7.2 Conservation of Total Energy

In the real world, there are no frictionless surfaces, ideal springs, or vacuums in which fluid friction is absent. Some type of frictional force influences nearly every real process, and frictional forces are nonconservative forces. Nevertheless, the law of conservation of energy is a fundamental law of nature even in the real world. To develop the law of conservation of energy and apply it to real processes, it is helpful to define and use the concept of a system.

Any object or group of objects can be defined as a **system**. Once a system is defined, forces are classified as internal or external forces. An **internal force** is any force exerted on any object in the system due to another object in the system. An **external force** is any force exerted by an object that is not part of the system on an object within the system.

Scientists classify systems according to their interaction with their surroundings, as illustrated in Figure 4.6. An **open system** can exchange both matter and energy with its surroundings. Matter does not enter or leave a **closed system,** but energy can enter or leave. Neither matter nor energy can enter or leave an **isolated system.**

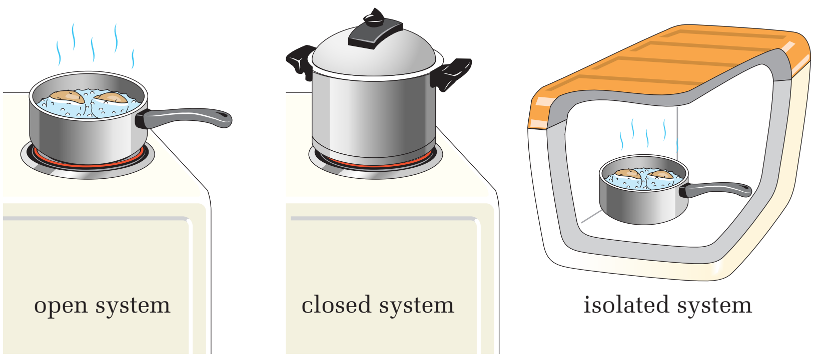


Figure 7.6 An open pot of potatoes boiling on the stove represents an open system, because heat is entering the pot and water vapour is leaving the system. A pressure cooker prevents any matter from escaping but heat is entering, so the pressure cooker represents a closed system. If the pot is placed inside a perfect insulator, neither heat nor water can enter or leave the system, making it an isolated system.

An external force can do work on a closed system, thus increasing the energy of the system. Conversely, a closed system can do work on its surroundings, thus decreasing the energy of the system. An isolated system, however, does not interact in any way with its surroundings. The law of conservation of (total) energy is based on an isolated system and stated in the following box.

SECTION OUTCOMES

- Analyze quantitatively the relationships among mass, speed, and thermal energy, using the law of conservation of energy.
- Compare empirical and theoretical values of total energy and account for discrepancies.
- Analyze common energy transformation situations using the closed system work-energy theorem.

KEY TERMS

- internal force
- external force
- open system
- closed system
- isolated system

MISCONCEPTION

A Closed System Is Not Isolated

Many people confuse the terms "closed" and "isolated" as they apply to systems. Although it might sound as though closed systems would not exchange anything with their surroundings, they do allow energy to enter or leave. Only isolated systems prevent the exchange of energy with the surroundings.

PHYSICS FILE

The first law of thermodynamics is another way of stating the law of conservation of energy. It states that the change in the energy of a system is equal to the amount of work done on or by the system plus the amount of heat that enters or leaves the system. The first law of thermodynamics can be stated mathematically as follows, where Q symbolizes heat.

$$\Delta E = W + Q$$

LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor destroyed, but it can be transformed from one form to another or transferred from one object to another. The total energy of an isolated system, including all forms of energy, always remains constant.

To bring the law of conservation of energy to a practical level, consider the example of skiers on a hill.

Friction is a non-conservative force. The amount of work done by a non-conservative force depends on the path taken by the force and the object. For example, the amount of energy transferred to the snow in Figure 7.7 depends on the path taken by the skier. The skier going straight down the slope should reach the bottom with a greater speed than the skier who is tracking back and forth across the slope.

