecules move (displacement) at different points along the tube. At the closed end of a tube, the air molecules are entirely prevented from moving. Thus, when the standing wave is set up, there has to be a **displacement node** at this closed end. (There will also be a pressure antinode at the same point because the closed end will experience the greatest variation in pressure over time.) At the open end of the air column, however, the air molecules are free to move back and forth relatively easily. Thus, when a standing wave is set up in the air column, there must be a **displacement antinode** at this open end.

PHYSICS FILE

Theories attempt to explain the complexities of real-world phenomena. In deriving the resonance lengths for a closed air column, it was assumed that the behaviour of the air in the column could be treated as being one-dimensional (like the standing waves in a spring). This is a very good assumption for the air inside the air tube, but is not as good at the open end, particularly if the diameter of the tube is large. As a result, the antinode at the open end of the tube actually lies a short distance inside the end. The nodes inside the tube, however, are spaced one half wavelength apart, regardless of the diameter of the tube.

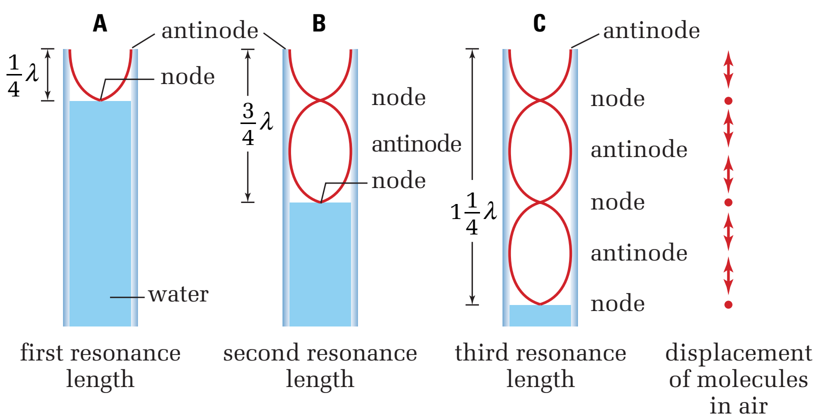


Figure 9.38 First three resonance lengths of a closed air column and their displacement standing wave patterns

TRY THIS...

Convince yourself that the equation in the box will give you the expressions for the resonance wavelengths by first solving for each *L* in the expressions on the right, and then by substituting the values one through four into the equation.

In Figure 9.38 (A), you can see that the first resonance length is equal to one quarter of a wavelength, or $\frac{\lambda}{4}$. In (B), the second resonance length is equal to three quarters of a wavelength, or $\frac{3\lambda}{4}$. Finally, in (C), the third resonance length is equal to one and a quarter wavelengths, or $\frac{5\lambda}{4}$. Each of the subsequent resonance lengths is half a wavelength longer than the previous one. Thus, for the first three resonance lengths, the pattern is $\lambda = 4L_1$, $\lambda = \frac{4L_2}{3}$, $\lambda = \frac{4L_3}{5}$, $\lambda = \frac{4L_4}{7}$. To find the resonance lengths, solve for L_n in the equations, where n is a positive integer.

RESONANCE LENGTHS OF A CLOSED AIR COLUMN

The resonance lengths of a closed air column are odd integer multiples of the first resonance length, $\frac{1}{4}\lambda$.

$$L_n = (2n - 1)\frac{\lambda}{4}$$

where n is a positive integer.

Music Link

Wind instrument players play different notes by varying the lengths of the air columns in their instruments. In a trombone, this is most evident as the slide is pushed in and out. In trumpets, tubas, and other brass instruments, different valves are used to open or close auxiliary lengths of tubing. In clarinets, saxophones, and other woodwinds, key pads are lifted off holes in the side of the instrument. In what other ways do these instruments vary?

Resonance Lengths of an Open Air Column

An air column that is open at both ends is called an open air column. At these open ends, the air molecules are free to move easily, so that when a standing wave is set up, there is a displacement antinode at each end, as shown in Figure 9.39.

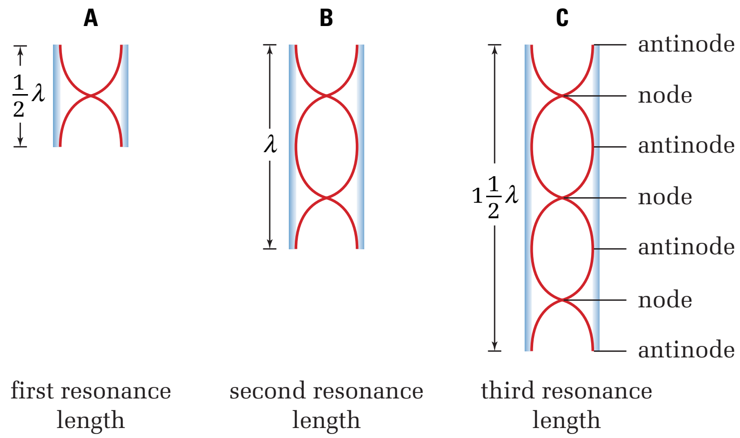


Figure 9.39 First three resonance lengths of an open air column and their displacement standing wave patterns

As you can see in Figure 9.39, the first three resonance lengths are $\frac{\lambda}{2}$, λ, $\frac{3\lambda}{2}$, with each of the subsequent resonance lengths half a wavelength longer. Thus, for the first three resonance lengths, the pattern is $\lambda = 2L_1$, $\lambda = L_2$, $\lambda = \frac{3L_3}{2}$.

RESONANCE LENGTHS OF AN OPEN AIR COLUMN

The resonance lengths of an open air column are integral multiples of the first resonance length, $\frac{1}{2}\lambda$.

$$L_n = \frac{n\lambda}{2}$$

MODEL PROBLEM

Resonance Lengths of a Closed Air Column

A vibrating tuning fork is held near the mouth of a narrow plastic pipe partially submerged in water. The pipe is raised, and the first loud sound is heard when the air column is 9.0 cm long. The temperature in the room is 20°C.

(a) Calculate the wavelength of the sound produced by the tuning fork.

(b) Calculate the length of the air column for the second and third resonances.

(c) Estimate the frequency of the tuning fork.

Frame the Problem

- Since one end of the pipe is submerged in water, it is a *closed air column.*
- The shortest *resonance length* is one quarter of the wavelength of the sound.
- The resonance lengths of a closed air column are $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.
- The *wave equation* applies to all kinds of waves.

Identify the Goal

(a) The wavelength, λ, of the sound

(b) The length of the air column for the second and third resonances, L_2 and L_3

(c) The frequency, f, of the tuning fork

Variables and Constants

Known	Unknown	
$L_1 = 9.0$ cm	L_2	v
$T = 20°C$	L_3	f
	λ	

continued ►

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Strategy

Calculations

You can find the wavelength of the sound because you know the first resonance length. Substitute this value into the equation.

$$L_1 = \frac{\lambda}{4}$$
$$\lambda = 4L_1$$
$$\lambda = 4\ (9.0\ \text{cm})$$
$$\lambda = 36\ \text{cm or } 0.36\ \text{m}$$

(a) The wavelength of the sound is 0.36 m.

Use the equation that relates the length of the pipe to the wavelength to find the second and third resonance lengths.

