## Conservation of Momentum

## SECTION

## OUTCOMES

- apply qualitatively the law of conservation of momentum to one-dimensional collisions and recoil.
- Determine experimentally whether a collision is elastic or inelastic


## K E Y

TERMS

- elastic collisions
- inelastic collisions
- conservation of momentum

When the cue ball hits the eight ball in billiards, the eight ball hits the cue ball. When a rock hits the ground, the ground hits the rock. In any collision, two objects exert forces on each other. Collisions are another type of process in which conservation laws play an essential role. As is the case with all processes, total energy is conserved in a collision. If the materials involved in the collision are perfectly elastic, mechanical energy - more specifically kinetic energy - is conserved. This condition divides collisions into two categories, elastic collisions in which kinetic energy is conserved and inelastic collisions in which kinetic energy is not conserved.

Another important conservation law is also extremely useful in analyzing collisions - the law of conservation of momentum. In Chapter 5, you discovered that rearranging Newton's second law led to the concepts of impulse and momentum. You learned about the impulse-momentum theorem which states that the impulse exerted on an object is equal to the change in the momentum of the object during a collision. The impulse-momentum theorem can be stated mathematically as $F \Delta t=m v_{2}-m v_{1}$. In this chapter, you will take the process one step further and apply Newton's third law to the impulse-momentum theorem.


Figure 7.9 The game of billiards offers many excellent examples of collisions.

## Newton's Third Law and Momentum

Newton's third law states that "For every action force on object B due to object $A$, there is a reaction force, equal in magnitude but opposite in direction, acting on object A due to object B." Unlike

Newton's second law focusses on the motion of one specific object but his third law deals with the interaction between two objects. When you apply Newton's third law to collisions, you discover one of the most important laws of physics - the law of conservation of momentum. The following steps, along with the diagram in Figure 7.10, show you how to derive the law of conservation of momentum by applying Newton's third law to a collision between two objects.


- Write the impulse-momentum theorem for each of two objects, A and B, that collide with each other.
- Apply Newton's third law to the forces that A and B exert on each other.
- The duration of the collision is the same for both objects. Therefore, you can multiply both sides of the equation above by $\Delta t$.
- Substitute the expressions for change in momentum in the first step into the equation in the third step and then simplify.
- Algebraically rearrange the last equation so that (1) the terms representing the before-collision conditions precede the equals sign and (2) the terms for the after-collision conditions follow the equals sign.

Figure 7.10 Object A exerts a force on object $B$, causing a change in $\mathrm{B}^{\prime} \mathrm{s}$ momentum. At the same time, object $B$ exerts a force equal in magnitude and opposite in direction on object A, changing A's momentum.

$$
\begin{aligned}
& \vec{F}_{\mathrm{A}} \Delta t=m_{\mathrm{A}}{\overrightarrow{V_{A}} 2}-m_{\mathrm{A}}{\overrightarrow{V_{\mathrm{A}} 1}} \\
& \vec{F}_{\mathrm{B}} \Delta t=m_{\mathrm{B}} \overrightarrow{\mathrm{~V}}_{\mathrm{B} 2}-m_{\mathrm{B}} \vec{V}_{\mathrm{B} 1}
\end{aligned}
$$

$$
\vec{F}_{\mathrm{A}}=-\vec{F}_{\mathrm{B}}
$$

$$
\vec{F}_{\mathrm{A}} \Delta t=-\vec{F}_{\mathrm{B}} \Delta t
$$

$$
m_{\mathrm{A}} \overrightarrow{\mathrm{~V}}_{\mathrm{A} 1}+m_{\mathrm{B}} \overrightarrow{\mathrm{~V}}_{\mathrm{B} 1}=m_{\mathrm{A}} \vec{v}_{\mathrm{A} 2}+m_{\mathrm{B}}{\overrightarrow{V_{\mathrm{B}}} 2}
$$

The last equation is a mathematical expression of the law of conservation of momentum, which states that the total momentum of two objects before a collision is the same as the total momentum of the same two objects after they collide.