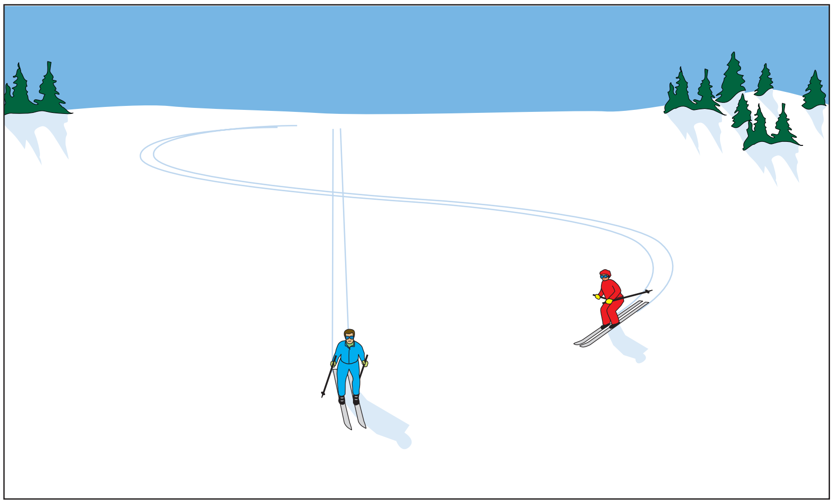


Figure 7.7 Although friction between the skis and the snow is small, friction nevertheless does some work on the skiers, slowing their velocity a little. The work done by friction is greater along the longer of the paths.

Friction causes the skier to do work on the environment. The snow heats up slightly and is moved around. For the skier, this is negative work — the skier is losing energy and cannot regain it as useful kinetic or potential energy. The sum of the skier's kinetic and gravitational potential energy at the end of the run will be less than it was at the start.

Wind pressure is another example of a nonconservative force. If the skier had the wind coming from behind, the wind (the environment) could be doing work on the skier. This would be positive work. The sum of the skier's kinetic and gravitational potential energies could increase beyond the initial total. However, the amount of energy transferred by the wind would depend to

ELECTRONIC LEARNING PARTNER

To enhance your understanding of energy transformation, refer to your Electronic Learning Partner.

a large extent on the path of the skier, so the wind would be a nonconservative force.

When dealing with nonconservative forces, the law of conservation of energy still applies. However, you must account for the energy exchanged between the moving object and its environment. One approach to this type of situation is to define the system as the skier and the local environment; that is, the skier, wind, and snow become the system.

All real mechanical processes involve some type of frictional forces that do work on the system, causing a reduction of the amount of mechanical energy in the system at the end of the process. You will often read that the system "lost" energy. Since energy cannot be created or destroyed, what happens to the energy? Work done by nonconservative forces transforms mechanical energy into thermal energy, which is the random kinetic energy of the molecules in a substance. Heat is the transfer of thermal energy during a process. You can now state the law of conservation of energy mathematically as shown in the following box.

CONSERVATION OF TOTAL ENERGY

The work done by nonconservative forces is the difference of the final mechanical energy and the initial mechanical energy of a system.

$$W_{nc} = E_{final} - E_{initial}$$

Quantity	Symbol	SI unit
work done by nonconservative forces	W_{nc}	J (joule)
mechanical energy of the system after the process	E_{final}	J (joule)
mechanical energy of the system before the process	$E_{initial}$	J (joule)

Unit Analysis

All of the units are joules.

In many processes, the temperature change in the objects involved is so small that it is almost imperceptible, making the calculations seem artificial. The following investigation, however, will make temperature changes resulting from a loss of mechanical energy seem real.

PHYSICS FILE

Quite often, you might not want your forces to be conservative. Without friction, many of your clothes would simply fall apart into strands as you moved. In addition, keep-fit programs would have to be greatly modified. A person who rides an exercise bicycle to lose mass (through chemical reactions that provide the energy) does not want the energy back. It simply is dissipated as sound and heat. Likewise, the weight lifter who does work to lift a bar bell does not expect to receive that energy back when the bar bell is lowered.