Substitute the wavelength into the equations for second and third resonance lengths.

$$L_n = (2n - 1)\frac{\lambda}{4}$$
$$L_2 = (2(2) - 1)\frac{\lambda}{4}$$
$$L_2 = (4 - 1)\frac{\lambda}{4}$$
$$L_2 = \frac{3\lambda}{4}$$
$$L_2 = \frac{3}{4}(36\ \text{cm})$$
$$L_2 = 27\ \text{cm or } 0.27\ \text{m}$$

$$L_n = (2n - 1)\frac{\lambda}{4}$$
$$L_3 = (2(3) - 1)\frac{\lambda}{4}$$
$$L_3 = (6 - 1)\frac{\lambda}{4}$$
$$L_3 = \frac{5\lambda}{3}$$
$$L_3 = \frac{5}{4}\ (36\ \text{cm})$$
$$L_3 = 45\ \text{cm or } 0.45\ \text{m}$$

(b) The second and third resonance lengths are 0.27 m and 0.45 m, respectively.

You can find the frequency of the tuning fork if you know the speed of sound in the room and the wavelength of the sound.

$$v = f\lambda$$

You can find the speed of sound if you know the temperature in the room.

$$v = 331 + 0.59T_C$$

Substitute the value of the temperature in the formula for the speed of sound.

$$v = 331\ \text{m/s} + 0.59\ \frac{\frac{\text{m}}{\text{s}}}{\cancel{^\circ\text{C}}}\ (20\cancel{^\circ\text{C}})$$
$$v = 331\ \text{m/s} + 11.8\ \text{m/s}$$
$$v = 342.8\ \text{m/s}$$

Substitute the values of the wavelength (in metres) and the speed of sound in the formula for the frequency.

Substitute first

$$v = f\lambda$$
$$342.8\ \text{m/s} = f(0.36\ \text{m})$$
$$f = \frac{342.8\ \frac{\cancel{\text{m}}}{\text{s}}}{0.36\ \cancel{\text{m}}}$$
$$f = 952.2\ \text{s}^{-1}$$

Solve for *f* first

$$v = f\lambda$$
$$f = \frac{v}{\lambda}$$
$$f = \frac{342.8\ \frac{\cancel{\text{m}}}{\text{s}}}{0.36\ \cancel{\text{m}}}$$
$$f = 952.2\ \text{s}^{-1}$$

(c) The frequency of the tuning fork is 9.5×10^2 Hz.

Validate

If the first resonance length is 9.0 cm, the second is three times the first (27 cm), and the third is five times the first (45 cm).

At a temperature of 20°C, the speed of sound is about 340 m/s. The wavelength of the sound is $4 \times 9 = 36$ cm, which is about $\frac{1}{3}$ m. The frequency is then approximately 340 m/s divided by $\frac{1}{3}$ m, or 1000 Hz.

PRACTICE PROBLEMS

16. A narrow plastic pipe is almost completely submerged in a graduated cylinder full of water, and a tuning fork is held over its open end. The pipe is slowly raised from the water. An increase in loudness of the sound is heard when the pipe has been raised 17 cm and again when it has been raised 51 cm.
    (a) Determine the wavelength of the sound produced by the tuning fork.
    (b) If the pipe continues to be raised, how far from the top of the pipe will the water level be when the next increase in loudness is heard?

17. The first resonance length of an air column, resonating to a fixed frequency, is 32 cm.
    (a) Determine the second and third resonance lengths, if the column is closed at one end.
    (b) Determine the second and third resonance lengths, if the column is open at both ends.

18. The third resonance length of a closed air column, resonating to a tuning fork, is 95 cm. Determine the first and second resonance lengths.

19. The second resonance length of an air column, open at both ends and resonating to a fixed frequency, is 64 cm. Determine the first and third resonance lengths.

20. A particular organ pipe, open at both ends, needs to resonate in its fundamental mode with a frequency of 128 Hz. The organ has been designed to be played at a temperature of 22˚C.
    (a) How long does the organ pipe need to be?
    (b) If this pipe is closed at one end by a stopper, at what fundamental frequency will it resonate?

Resonance Frequencies for Fixed-Length Air Columns

How can a bugler play a melody when there are no valves, keys, or slides? The length of the air column is fixed, so the fundamental frequency is fixed. When trumpeters and trombonists blow into the mouthpiece, the vibration of their lips creates a range of frequencies. The length of the air column determines the fundamental frequency and overtones, which it "amplifies" by setting up a standing wave. The resonating air column is, in fact, what makes the sound musical.

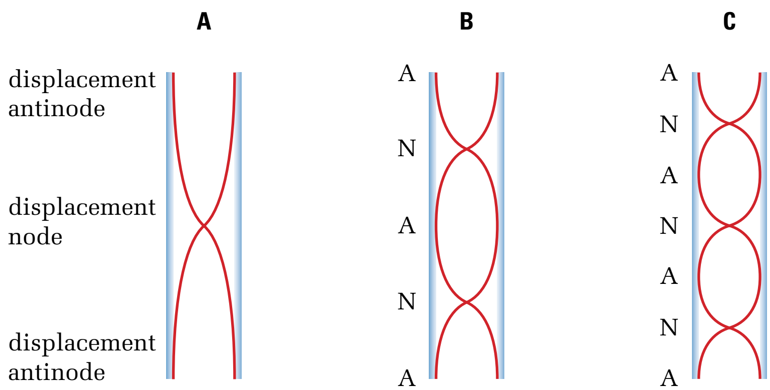
The pitch of a wind instrument can be changed not only by increasing or decreasing the length of its air column, but also by increasing or decreasing the tension in the player's lips (the embouchure). This change in the player's embouchure produces a different range of frequencies as the player blows through the mouthpiece. High tension of the lips creates only frequencies that are higher than the fundamental. The result is that the pitch is

Figure 9.40 Bugles have no valves, keys, or slides, so how can the bugler play a tune?

perceived as the lowest frequency of the overtone frequencies that set up standing waves in the air column. Although buglers have no means of varying the length of the air columns, they can still play a melody by varying their embouchures. They can access a range of different notes when playing "Reveille," "Last Post," or "Taps."

Open Air Columns

Most wind instruments, such as the bugle and the flute, behave the same as air columns that are open at both ends. The major exceptions are reed instruments such as the clarinet, oboe, and bassoon. They behave like closed air columns. For an open air column of given length L, the fundamental and overtone frequencies can be calculated in a straightforward way. The wave equation tells you that the frequency is the quotient of the speed of the wave and the wavelength, $f = \frac{v}{\lambda}$. Thus, you can find the relationship between the frequency and the length of the air column by substituting the expression for wavelength in terms of air column length, L, as shown in Figure 9.41.



$\lambda_1 = 2L$ $\qquad$ $\lambda_2 = L$ $\qquad$ $\lambda_3 = \frac{2L}{3}$

$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ $\qquad$ $f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$ $\qquad$ $f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{2L}{3}} = \frac{3v}{2L} = 3f_1$

Figure 9.41 **(A)** Fundamental mode or first harmonic; **(B)** first overtone or second harmonic; **(C)** second overtone or third harmonic

In general, the harmonics of an open air column form the series f_1, $2f_1$, $3f_1$, $4f_1$, $5f_1$, and so on. That is, all integer multiples of the fundamental frequency are produced when an air column is open at both ends. Thus, the effect of these frequencies sounding together is music rather than noise. This is why all of these frequencies are referred to as harmonics — the fundamental frequency as the first harmonic, the first overtone as the second harmonic, and so on.