$$
\begin{aligned}
& m_{\mathrm{A}} \overrightarrow{\mathrm{~V}}_{\mathrm{A} 2}-m_{\mathrm{A}} \overrightarrow{\mathrm{~V}}_{\mathrm{A} 1}=-\left(m_{\mathrm{B}}{\overrightarrow{V_{B}} 2}-m_{\mathrm{B}}{\overrightarrow{V_{B}}}\right) \\
& m_{\mathrm{A}}{\overrightarrow{V_{\mathrm{A}}} 2}-m_{\mathrm{A}}{\overrightarrow{V_{\mathrm{A}}} 1}=-m_{\mathrm{B}}{\overrightarrow{V_{\mathrm{B}}} 2}+m_{\mathrm{B}}{\overrightarrow{V_{\mathrm{B}}} 1}
\end{aligned}
$$

## PHYSICS FILE

When working with collisions, instead of using subscripts such as "2," physicists often use a superscript symbol called a "prime," which looks like an apostrophe, to represent the variables after a collision. The variable is said to be "primed." Look for this notation in the box on the right.

COURSE CHALLENGE: SCANNING TECHNOLOGIES

## Momentum

A moving planet, bowling ball, and an electron share the property of momentum. Learn more about momentum conservation as it relates to the Course Challenge in your e-book.

## LAW OF CONSERVATION OF MOMENTUM

The sum of the momenta of two objects before a collision is equal to the sum of their momenta after they collide.

$$
m_{\mathrm{A}}{\overrightarrow{v_{\mathrm{A}}}}+m_{\mathrm{B}}{\overrightarrow{v_{\mathrm{B}}}}=m_{\mathrm{A}} \vec{v}_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} \vec{V}_{\mathrm{B}}^{\prime}
$$

## Quantity

mass of object A
mass of object B
velocity of object $A$ before the collision velocity of object $B$ before the collision velocity of object $A$ after the collision velocity of object B after the collision

| Symbol | SI unit |
| :---: | :--- |
| $m_{\mathrm{A}}$ | kg (kilograms) |
| $m_{\mathrm{B}}$ | kg (kilograms) |

$\vec{V}_{\mathrm{A}} \quad \frac{\mathrm{m}}{\mathrm{s}}$ (metres per second)
$\vec{V}_{B} \quad \frac{m}{\mathrm{~S}}$ (metres per second)
$\stackrel{\rightharpoonup}{V}_{\mathrm{A}}^{\prime} \quad \frac{\mathrm{m}}{\mathrm{s}}($ metres per second $)$
$\vec{V}_{\mathrm{B}}^{\prime} \quad \frac{\mathrm{m}}{\mathrm{s}}($ metres per second $)$

The law of conservation of momentum can be broadened to more than two objects by selecting all of the objects in an isolated system. To demonstrate that the momentum of an isolated system is conserved, start with the impulse-momentum theorem, where $\vec{p}_{\text {sys }}$ represents the total momentum of all of the objects within the system.

- An impulse on a system due to an external force causes a change in the momentum of the system.

$$
\vec{F}_{\text {ext }} \Delta t=\Delta \vec{p}_{\text {sys }}
$$

- If a system is isolated, the net external force acting on the system is zero.

$$
(0.0 \mathrm{~N}) \Delta \mathrm{t}=\Delta \vec{p}_{\text {sys }}
$$

- If the impulse is zero, the change in momentum must be zero.

$$
\Delta \stackrel{\rightharpoonup}{p}_{\mathrm{sys}}=0.0 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}
$$

The last expression is an alternative form of the equation for the conservation of momentum. The equation states that the change in momentum of an isolated system is zero. The particles or objects within the system might interact with each other and exchange momentum, but the total momentum of the isolated system does not change.

In reality, systems are rarely perfectly isolated. In nearly all real situations, immediately after a collision, frictional forces and interactions with other objects change the momentum of the objects involved in the collision. Therefore, it might appear that the law of conservation of momentum is not very useful. However, the law always applies to a system from the instant before to the instant after a collision. If you know the conditions just before a collision, you can always use conservation of momentum to determine the momentum and, thus, velocity of an object at the instant after a collision. Often, these values are all that you need to know.