PHYSICS FILE

One of James Joule's scientific goals was to determine the "mechanical equivalent of heat." A story is often told that, while Joule was on his honeymoon in Switzerland, he spent his time measuring the temperature of water at the top and at the bottom of waterfalls. By determining the difference in the water temperature, he could determine how much thermal energy the water would gain by falling a certain distance.

INVESTIGATION 7-C

Mechanical and Thermal Energy

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Identifying variables

Before 1800, physicists and chemists did not know that a relationship existed between mechanical energy and heat. Count Rumford (Benjamin Thompson: 1753–1814) was the first to observe such a relationship, followed by Julius Robert Mayer (1814–1878). Rumford and Mayer made some very important discoveries. Mayer was unable to express himself clearly in writing, however, so his discoveries were overlooked. Eventually, James Prescott Joule (1818–1889) was credited with the determination of the mechanical equivalence of heat. In this investigation you will perform experiments similar to those of Mayer and Joule.

Problem

How much heat is produced when a mass of lead pellets is repeatedly lifted and dropped through a known distance?

Equipment

- balance
- thermometer (°C)
- lead shot
- cardboard or plastic tube with a small hole in the side, close to one end; the ends must be able to be closed
- metre stick
- small amount of masking or duct tape

Procedure

1. Determine the mass of the lead shot.
2. Place the lead shot into the tube and close up the tube. Let the tube sit upright on a desk for several minutes to allow the tube and its contents to come to room temperature. Make sure that the hole is close to the bottom of the tube.
3. Insert the thermometer or temperature probe through the hole in the tube and nestle the end in the lead shot. Measure and record the temperature.
4. Close the hole.
5. Measure the length of the tube.
6. Repeatedly invert the tube for several minutes, waiting only to allow the lead shot to fall to the bottom on each inversion. Keep track of the number of inversions.
7. Finish the inversions with the hole near the bottom of the tube. Remove the tape and measure the temperature of the lead shot. Record the final temperature.

Analyze and Conclude

1. What were the initial and final temperatures of the lead shot? What was the total mass of the lead shot?
2. Determine the quantity of heat gained by the lead shot using the formula, $Q = mc\Delta T$, where Q is the quantity of heat, m is the mass of the lead shot, c is the specific heat capacity of lead (128 J/kg·C°), and ΔT is the temperature change.
3. Determine the total distance through which the lead shot was lifted by the inversions and calculate the total gain in gravitational potential energy of the lead.
4. Determine the percentage of the gravitational potential energy that was converted into heat.
5. If the conversion into heat does not account for all of the gravitational potential energy gained by the lead shot, where else might some of the energy have gone?

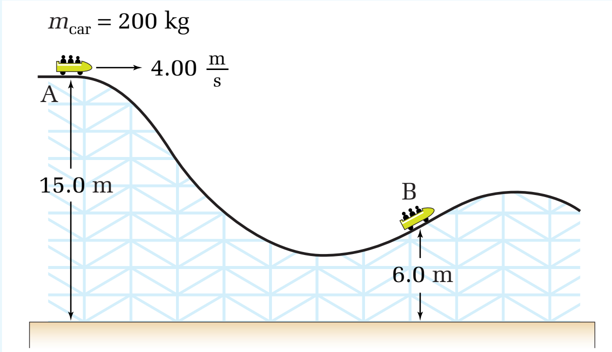
Apply and Extend

6. How could this investigation be improved? Try to design a better apparatus and, if possible, carry out the investigation again.
7. Do research and write a summary of the work of Rumford, Mayer, and Joule on the mechanical equivalence of heat.