RESONANCE FREQUENCIES OF A FIXED-LENGTH OPEN AIR COLUMN

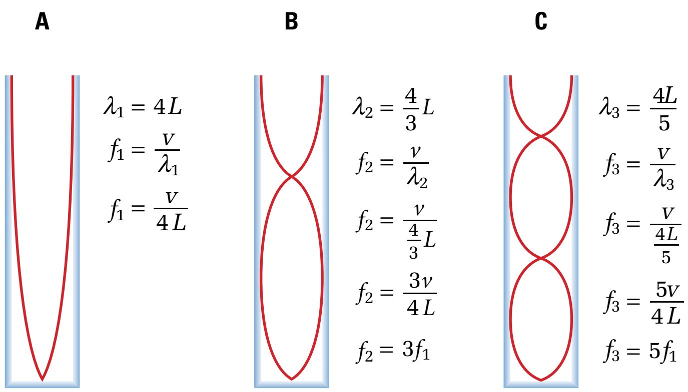
The resonance frequencies of a fixed-length open air column are integral multiples of the first resonance frequency, f_1.

$$f_n = nf_1$$

where $f_1 = \frac{v}{2L}$

Closed Air Columns

Just as the pattern of resonant lengths for closed air columns differs from that of open columns, so also does the pattern of resonant frequencies. Figure 9.42 shows you how to determine the relationship for resonant frequencies in closed air columns.



A: $\lambda_1 = 4L$, $f_1 = \frac{v}{\lambda_1}$, $f_1 = \frac{v}{4L}$

B: $\lambda_2 = \frac{4}{3}L$, $f_2 = \frac{v}{\lambda_2}$, $f_2 = \frac{v}{\frac{4}{3}L}$, $f_2 = \frac{3v}{4L}$, $f_2 = 3f_1$

C: $\lambda_3 = \frac{4L}{5}$, $f_3 = \frac{v}{\lambda_3}$, $f_3 = \frac{v}{\frac{4L}{5}}$, $f_3 = \frac{5v}{4L}$, $f_3 = 5f_1$

Figure 9.42 **(A)** Fundamental mode or first harmonic; **(B)** first overtone or second harmonic; **(C)** second overtone or third harmonic

In general, the harmonics of a closed air column form the series f_1, $3f_1$, $5f_1$, $7f_1$, $9f_1$, and so on. That is, only odd integral multiples of the fundamental frequencies are produced when the air column is closed at one end.

RESONANCE FREQUENCIES OF A FIXED-LENGTH CLOSED AIR COLUMN

The resonance frequencies of a fixed-length closed air column are odd integer multiples of the first resonance frequency, f_1.

$$f_n = (2n - 1)f_1$$

where $f_1 = \frac{v}{4L}$

Math Link

By doubling the frequency of a note, a wind player jumps an octave in pitch. For brass players, this is easily accomplished by tightening up their embouchure to move from the first to the second harmonic of an open air column. For a clarinet player, however, this will not work because of the odd integer harmonic structure of a closed air column. Use the equation for resonance frequencies in closed air columns to show why this is true.

Clarinet players cannot jump an octave by changing their embouchure.

INVESTIGATION 9-B

Determining the Speed of Sound in Air

TARGET SKILLS

- Predicting
- Performing and recording
- Identifying variables
- Communicating results

Problem

How can resonance in air columns be used to obtain a more accurate value for the speed of sound in air?

Prediction

Make a prediction about the relative precision of measurements of the speed of sound. Will measuring the speed of sound using air columns have greater precision than the method you used in the Quick Lab on page 390? On what do you base your prediction?

Equipment

- 512 Hz and 1024 Hz tuning forks
- rubber mallet
- hollow plastic pipe (do *not* use a glass pipe)
- 1000 mL graduated cylinder
- water at room temperature
- thermometer
- metre stick

CAUTION Use only plastic pipes with tuning forks. If glass pipes are used, there is a danger of flying glass if the vibrating tuning fork touches the pipe.

Procedure

1. After placing the plastic pipe in the graduated cylinder, fill the cylinder with water, as near to the top as possible. Using the thermometer, measure the temperature of the air in the room.
2. Sound the 512 Hz tuning fork, and hold it over the open end of the plastic pipe. Raise the pipe slowly out of the water while keeping the tuning fork positioned over the end of the plastic pipe. Locate the positions for which the sound increases dramatically. (Ignore positions of slightly increased sound of different frequencies from your tuning fork.) Have your partner carefully measure the length of the air column for each of these resonance points.
3. Arrange your data in four columns, with the headings: Resonance length, Change in resonance length, Wavelength, Speed of sound ($v = (512\ \text{Hz})\lambda$).

   Frequency = 512 Hz
   Temperature = __________
4. Repeat the procedure with the 1024 Hz tuning fork. Arrange your data and calculations in a similar table.

Analyze and Conclude

1. Using the data you obtained with the 512 Hz tuning fork, calculate an average value for the speed of sound in air. Estimate the precision of this value.
2. Using the data you obtained with the 1024 Hz tuning fork, calculate an average value for the speed of sound in air. Estimate the precision of this value.
3. Using your measurement of the temperature of the air, calculate a value for the speed of sound in air, using $v = 331 + 0.59T_C$.
4. Earlier in this chapter, you used an echolocation procedure to determine the speed of sound in air. Compare the two values you obtained in this experiment with each other and with your earlier result. Compare the precisions of the three results. Which result do you think is the most accurate? Why?
5. How do your experimental results for the speed of sound in air compare with the calculated value? Which do you trust the most? Why?

MODEL PROBLEMS

Harmonics in a Fixed-Length Air Column

1. An air column, open at both ends, has a first harmonic of 330 Hz.

(a) What are the frequencies of the second and third harmonics?

(b) If the speed of sound in air is 344 m/s, what is the length of the air column?

Frame the Problem

- The air column is *open at both ends*, so the *harmonics* are integral multiples of f_1.
- The frequency of the first harmonic, or f_1, is 330 Hz.
- The frequency of the first harmonic is equal to the speed of sound in air divided by twice the length of the air column.

Identify the Goal

(a) The frequencies of the second and third harmonics, f_2 and f_3

(b) The length, L, of the air column

Variables and Constants

Known	Unknown
$f_1 = 330$ Hz	f_2
$v = 344$ m/s	f_3
	L

Strategy

You can find the frequencies of the second and third harmonics because you know the frequency of the first harmonic.

Substitute this value into the equations.

Calculations

$f_n = nf_1$

$f_2 = 2f_1$

$f_2 = 2(330\text{ Hz})$

$f_2 = 660\text{ hz}$

$f_3 = 3f_1$

$f_3 = 3(330\text{ Hz})$

$f_3 = 990\text{ Hz}$

(a) The frequencies of the second and third harmonics are 660 Hz and 990 Hz, respectively.

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Strategy

You can find the length of the air column because you know the speed of sound in air and the frequency of the first harmonic.

Substitute these values into the equation.

Calculations

Substitute first

$$f_1 = \frac{v}{2L}$$

$$330\ \text{Hz} = \frac{344\ \frac{\text{m}}{\text{s}}}{2L}$$

$$330\ s^{-1}\,(2L) = \frac{344\ \frac{\text{m}}{\text{s}}}{\cancel{2L}}\,(\cancel{2L})$$

$$\frac{\cancel{660\ \text{s}^{-1}}\,L}{\cancel{660\ \text{s}^{-1}}} = \frac{344\ \frac{\text{m}}{\cancel{s}}}{660\ \cancel{s^{-1}}}$$

$$L = 0.5212\ \text{m}$$

Solve for *L* first

$$f_1 = \frac{v}{2L}$$

$$f_1 L = \frac{v}{2\cancel{L}}(\cancel{L})$$

$$\frac{\cancel{f_1}L}{\cancel{f_1}} = \frac{v}{2f_1}$$

$$L = \frac{344\ \frac{\text{m}}{\cancel{s}}}{2\left(330\ \cancel{s^{-1}}\right)}$$

$$L = 0.5212\ \text{m}$$

(b) The length of the air column is 0.52 m.