## Collisions in One Dimension

Since momentum is a vector quantity, both the magnitude and the direction of the momentum must be conserved. Therefore, momentum is conserved in each dimension, independently. For complex situations, it is often convenient to separate the momentum into its components and work with each dimension separately. Then you can combine the results and find the resultant momentum of the objects in question. Solving problems that involve only one dimension is good practice for tackling more complex problems.


Figure 7.11 A moving car and its occupants can be defined as being a system. The children in the car might be exerting forces on each other or on objects that they are handling. Although they are exchanging momentum between themselves and the objects, these changes have no effect on the total momentum of the system.

## MODEL PROBLEM

## Analyzing a Collision between Boxcars

A $1.75 \times 10^{4} \mathrm{~kg}$ boxcar is rolling down a track toward a stationary boxcar that has a mass of $2.00 \times 10^{4} \mathrm{~kg}$. Just before the collision, the first boxcar is moving east at $5.45 \mathrm{~m} / \mathrm{s}$. When the boxcars collide, they lock together and continue down the track. What is the velocity of the two boxcars immediately after the collision?

## Frame the Problem

- Make a sketch of the momentum vectors representing conditions just before and just after the collision.
- Before the collision, only one boxcar (A) is moving and therefore has momentum.
- At the instant of the collision, momentum is conserved.


$$
\begin{array}{lll}
\vec{V}_{\mathrm{A}}=5.45 \frac{\mathrm{~m}}{\mathrm{~s}} & \vec{V}_{\mathrm{B}}=0.00 \frac{\mathrm{~m}}{\mathrm{~s}} & \stackrel{\rightharpoonup}{V}_{\mathrm{AB}}^{\prime}=? \\
m_{\mathrm{A}}=1.75 \times 10^{4} \mathrm{~kg} & m_{\mathrm{B}}=2.00 \times 10^{4} \mathrm{~kg} &
\end{array}
$$

- After the collision, the two boxcars (A and B) move as one mass, with the same velocity.


## Identify the Goal

The velocity, $\vec{v}_{A B}^{\prime}$, of the combined boxcars immediately after the collision

## Variables and Constants

## Known

$m_{\mathrm{A}}=1.75 \times 10^{4} \mathrm{~kg}$
$m_{\mathrm{B}}=2.00 \times 10^{4} \mathrm{~kg}$
$\vec{V}_{\mathrm{A}}=5.45 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]$

## Implied

$\overrightarrow{V_{B}}=0.00 \frac{\mathrm{~m}}{\mathrm{~s}}$

## Unknown

$\vec{V}^{\prime}{ }_{\text {AB }}$

## Strategy

Apply the law of conservation of momentum.

After the collision, the two masses act as one, with one velocity. Rewrite the equation to show this condition.
Solve for $\vec{V}_{A B}^{\prime}$.

Substitute values and solve.

## Calculations

$$
\begin{aligned}
& m_{A} \vec{V}_{\mathrm{A}}+m_{\mathrm{B}}{\overrightarrow{V_{B}}}_{\mathrm{B}}=m_{\mathrm{A}}{\overrightarrow{V_{A}^{\prime}}}^{\prime}+m_{\mathrm{B}}{\overrightarrow{V_{B}}}^{\prime} \\
& m_{A} \vec{V}_{A}+m_{B}{\overrightarrow{V_{B}}}_{B}=\left(m_{A}+m_{B}\right) \vec{v}_{A B}^{\prime}
\end{aligned}
$$

$$
\begin{aligned}
& \vec{V}_{A B}^{\prime}=\frac{\left(1.75 \times 10^{4} \mathrm{~kg}\right)\left(5.45 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]\right)+\left(2.00 \times 10^{4} \mathrm{~kg}\right)\left(0.00 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{\left(1.75 \times 10^{4} \mathrm{~kg}+2.00 \times 10^{4} \mathrm{~kg}\right)} \\
& \vec{V}_{A B}^{\prime}=\frac{9.5375 \times 10^{4} \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]}{3.75 \times 10^{4} \mathrm{~kg}} \\
& \vec{V}_{A B}^{\prime}=2.543 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}] \\
& \vec{V}_{\mathrm{AB}}^{\prime} \cong 2.54 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]
\end{aligned}
$$

The locked boxcars were rolling east down the track at $2.54 \mathrm{~m} / \mathrm{s}$.

## Validate

The combined mass of the boxcars was nearly double the mass of the boxcar that was moving before the collision. Since the exponents of mass and velocity are always one, making the relationships linear, you would expect that the velocity of the combined boxcars would be just under half of the velocity of the single boxcar before the collision. Half of $5.45 \mathrm{~m} / \mathrm{s}$ is approximately $2.7 \mathrm{~m} / \mathrm{s}$. The calculated value of $2.54 \mathrm{~m} / \mathrm{s}$ is very close to what you would expect.

## PRACTICE PROBLEMS

25. Claude and Heather are practising pairs skating for a competition. Heather ( 47 kg ) is skating with a velocity of $2.2 \mathrm{~m} / \mathrm{s}$. Claude ( 72 kg ) is directly behind her, skating with a velocity of $3.1 \mathrm{~m} / \mathrm{s}$. When he reaches her, he holds her waist and they skate together. At the instant after he takes hold of her waist, what is their velocity?
26. Two amusement park "wrecker cars" are heading directly toward each other. The combined mass of car A plus driver is 375 kg and it is moving with a velocity of $+1.8 \mathrm{~m} / \mathrm{s}$. The combined mass of car B plus driver is 422 kg and it is moving with a velocity of $-1.4 \mathrm{~m} / \mathrm{s}$. When they collide, they attach and continue moving along the same straight line. What is their velocity immediately after they collide?

## Recoil

Imagine yourself in the situation illustrated in Figure 7.12. You are in a small canoe with a friend and you decide to change places. Assume that the friction between the canoe and the water is negligible. While the canoe is not moving in the water, you very carefully stand up and start to take a step. You suddenly have the sense that the boat is moving under your feet. Why?


Figure 7.12 If you start to step forward in a canoe, the canoe recoils under your feet.

When you stepped forward, your foot pushed against the bottom of the canoe and you started to move. You gained momentum due to your velocity. Momentum of the system - you, your friend, and the canoe - must be conserved, so the canoe started to move in the opposite direction. The interaction that occurs when two stationary objects push against each other and then move apart is called recoil. You can use the equation for conservation of momentum to solve recoil problems, as the following problem illustrates.

## MODEL PROBLEM

## Recoil of a Canoe

For the case described in the text, find the velocity of the canoe and your friend at the instant that you start to take a step, if your velocity is $0.75 \mathrm{~m} / \mathrm{s}$ [forward]. Assume that your mass is 65 kg and the combined mass of the canoe and your friend is 115 kg .

## Frame the Problem

- Make a simple sketch of the conditions before and after you took a step.

| before |  |  |
| :--- | :--- | :--- |
| $m_{\mathrm{A}}=65 \mathrm{~kg}$ | $m_{\mathrm{B}}=115 \mathrm{~kg}$ | $m_{\mathrm{A}}=65 \mathrm{~kg}$ |
| $\vec{V}_{\mathrm{A}}=0.00 \frac{\mathrm{~m}}{\mathrm{~s}}$ | $\vec{V}_{\mathrm{B}}=0.00 \frac{\mathrm{~m}}{\mathrm{~s}}$ | $\vec{V}_{\mathrm{B}}^{\prime}=115 \mathrm{~kg}$ |
|  |  |  |

- The canoe was not moving when you started to take a step.
- You gained momentum when you started to move. Label yourself "A" and consider the direction of your motion to be positive.
- The canoe had to move in a negative direction in order to conserve momentum. Label the canoe and your friend "B."