Validate

For an open air column, the harmonics are all integral multiples of the first harmonic. Thus, the frequency of the second harmonic is $2 \times 330 = 660$ Hz, and the frequency of the third harmonic is $3 \times 330 = 990$ Hz.

For the first harmonic, the resonance length is $\frac{1}{2}\lambda$ and the wavelength can also be calculated from $\lambda = \frac{v}{f}$. The length is equal to $\frac{1}{2} \times \frac{344}{330}$ or approximately $\frac{1}{2} \times 1$ or 0.5 m. These values are very close to the calculated values.

2. An air column, closed at one end, has a first harmonic of 330 Hz. If the speed of sound in air is 344 m/s, what is the length of the air column?

Frame the Problem

- The air column is *closed at one end*, so the *harmonics* are f_1, $3f_1$, $5f_1$, $7f_1$, $9f_1$, ...
- The frequency of the first harmonic, or f_1, is 330 Hz.
- The frequency of the first harmonic is equal to the speed of sound in air divided by four times the length of the air column.

Identify the Goal

The length, L, of the air column

Variables and Constants

Known	Unknown
$f_1 = 330$ Hz	L
$v = 344$ m/s	

Strategy

You can find the length of the air column because you know the speed of sound in air and the frequency of the first harmonic.

Substitute these values into the equation.

The length of the air column is 0.26 m.

Calculations

Substitute first

$$f_1 = \frac{v}{4L}$$

$$330\ \text{Hz} = \frac{344\,\frac{\text{m}}{\text{s}}}{4L}$$

$$330\ \text{Hz}\,(L) = \frac{344\,\frac{\text{m}}{\text{s}}}{4\cancel{L}}\,(\cancel{L})$$

$$\frac{\cancel{330\ \text{s}^{-1}}\,L}{\cancel{330\ \text{s}^{-1}}} = \frac{86\,\frac{\text{m}}{\cancel{\text{s}}}}{330\ \cancel{\text{s}^{-1}}}$$

$$L = 0.2606\ \text{m}$$

Solve for *L* first

$$f_1 = \frac{v}{4L}$$

$$f_1 L = \frac{v}{4\cancel{L}}(\cancel{L})$$

$$\frac{\cancel{f_1}L}{\cancel{f_1}} = \frac{v}{4f_1}$$

$$L = \frac{344\,\frac{\text{m}}{\cancel{\text{s}}}}{4\left(330\ \cancel{\text{s}^{-1}}\right)}$$

$$L = 0.2606\ \text{m}$$

Validate

For the first harmonic of a closed air column, the resonance length is $\frac{1}{4}\lambda$ and the wavelength can be calculated from $\lambda = \frac{v}{f}$. Thus, the length is $\frac{1}{4} \times \frac{344}{330}$, or approximately $\frac{1}{4} \times 1$, or 0.25 m.

PRACTICE PROBLEMS

21. An air column, open at both ends, resonates with a fundamental frequency of 256 Hz. Determine the frequencies of its first and second overtones (second and third harmonics).

22. A bugle is essentially a 2.65 m pipe that is open at both ends.

(a) Determine the lowest frequency note that can be played on a bugle.

(b) Determine the next two higher frequencies that will produce resonance.

23. A trombone is playing F (87.3 Hz) as its first harmonic. A trombone functions as an air column that is open at both ends.

(a) Determine the second harmonic.

(b) If the speed of sound is 344 m/s, what is the length of the tubing being used? Why would this note be difficult to play?

Hearing Interference

The standing waves that generate musical tones are one dimensional waves. When you investigate sound waves in two dimensions, you can find some curious ways to detect interference. The silence in the following Quick Lab will demonstrate interference of sound.

QUICK LAB

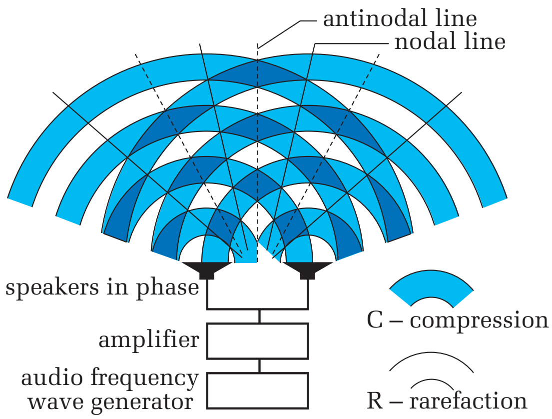
Locating Interference Between Two Loudspeakers

TARGET SKILLS

- **Performing and recording**
- **Predicting**
- **Communicating results**

Two point wave sources vibrating in phase produce a characteristic interference pattern of nodal and antinodal lines. To create sound interference, you can use two loudspeakers placed side by side, driven by the same audio frequency generator, as two in-phase sound sources. Since destructive interference is produced when one sound wave travels half a wavelength farther than the other (the crest of one wave meets the trough of the second wave), you should be able to locate a point of destructive interference. A frequency of 340 Hz will produce a wavelength of approximately 1 m, so you are looking for a point that is 0.5 m farther from one loudspeaker than the other. By walking across the expected pattern, you should be able to hear a variation of loudness as you walk through nodal and antinodal lines. Set up speakers as shown in the illustration and locate the nodal and antinodal line. Walk back and forth in front of the speakers until you are convinced that you have heard interference in sound.

(**Note:** Unwanted reflections of sound waves from hard surfaces in the room can complicate the sound pattern produced and make it difficult to hear clear points of destructive interference. Try to keep your loudspeakers away from the walls of the room or any other potential sound reflectors.)



Sound interference produced by two loudspeakers vibrating in phase

Analyze and Conclude

1. What do you expect to hear as you cross the centre line (perpendicular bisector) of a line between the two loudspeakers?
2. How much farther along your path do you expect the first quiet point to be?
3. How far apart do you expect two adjacent quiet points to be?
4. How could you change your walking path to make these quiet points farther apart and, hence, easier to identify?

A second way to hear interference is to generate two pure tones that have frequencies that are close together but not identical. You will hear the intensity of the sound wavering from loud to quiet and back to loud. This phenomenon is known as **beats** and is the consequence of two sources of similar (but not identical) frequency producing sound waves at the same time. One beat is a complete cycle from loud to quiet to loud, and the **beat frequency** is the number of cycles of loud-quiet-loud produced per second. The beat frequency depends on the difference in frequency of the two sounds. A larger difference in frequency produces a greater beat frequency, and a smaller difference in frequency produces a smaller beat frequency.

PROBEWARE

www.mcgrawhill.ca/links/atlphysics

If your school has probeware equipment, go to the above Internet site for interesting activities on the speed of sound.

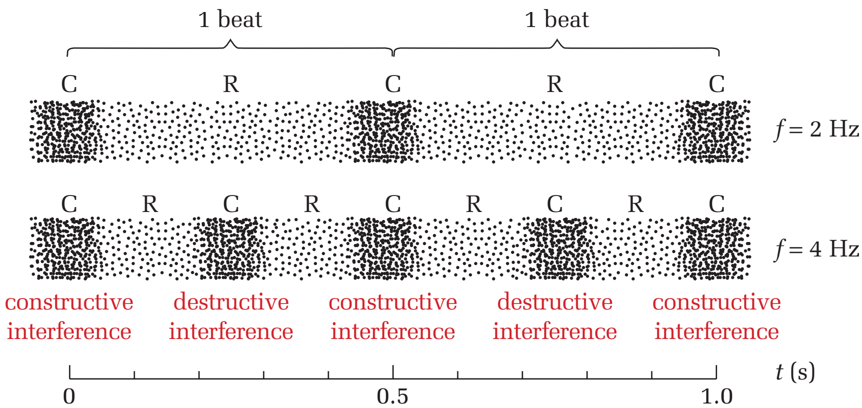


Figure 9.43 Beats are produced by alternating instances of constructive and destructive interference over time.

Using the principle of superposition, the resultant pressure wave produced by two component sound waves of similar frequency can be constructed, as shown above in Figure 9.43. This resultant wave clearly illustrates the regular variations in loudness characteristic of beats. A second method of visualizing beats is shown in Figure 9.44. Here, air pressure is plotted against time.

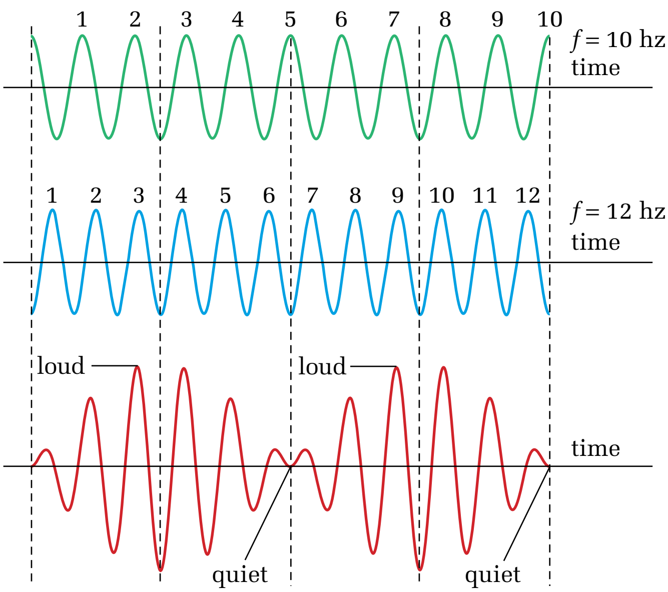


Figure 9.44 The superposition of two similar sound waves produces a resultant sound wave with an intensity that alternates between loud and quiet.

TRY THIS...

Sketch several pairs of component waves and their resultant wave to convince yourself that the beat frequency is equal to the difference in frequency of the two component waves.

Music Link

When musicians playing in an ensemble are tuning their instruments, they listen for beats produced by the note they are sounding interfering with a reference note sounded by one musician. They then adjust their own instruments to reduce the beat frequency to zero. Which instrument in an orchestra plays the reference note, and why?

BEAT FREQUENCY

The beat frequency is the absolute value of the difference of the frequencies of the two component waves.

$$f_{beat} = |f_2 - f_1|$$

Quantity	Symbol	SI unit
beat frequency	f_{beat}	Hz (hertz)
frequency of one component wave	f_1	Hz (hertz)
frequency of other component wave	f_2	Hz (hertz)

MODEL PROBLEM

Finding the Unknown Frequency

A tuning fork of unknown frequency is sounded at the same time as one of frequency 440 Hz, resulting in the production of beats. Over 15 s, 46 beats are produced. What are the possible frequencies of the unknown-frequency tuning fork?

Frame the Problem

- Two tuning forks of different *frequency* are sounding at the same time.
- This results in the production of *beats*.
- The absolute value of the difference between the two *frequencies* is equal to the beat *frequency*.

Identify the Goal

The possible frequencies, f_2, of the unknown-frequency tuning fork

Variables and Constants

Known	Unknown
$f_1 = 440$ Hz	f_2
$N = 46$	f_{beat}
$\Delta t = 15$ s	

Strategy

You can find the beat *frequency* because you know the number of beats and the time interval. Substitute these values into the equation.

The absolute value of the difference between the two frequencies is equal to the beat frequency. Substitute the known values into the equation.

Simplify. Notice that when you remove the absolute value sign, you cannot know whether the value is positive or negative. Therefore, use both possibilities.

Using the positive value

Using the negative value

The frequency of the second tuning fork is either 443 Hz or 437 Hz.

Calculations

$$f_{beat} = \frac{N}{\Delta t}$$

$$f_{beat} = \frac{46\text{ beats}}{15\text{ s}}$$

$$f_{beat} = 3.1\text{ Hz}$$

$$f_{beat} = |f_2 - f_1|$$

$$3.1\text{ Hz} = |f_2 - 440\text{ Hz}|$$

$$|f_2 - 440\text{ Hz}| = \pm 3.1\text{ Hz}$$

$$f_2 - 440\text{ Hz} = 3.1\text{ Hz}$$
$$f_2 = 440\text{ Hz} + 3.1\text{ Hz}$$
$$f_2 = 443.1\text{ Hz}$$

$$f_2 - 440\text{ Hz} = -3.1\text{ Hz}$$
$$f_2 = 440\text{ Hz} - 3.1\text{ Hz}$$
$$f_2 = 436.9\text{ Hz}$$

Validate

Having 46 beats in 15 s gives a beat frequency of about 3 Hz, but the beat frequency is equal to the absolute value of the difference between the two tuning forks. Thus, the unknown frequency is either 3 Hz above 440 Hz or 3 Hz below 440 Hz. It is either 443 Hz or 437 Hz.

PROBLEM TIP

The beat frequency provides you with only the *difference* in frequency between two sounds. If the frequency of one of the sounds is known, then the other sound could have one of two possible frequencies — one higher and one lower than the known frequency. You would need some additional information to determine which of these two possible frequencies is the right one.

PRACTICE PROBLEMS

24. Two tuning forks of frequencies 512 Hz and 518 Hz are sounded at the same time.

(a) Describe the resultant sound.

(b) Calculate the beat frequency.

25. A 440 Hz tuning fork is sounded at the same time as a 337 Hz tuning fork. How many beats will be heard in 3.0 s?

26. A trumpet player sounds a note on her trumpet at the same time as middle C is played on a piano. She hears 10 beats over 2.0 s. If the piano's middle C has been tuned to 256 Hz, what are the possible frequencies of the note she is sounding?

27. A string on an out-of-tune piano is struck at the same time as a 440 Hz tuning fork is sounded. The piano tuner hears 12 beats in 4.0 s. He then slightly increases the tension in the string in order to increase the pitch of the note. Now he hears 14 beats in 4.0 s.

(a) What was the original frequency of the string on the out-of-tune piano?

(b) Is the piano more or less in tune after he tightens the string? Explain.

Figure 9.45 You can usually hear a siren long before you see the emergency vehicle, because sound can "bend" around corners.

Diffraction of Sound and Light

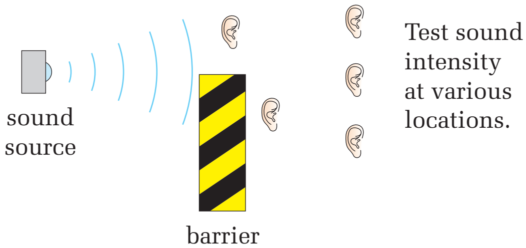
A siren pierces the serenity of a quiet evening. Although you are unable to see the emergency vehicle generating the noise, you are certainly able to hear it. The sound is able to bend around corners, pass through doorways, and eventually reach your ears. The ability of sound energy to bend around corners and spread around barriers is not only a property of sound, but is also a property of all waves. However, it is much more obvious for sound.