## Identify the Goal

The initial velocity, $\vec{V}_{B}^{\prime}$ of the canoe and your friend

## Variables and Constants

## Known

$$
\begin{aligned}
& m_{\mathrm{A}}=65 \mathrm{~kg} \quad \vec{v}_{\mathrm{A}}^{\prime}=0.75 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& m_{\mathrm{B}}=115 \mathrm{~kg}
\end{aligned}
$$

## Strategy

Apply conservation of momentum.
Velocities before the interaction were zero; therefore, the total momentum before the interaction was zero. Set these values equal to zero and solve for the velocity of $B$ after the reaction.

$$
\begin{array}{ll}
\text { Implied } & \text { Unknown } \\
\vec{V}_{A}=0.00 \frac{\mathrm{~m}}{\mathrm{~s}} & \vec{V}_{B}^{\prime} \\
\vec{V}_{\mathrm{B}}=0.00 \frac{\mathrm{~m}}{\mathrm{~s}} &
\end{array}
$$

## Calculations

$$
\begin{aligned}
& m_{\mathrm{A}} \vec{V}_{\mathrm{A}}+m_{\mathrm{B}} \vec{V}_{\mathrm{B}}=m_{\mathrm{A}} \vec{V}_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} \vec{V}_{\mathrm{B}}^{\prime} \\
& 0.0 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}=m_{\mathrm{A}} \vec{V}_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} \vec{V}_{\mathrm{B}}^{\prime} \\
& m_{\mathrm{A}} \vec{V}_{\mathrm{A}}^{\prime}=-m_{\mathrm{B}} \vec{V}_{\mathrm{B}}^{\prime} \\
& \vec{V}_{\mathrm{B}}^{\prime}=-\frac{m_{\mathrm{A}} \vec{V}_{\mathrm{A}}^{\prime}}{m_{\mathrm{B}}}
\end{aligned}
$$

Substitute values and solve.

$$
\begin{aligned}
& \vec{V}_{\mathrm{B}}^{\prime}=-\frac{(65 \mathrm{~kg})\left(0.75 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{115 \mathrm{~kg}} \\
& {\overrightarrow{V_{B}^{\prime}}}^{\prime}=-0.4239 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& {\overrightarrow{V_{B}^{\prime}}}^{\prime} \cong-0.42 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

The velocity of the canoe and your friend, immediately after you started moving, was $-0.42 \mathrm{~m} / \mathrm{s}$.

## Validate

Since the mass of the canoe plus your friend was larger than your mass, you would expect that the magnitude of their velocity would be smaller, which it was. Also, the direction of the velocity of the canoe plus your friend must be negative, that is, in a direction opposite to your direction. Again, it was.

## PRACTICE PROBLEMS

27. A 1385 kg cannon containing a 58.5 kg cannon ball is on wheels. The cannon fires the cannon ball, giving it a velocity of $49.8 \mathrm{~m} / \mathrm{s}$ north. What is the initial velocity of the cannon the instant after it fires the cannon ball?

28. While you are wearing in-line skates, you are standing still and holding a 1.7 kg rock. Assume that your mass is 57 kg . If you throw the rock directly west with a velocity of $3.8 \mathrm{~m} / \mathrm{s}$, what will be your recoil velocity?
29. The mass of a uranium- 238 atom is $3.95 \times 10^{-25} \mathrm{~kg}$. A stationary uranium atom emits an alpha particle with a mass of $6.64 \times 10^{-27} \mathrm{~kg}$. If the alpha particle has a velocity of $1.42 \times 10^{4} \mathrm{~m} / \mathrm{s}$, what is the recoil velocity of the uranium atom?

TARGET SKILLS
Predicting

- Performing and recording
- Analyzing and interpreting

Are there characteristics that allow you to predict whether a collision will be elastic? In this investigation, you will observe and analyze several collisions and draw conclusions regarding whether a type of collision will be elastic or inelastic. If you do not have the necessary equipment, you can find excellent simulations on the Internet.

## Problem

What are the characteristics of elastic and inelastic collisions?