QUICK LAB: Diffraction of Sound

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

Diffraction is readily observed in mechanical waves. In this activity, you will study some variables associated with diffraction of sound waves.

Using the wave model for sound, predict how sound intensity will vary (e.g., sharply, gradually) behind the edge of a solid barrier and how changes in wavelength will influence the results. Use an audio frequency generator connected to a single speaker as a sound source. Use a relatively sound-proof barrier, such as a wall with a wide door, to test your predictions. A door opening into a large open space, as shown in the diagram, will reduce the amount of reflection from walls and will therefore yield the best results. Select and maintain a single, relatively low intensity (volume) to reduce effects produced by reflection of sound off nearby objects.



Carefully analyze how the intensity of the source varies at different locations behind the barrier, as shown in the diagram. Experiment to see how wavelength affects the amount of diffraction.

Analyze and Conclude

1. Were your predictions about the diffraction of sound accurate? Explain.
2. Describe and illustrate how the sound intensity varied at different locations behind the edge of the barrier.
3. Does varying the wavelength of the source affect the amount of sound wave diffraction? Explain and provide evidence.
4. Do your results validate the wave model of sound? Explain.
5. Suggest why only one speaker is used in this activity. Include the principle of superposition of waves in your answer.

A Definitive Experiment

Bending around corners — a form of diffraction — is a property of all waves. However, scientists studying light at the time of Newton were not able to detect any significant diffraction of light. To determine with confidence whether light behaved like a wave or a particle, scientists needed a carefully planned experiment that could clearly show evidence or lack of evidence of interference of light. To visualize the type of experiment that would be definitive, observe the pattern of water waves in Figure 9.46, which results from two point sources creating a periodic disturbance.

Figure 9.46 Nodal lines, resulting from total destructive interference, are clearly visible, radiating outward from between the two sources.

In Figure 9.46, you can see lines emanating from the sources that show constructive interference — standing waves — and destructive interference — no movement of the water. If light behaves like a wave, a similar experiment should reveal bright areas — constructive interference — and dark areas — destructive interference — on a screen. Figure 9.47 shows how interference resulting from two light sources would create light and dark regions.

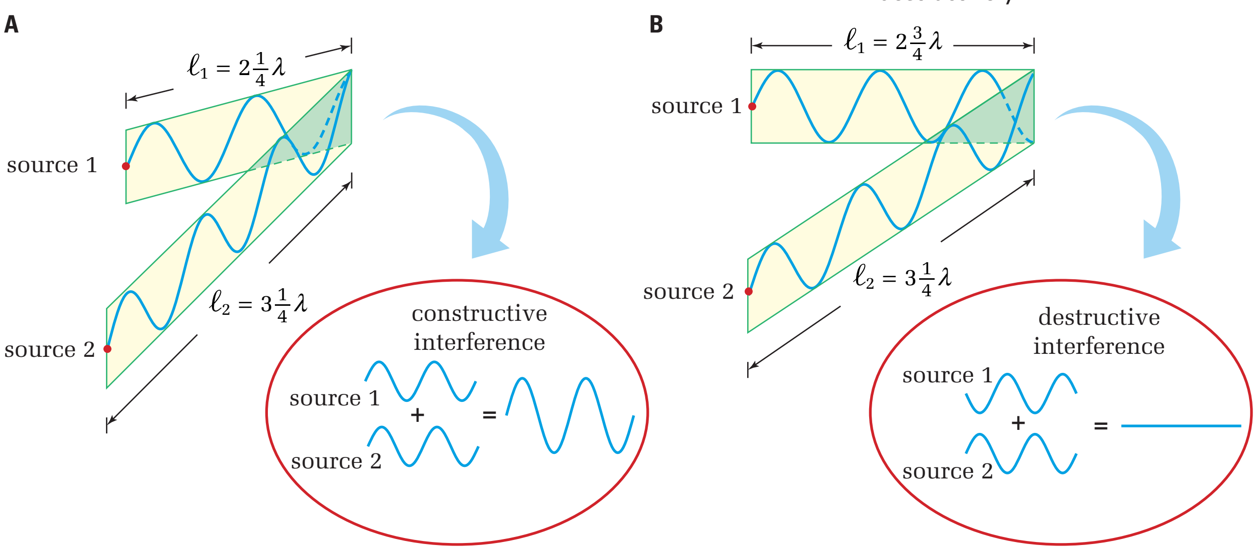


Figure 9.47 **(A)** Waves from each source with a path difference of whole-number multiples of wavelength interfere constructively. **(B)** Waves from each source with a path difference of multiples of one half wavelength interfere destructively.

PHYSICS FILE

In the seventeenth century, Huygens, a Dutch physicist, developed the first significant theory proposing that light travelled as a wave. Scientists supporting this theory believed that light — now known to be electromagnetic waves — travelling through space from the Sun must also travel through a medium. By the mid-nineteenth century the wave theory was well established, and scientists "invented" a medium, called "ether," that was considered to permeate the entire universe. The electromagnetic waves were called "ether waves." In 1887, a series of brilliant experiments by Michelson and Morley proved conclusively that ether did not exist, and it was established that electromagnetic waves like light could, in fact, travel through a vacuum without the need of a medium of any kind.

Examination of Figure 9.47 reveals two important features that must be designed into the experiment. First, the sources must produce coherent waves. **Coherent** sources produce waves of the same frequency and in phase with each other. Second, the distance between the sources must be of the order of magnitude of the wavelength of the waves. If the sources are placed too far apart, the light and dark areas on a screen will be too close together to be observed. (As a rule of thumb, the sources must be no farther than 10 wavelengths apart.)

These conditions were exceedingly difficult for scientists to create in the 1700s. Physicists could produce light of one frequency by passing it through a prism but, before the invention of the laser, coherent light sources did not exist. The phases of light emanating from a source were random. Therefore, constructive or destructive interference would occur in a random way and the effects would be an average of light and dark, so that they appeared to be uniform. In addition, since physicists did not even know whether light behaved like a particle or a wave, they had no way of knowing what the wavelength might be. It took nearly 100 years after Newton presented his model of light for the debate to be resolved.

Young's Double-Slit Experiment

Thomas Young (1773–1829) devised an ingenious experiment, as illustrated in Figure 9.48, that produced an interference pattern with light. Using one monochromatic light source, Young allowed the light to fall onto an opaque material with a single, narrow slit that acted as a new source. The light passing through the single slit spread as it travelled to a second opaque barrier. The second barrier had two narrow slits placed very closely together.

In part (B) of Figure 9.48, you can see that two parts of the same wavefront from the single slit reach the double slits at the same time. Since these two parts of the same wavefront behave as new sources at the double slits, the light leaving the double slits is essentially coherent. Young experimented with this set-up for more than two years before he realized that the double slits had to be so close together that they almost appeared to be one slit to the unaided eye. The light that passed through the double-slit barrier fell on a nearby screen, producing the historic pattern of light and dark lines caused by the interference of light waves. Young's results catapulted the wave model for light into centre stage, where it would remain unchallenged for more than 100 years.