## Equipment

- air track (with source of compressed air)
- 2 gliders (identical, either middle- or large-sized)
- 2 photogate timers
- laboratory balance
- 4 glider bumper springs
- 2 Velcro ${ }^{\text {TM }}$ bumpers (or a needle and a piece of wax)
- 2 velocity flags ( 10 cm ) (or file cards cut to a 10 cm length)
- modelling clay


## Procedure

1. Set up the air track and adjust the levelling screw to ensure that the track is horizontal. You can test whether the track is level by turning on the air pressure and placing a glider on the track. Hold the glider still and then release it. If the track is level, the glider will remain in place. If the glider gradually starts moving, the air track is not level.
2. Attach a velocity flag (or 10 cm card) and two bumper springs to each glider. If only one bumper spring is attached, the glider might not be properly balanced.
3. Position the photogates about one fourth the length of the track from each end, as shown in the diagram. Adjust the height of the
photogates so that the velocity flags will pass through the gates smoothly but will trigger the gates.

4. Label one glider "A" and the other glider "B." Use the laboratory balance to determine accurately the mass of each glider.
5. With the air flowing, place glider A on the left end of the air track and glider B in the centre.
6. Perform a test run by pushing glider A so that it collides with glider B. Ensure that the photogates are placed properly so that the flags are not inside the gates when the gliders are in contact. Adjust the positions of the photogates, if necessary.
7. The first set of trials will be like the test run, with glider A on the left end of the track and glider B in the centre. Turn on the photogates and press the reset button. Push glider A and allow it to collide with glider B. Allow both gliders to pass through a photogate after the collision, then catch them before they bounce back and pass through a photogate again. Record the data in a table similar to the one shown on the next page. Since all of the motion will be in one dimension, only positive and negative signs will be needed to indicate direction. Vector notations will not be necessary.

The displacement, $\Delta d$, is the distance that the gliders travelled while passing through the photogates. This displacement is the length of the flag. Time $\Delta t_{\mathrm{i}}$ is the time that a glider spent in the photogate before the
collision, while $\Delta t_{\mathrm{f}}$ is the time the glider took to pass through the photogate after the collision. Calculate velocity, $v$, from the displacement and the time interval. Be sure to include positive and negative signs.

| Glider A (mass =?) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Trial | $\Delta \vec{d}$ | $\Delta t_{\mathrm{i}}$ | $\overrightarrow{v_{\mathrm{i}}}$ | $\Delta t_{\mathrm{f}}$ | $\overrightarrow{v_{\mathrm{f}}}$ |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |


| Glider B (mass = ?) |  |  |  |
| :---: | :---: | :---: | :---: |
| Trial | $\Delta \vec{d}$ | $\Delta t_{\mathrm{f}}$ | $\overrightarrow{v_{f}}$ |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |

8. Increase the mass of glider B by attaching some modelling clay to it. Be sure that the clay is evenly distributed along the glider. If the glider tips to the side or to the front or back, the motion will not be smooth. Determine the mass of glider B. Repeat step 7 for the two gliders, which are now of unequal mass.
9. Exchange the gliders and their labels. That is, the glider with the extra mass is on the left and becomes glider "A." The glider with no extra mass should be in the centre and labelled "B." Repeat step 7 for the new arrangement of gliders.
10. Remove the clay from the glider. Place one glider at each end of the track. Practise starting both gliders at the same time, so that they collide near the middle of the track. The collision must not take place while either glider is in a photogate. When you have demonstrated that you can carry out
the collision correctly, perform three trials and record the data in a table similar to the ones shown here. This table will need two additional columns - one for the initial time for glider B and a second for the initial velocity of glider B.
11. Remove the bumper springs from one end of each glider and attach the Velcro ${ }^{\text {TM }}$ bumpers. (If you do not have Velcro ${ }^{\text {TM }}$ bumpers, you can attach a large needle to one glider and a piece of wax to the other. Test to ensure that the needle will hit the wax when the gliders collide.)
12. With the Velcro ${ }^{\mathrm{TM}}$ bumpers attached, perform three sets of trials similar to those in steps 7, 8, and 9. You might need to perform trial runs and adjust the position of the photogates so that both gliders can pass through the photogate before reaching the right-hand end of the air track. Record the data in tables similar to those you used previously.