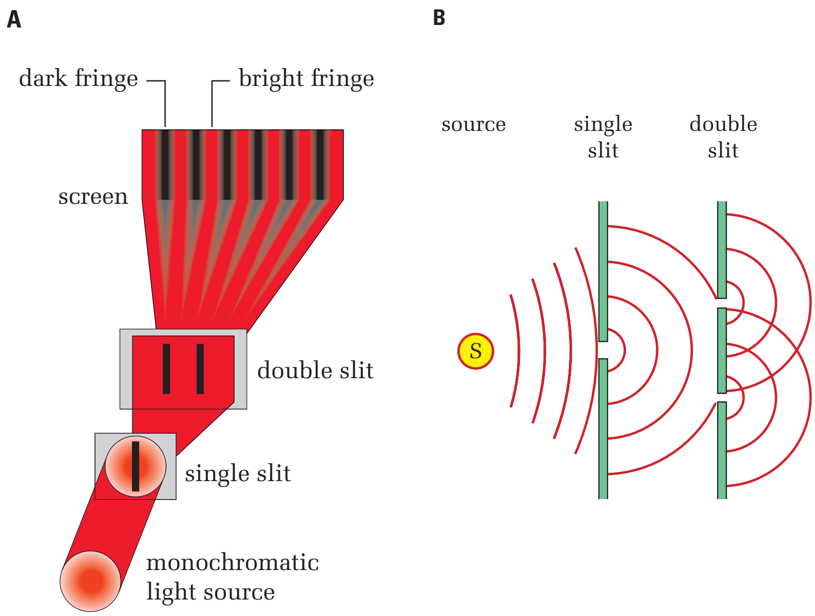


Figure 9.48 **(A)** Young's experiment used a single incandescent bulb and two narrow slits to produce coherent sources. He successfully showed that light could form an interference pattern similar to those produced with mechanical waves. **(B)** The wavefronts emanating from the double slits resemble the water waves generated by two point sources.

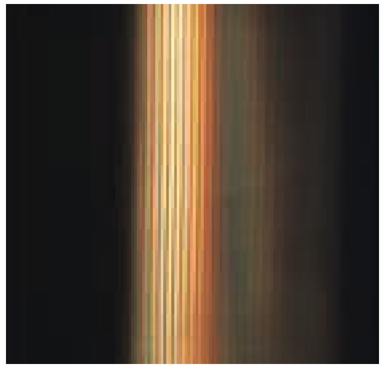


Figure 9.49 Photograph of an interference pattern from Young's experiment (notice how the intensity reduces toward the edges)

Young was successful where others had failed for several reasons.

- He used a monochromatic (single wavelength) light source.
- The double slits acted as two sources and were spaced much more closely together than was possible if two separate light sources were used.
- The light passing through the initial single slit acted as a point source. When a wavefront from the point source reached the double slits, two parts of the same wavefront became new sources for the double slits and were therefore coherent.

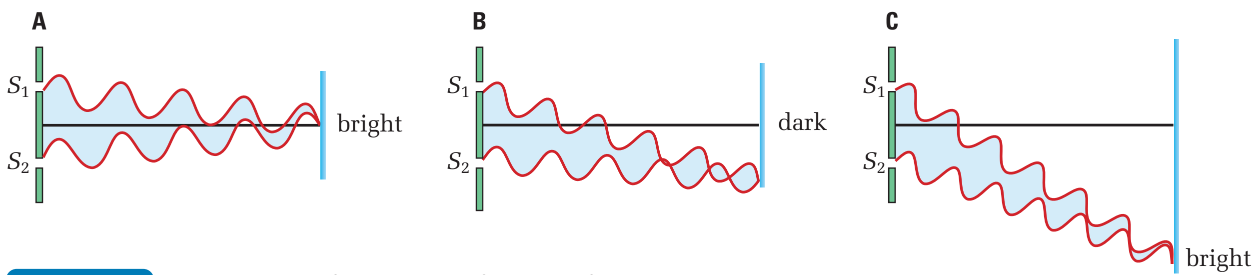


Figure 9.50 Light and dark fringes result from interfering waves.

To understand the mathematical analysis of the pattern produced by Young's double-slit experiment, examine Figure 9.51. In part (A), you see coherent light waves entering the two slits and passing through. Light leaves the slits in all directions, but you can study one direction at a time.

Since the distance between the slits and the screen (labelled x) is approximately a million times larger than the distance between the slits, you can assume that the waves leaving the slits parallel to each other will hit the screen at the same point. Part (B) is drawn to the scale of the slit-to-screen distance and therefore the two parallel rays appear as one line.

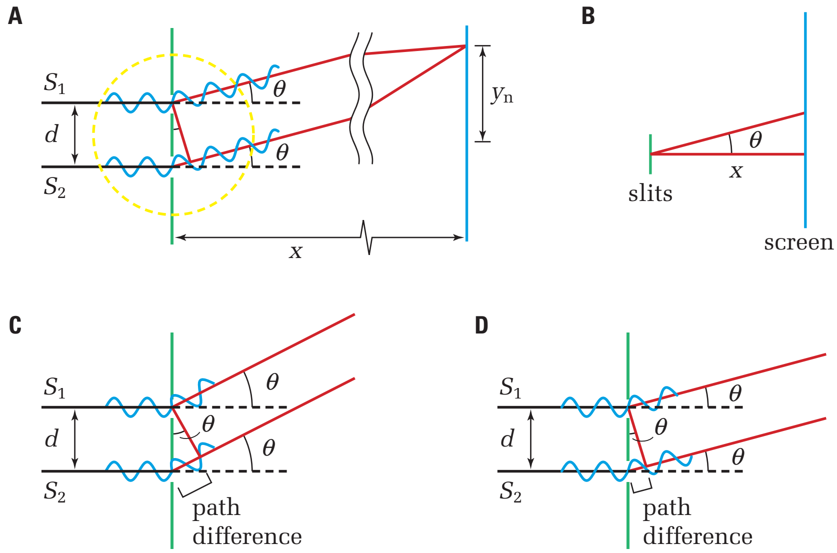


Figure 9.51 The path difference between slits that light travels to reach the screen is given by $d \sin \theta$.

Parts (C) and (D) of Figure 9.51 illustrate two special cases — constructive interference and destructive interference. Inspection of the right triangle in part (C) shows that the hypotenuse is the distance, d, between the two slits. One side is formed by a line drawn from slit 1 that is perpendicular to the ray leaving slit 2. The third side of the triangle is the distance that ray 2 must travel farther than ray 1 to reach the screen. When this path difference, PD, is exactly one wavelength, the two waves continue from the slit in phase and therefore experience constructive interference.

When the two rays reach the screen, they will produce a bright spot on the screen, called a bright **fringe**. Using trigonometry, you can see that the path difference is equal to $d \sin \theta$. In fact, if the path difference is any integer number of full wavelengths, the waves will remain in phase and will create a bright fringe on the screen. Notice that, from the geometry of the apparatus, the angle θ, formed by the slit separation and the perpendicular line between the light rays, is the same as the angle between the horizontal line going to the screen and the direction of the rays going toward the screen. The result of this analysis can be expressed mathematically as shown in the following box.

PROBEWARE

www.mcgrawhill.ca/links/atlphysics

If your school has probeware equipment, visit the above Internet site for an in-depth activity about the interference effects of light.