## Analyze and Conclude

1. For each glider in each trial, calculate the initial momentum (before the collision) and the final momentum (after the collision).
2. For each trial, calculate the total momentum of both gliders before the collision and the total momentum of both after the collision.
3. For each trial, compare the momentum before and after the collision. Describe how well the collisions demonstrated conservation of momentum.
4. In any case for which momentum did not seem to be conserved, provide possible explanations for errors.
5. Calculate the kinetic energy of each glider in each trial. Then calculate the total kinetic energy of both gliders before and after the collision for each trial.
6. Compare the kinetic energies before and after the collisions and decide which collisions were elastic and which were inelastic. Due to measurement errors, do not expect the kinetic energies to be identical before and after a collision. Decide if the values appear to be close enough that the differences could be attributed to measurement errors.
7. Examine the nature of the collisions that you considered to be elastic and those that you classed as inelastic. Look for a trend that would permit you to predict whether a collision would be elastic or inelastic. Discuss your conclusions with the rest of class. How well did your conclusions agree with those of other class members?

## MODEL PROBLEM

## Classifying a Collision

A 0.250 kg billiard ball moving at $5.00 \mathrm{~m} / \mathrm{s}$ collides head on with a stationary, 0.800 kg steel ball. If the billiard ball bounces directly backwards at $2.62 \mathrm{~m} / \mathrm{s}$, was the collision elastic?

## Frame the Problem

- Momentum is always conserved in a collision.
- If the collision is elastic, kinetic energy must also be conserved.
- The motion is in one dimension so only positive and negative signs will be used to indicate direction.
- If the billiard ball bounced directly backwards, its velocity after the collision must be negative.
- Let subscript b represent the billiard ball and subscript s represent the steel ball.


## Identify the Goal

Is the collision elastic? If it is elastic, then $E_{\mathrm{kb}}=E_{\mathrm{kb}}^{\prime}+E_{\mathrm{ks}}$

## Variables and Constants

| Known |  | Implied | Unknown |  |
| :--- | :--- | :--- | :--- | :--- |
| $m_{\mathrm{b}}=0.250 \mathrm{~kg}$ | $v_{\mathrm{b}}=5.00 \frac{\mathrm{~m}}{\mathrm{~s}}$ | $V_{\mathrm{s}}=0.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ | $V_{\mathrm{s}}^{\prime}$ | $E_{\mathrm{kb}}^{\prime}$ |
| $m_{\mathrm{s}}=0.800 \mathrm{~kg}$ | $v_{\mathrm{b}}^{\prime}=-2.62 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  | $E_{\mathrm{kb}}$ | $E_{\mathrm{ks}}^{\prime}$ |

## Strategy

Since momentum is conserved in all collisions, use the law of conservation of momentum to find the velocity of the steel ball after the collision.

Calculate the total kinetic energy before the collision. This is just the kinetic energy of the billiard ball since the steel ball was not moving.

Calculate the sum of the kinetic energies of the billiard ball and the steel ball. This sum is the total kinetic energy after the collision.