CONSTRUCTIVE INTERFERENCE

A bright fringe will appear on a screen when an integer number of wavelengths of light is equal to the product of the slit separation and the sine of the angle between the slit separation and the line perpendicular to the light rays leaving the slits.

$$n\lambda = d\sin\theta \qquad \text{where } n = 0, 1, 2, 3, \ldots .$$

Quantity	**Symbol**	**SI unit**
integer number of full wavelengths	n	none
wavelength of light	λ	m (metres)
distance between slits	d	m (metres)
angle between slit separation and line perpendicular to light rays	θ	unitless (degrees are not a unit)

Unit Analysis

metre = metre   m = m

Inspection of part (D) of Figure 9.51 shows that when the path difference is a half wavelength, the light waves that leave the slits are out of phase and experience destructive interference. When the waves reach the screen, they will cancel each other and the screen will be dark. Between the bright and dark fringes, the screen will appear to be shaded. A complete analysis shows that when the path difference is exactly half a wavelength more than any number of full wavelengths, the waves will destructively interfere and a dark spot or dark fringe will appear on the screen. This condition can be described mathematically as

$$\left(n - \tfrac{1}{2}\right)\lambda = d\sin\theta \qquad \text{where } n = 1, 2, 3, \ldots$$

In a typical experiment, you would not be able to measure the path difference or the angle θ. Instead, you would measure the distance y_n between the central bright fringe and another bright fringe of your choice. You could then determine the angle θ by applying trigonometry to part (B) of Figure 9.51, which gives

$$\tan\theta = \frac{y_n}{x}$$

In this expression, the variable n has the same meaning as it does in the previous relationships. When $n = 1$, the path difference is one full wavelength and y_1 describes the distance to the first bright fringe.

For very small angles, you can make an approximation that combines the two relationships above, as shown below.

- For very small angles, the sine of an angle is approximately equal to the tangent of the angle. $\sin\theta \cong \tan\theta$
- Using this approximation, you can write the expression for the wavelength, as shown. $n\lambda \cong d\tan\theta$
- Substitute the expression for $\tan\theta$ and substitute into the equation for the wavelength. $\tan\theta = \frac{y_n}{x}$ $\quad n\lambda \cong d\frac{y_n}{x}$

You can set n equal to 1 by using the distance between adjacent fringes and obtain the relationship shown in the following box.

APPROXIMATION OF THE WAVELENGTH OF LIGHT

The wavelength of light is approximately equal to the product of the distance between fringes and the distance between slits, divided by the slit-to-screen distance.

$$\lambda \cong \frac{\Delta y d}{x}$$

Quantity	Symbol	SI unit
wavelength	λ	m (metres)
distance separating adjacent fringes	Δy	m (metres)
distance between slits	d	m (metres)
distance from source to screen	x	m (metres)

Note 1: The distance between nodal line centres is identical to the distance between bright fringe centres. Therefore, this relationship applies equally to dark fringes (nodal lines) and bright fringes.

Note 2: This relationship is based on an approximation. Use it only for very small angles.

Young's double-slit experiment convinced physicists that light does, in fact, travel like a wave. The wave theory of light went undisputed for about two hundred years before some observations were made that could not be explained by the wave theory. You will learn about these observations in Chapter 18.

INVESTIGATION 9-C

Young's Double-Slit Experiment

TARGET SKILLS
- Predicting
- Identifying variables
- Communicating results

Young's ingenious double-slit experiment is readily duplicated with only simple equipment. In this investigation, you will reproduce results similar to those that Young produced — the results that convinced the scientific community that light was a wave.

Problem

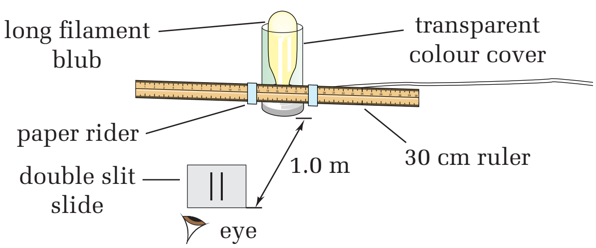
Is it possible to produce an interference pattern using light?

Prediction

- Make a prediction about the requirements of the experimental design that will be necessary to produce an interference pattern based on the wavelength of light and the nature of incandescent light sources.
- Make a second prediction about the nature of an interference pattern produced with short wavelength light (such as blue or green) compared to longer wavelength light (such as yellow or red). Which wavelength will allow you to make the most accurate measurements? Explain your prediction in detail.

Equipment

- long filament light source
- magnifying glass
- double-slit slides
- transparent colour light covers
- metre stick
- ruler with fractions of a millimetre markings
- 30 cm ruler



Procedure

1. Using a magnifying glass and finely ruled ruler, measure and record the centre-to-centre width of the slit separation.
2. Cut two paper riders for the 30 cm ruler to mark the width of the observed interference pattern.
3. Place a transparent colour cover over the portion of the light bulb with the straightest filament.
4. Place the 30 cm ruler with paper riders in front of the bulb. With your eye exactly 1 m from the bulb, observe the filament through the double slits.
5. Count the number of bright *or* dark fringes that you are able to clearly distinguish. Use the paper riders to mark off the edges of the observed fringes.
6. Repeat the experiment, varying the slit width and the wavelength of light.

Analyze and Conclude

1. Describe the effect on the observed interference pattern of **(a)** altering the slit width and **(b)** altering the wavelength of light.
2. Use your data to determine the wavelength of light used for each trial. How well did your calculated wavelength compare to expected values?
3. Was it easier to obtain data on one wavelength than on the others? If so, was your original prediction accurate? Explain.

Apply and Extend

CAUTION *Do not* look directly into the laser.

4. If time permits, use a helium-neon laser to verify the double-slit equation. Place a double-slit slide of known width in front of the laser beam. Observe the interference pattern on a screen a known distance from the laser. Calculate the wavelength of laser light. Determine the percentage deviation of the calculated value and your experimental value.

CAREERS IN PHYSICS

Sounds from the Seabed

The survey ship glides across the inky-blue ocean. Sensors on its hull sweep back and forth, sending sound waves to the seabed and receiving the sound waves that bounce back. This is multi-beam sonar (SOund Navigation And Ranging). The exact amount of sound energy bouncing back is determined by what is on the seabed. Harder materials send back more energy than softer ones. The sensors on the hull, called transducers, convert the sound waves they receive into electrical signals. These are processed by computer to produce three-dimensional, photographic-quality maps of what is on and beneath the seabed, perhaps an offshore oil pipeline, debris from a downed aircraft, or a sea cage used in fish farming.

Scientists in the University of New Brunswick's Geodesy and Geomatics Engineering Department (GGE) are among those who collect, process, and interpret such data. Dr. Susan E. Nichols and her colleagues apply their expertise to projects involving ocean governance. For example, countries sometimes disagree over offshore boundaries. Fish farmers sometimes conflict with people in traditional fisheries. Federal, provincial, and municipal governments want to ensure that petroleum development will not conflict with property rights or harm environmentally sensitive areas. In such cases, three-dimensional maps like those from the GGE are a valuable basis for discussing and resolving conflicts.

Besides ocean mapping, other uses for sonar include finding and sizing schools of fish, detecting submarines or icebergs, and determining whether there are valuable gravel, gold, or other mineral deposits in the ocean floor.

9.3 Section Review

1. C Explain the difference between music and noise.
2. K/U In a closed air column that is generating only the fundamental frequency, what is the relationship between the length of the air column and the wavelength of the sound?
3. K/U What are the conditions for resonance at the ends of an open air column?
4. C Explain the difference between a harmonic and an overtone.
5. C Using diagrams, explain how beats are created.
6. K/U How is the beat frequency related to the frequencies of the two individual tones?
7. MC How do orchestra musicians use the concept of beats to tune their instruments?
8. C Explain why it was so difficult to demonstrate interference of light waves for physicists in the 1700s.
9. C How did Young overcome the problem caused by the lack of a source of coherent light?
10. I Do research to learn about diffraction gratings. How is the pattern created by a diffraction grating related to the pattern created by a double slit?