## Calculations

$$
\begin{aligned}
& m_{\mathrm{b}} V_{\mathrm{b}}+m_{\mathrm{s}} V_{\mathrm{s}}=m_{\mathrm{b}} v_{\mathrm{b}}^{\prime}+m_{\mathrm{s}} V_{\mathrm{s}}^{\prime} \\
& m_{\mathrm{b}} V_{\mathrm{b}}+0.0-m_{\mathrm{b}} V_{\mathrm{b}}^{\prime}=m_{\mathrm{s}} V_{\mathrm{s}}^{\prime} \\
& v_{\mathrm{s}}^{\prime}=\frac{m_{\mathrm{b}} V_{\mathrm{b}}-m_{\mathrm{b}} V_{\mathrm{b}}^{\prime}}{m_{\mathrm{s}}} \\
& V_{\mathrm{s}}^{\prime}=\frac{(0.250 \mathrm{~kg})\left(5.00 \frac{\mathrm{~m}}{\mathrm{~s}}\right)-(0.250 \mathrm{~kg})\left(-2.62 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{0.800 \mathrm{~kg}} \\
& V_{\mathrm{s}}^{\prime}=\frac{1.25 \frac{\mathrm{~m}}{\mathrm{~s}}-\left(-0.655 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{0.800} \\
& v_{\mathrm{s}}^{\prime}=\frac{1.905 \frac{\mathrm{~m}}{\mathrm{~s}}}{0.800} \\
& v_{\mathrm{s}}^{\prime}=2.38125 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& E_{\mathrm{kb}}=\frac{1}{2} m_{\mathrm{b}} v_{\mathrm{b}}^{2} \\
& E_{\mathrm{kb}}=\frac{1}{2}(0.250 \mathrm{~kg})\left(5.00 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \\
& E_{\mathrm{kb}}=\frac{1}{2}\left(6.25 \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}}\right) \\
& E_{\mathrm{kb}}=3.125 \mathrm{~J} \\
& E_{\mathrm{kb}}^{\prime}=\frac{1}{2} m_{\mathrm{b}} V_{\mathrm{b}}^{2} \\
& E_{\mathrm{kb}}^{\prime 2}=\frac{1}{2}(0.250 \mathrm{~kg})\left(2.62 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \\
& E_{\mathrm{kb}}^{\prime}=\frac{1}{2}\left(1.7161 \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}}\right) \\
& E_{\mathrm{kb}}^{\prime}=0.85805 \mathrm{~J} \\
& E_{\mathrm{ks}}^{\prime}=\frac{1}{2} m_{\mathrm{s}} V_{\mathrm{s}}^{\prime 2} \\
& E_{\mathrm{ks}}^{\prime}=\frac{1}{2}(0.800 \mathrm{~kg})\left(2.38125 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \\
& E_{\mathrm{ks}}^{\prime}=\frac{1}{2}\left(4.536281 \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}}\right) \\
& E_{\mathrm{ks}}^{\prime}=2.26814 \mathrm{~J} \\
& E_{\mathrm{kb}}^{\prime}+E_{\mathrm{ks}}^{\prime}=0.85805 \mathrm{~J}+2.26814 \mathrm{~J} \\
& E_{\mathrm{kb}}^{\prime}+E_{\mathrm{ks}}^{\prime}=3.12619 \mathrm{~J}
\end{aligned}
$$

The total kinetic energies before and after the collision are the same to the third decimal place. Therefore, the collision was elastic.

## Validate

Although the kinetic energies before and after the collision differ in the fourth decimal place, the difference is less than $1 \%$. Since the data contained only three significant digits, this difference could easily be due to the precision of the measurement. Therefore, it is fair to say that the collision was elastic.

## PRACTICE PROBLEMS

30. A billiard ball of mass 0.155 kg moves with a velocity of $12.5 \mathrm{~m} / \mathrm{s}$ toward a stationary billiard ball of identical mass and strikes it in a head on collision. The first billiard ball comes to a complete stop. Determine whether the collision was elastic.
31. Car A, with a mass of 1735 kg , was travelling north at $55.5 \mathrm{~km} / \mathrm{h}$ and Car B, with a mass of 2540 kg , was travelling north at $37.7 \mathrm{~km} / \mathrm{h}$ when Car A struck Car B from behind. If the cars stuck together after the collision and continued in a straight line, what was their combined momentum after the collision? Was the collision elastic or inelastic?

### 7.3 Section Review

1. © Explain qualitatively how Newton's third law is related to the law of conservation of momentum.
2. K/U What is the difference between an internal force and an external force?
3. K/U How does a closed system differ from an isolated system?
4. K/U Under what circumstances is the change in momentum of a system equal to zero?
5. K/O Define and give an example of recoil.
6. K/U What is the difference between an elastic collision and an inelastic collision?
7. © Describe an example of an elastic collision and an example of an inelastic collision that were not discussed in the text.
8. C Given a set of data for a collision, describe a step-by-step procedure that you could use to determine whether the collision was elastic.