MODEL PROBLEMS

Energy Conversions on a Roller Coaster

1. A roller-coaster car with a mass of 200.0 kg (including the riders) is moving to the right at a speed of 4.00 m/s at point A in the diagram. This point is 15.00 m above the ground. The car then heads down the slope toward point B, which is 6.00 m above the ground. If 3.40×10^3 J of work are done by friction between points A and B, determine the speed of the car at point B.



Frame the Problem

- As the roller-coaster car *moves* down the track, most of the *gravitational potential energy* is converted into *kinetic energy*, but some is lost as *heat* due to *friction*.
- The law of conservation of total energy applies.
- The *work* done by friction is *negative* because it *removed energy* from the system.

Identify the Goal

The speed of the car at point B, v_B

Variables and Constants

Known		**Implied**	**Unknown**
$h_A = 15.00$ m	$W_{nc} = -3.40 \times 10^3$ J	$g = 9.81 \ \frac{m}{s^2}$	v_B
$h_B = 6.00$ m	$m = 200.0$ kg		
$v_A = 4.00 \ \frac{m}{s}$			

Strategy

Calculations

$$W_{nc} = E_{final} + E_{initial}$$

Write the law of conservation of energy.

$$W_{nc} = E'_k + E'_g - E_k - E_g$$

Rearrange to solve for E'_k.

$$E'_k = W_{nc} - E'_g + E_k + E_g$$

Expand by substituting the expressions for the forms of energy. Solve for the speed of the car at point B

$$\frac{1}{2}mv_B^2 = W_{nc} - mgh_B + \frac{1}{2}mv_A^2 + mgh_A$$

$$v_B^2 = \frac{2(W_{nc} - mgh_B + \frac{1}{2}mv_A^2 + mgh_A)}{m}$$

$$v_B = \sqrt{\frac{2(W_{nc} - mgh_B + \frac{1}{2}m(v_A)^2 + mgh_A)}{m}}$$

$$v_B = \sqrt{\frac{2\left[-(3.40 \times 10^3 \text{ J}) - \left(1.1772 \times 10^4 \ \frac{kg \cdot m^2}{s^2}\right) + \left(1.6 \times 10^3 \ kg\frac{m^2}{s^2}\right) + \left(2.943 \times 10^4 \ \frac{kg \cdot m^2}{s^2}\right)\right]}{200.0 \text{ kg}}}$$

$$v_B = \sqrt{\frac{3.1716 \times 10^4 \ \frac{kg \cdot m^2}{s^2}}{200.0 \text{ kg}}}$$

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Since the problem only requires magnitude, choose the positive value.

$$v_B = \sqrt{1.5858 \times 10^2\ \frac{m^2}{s^2}}$$
$$v_B = \pm 1.2593 \times 10^1\ \frac{m}{s}$$
$$v_B \cong 12.6\ \frac{m}{s}$$

The speed of the car at point B will be 12.6 m/s.

Validate

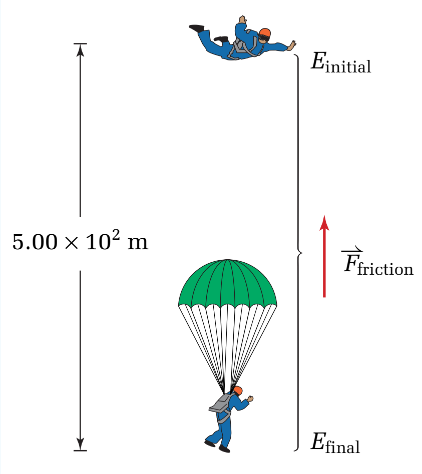
The speed at point B is expected to be larger than its speed at point A.

2. Work Done by Air Friction
A 65.0 kg skydiver steps out from a hot air balloon that is 5.00×10^2 m above the ground. After free-falling a short distance, she deploys her parachute, finally reaching the ground with a velocity of 8.00 m/s (approximately the speed with which you would hit the ground after having fallen a distance of 3.00 m).

(a) Find the gravitational potential energy of the skydiver, relative to the ground, before she jumps.

(b) Find the kinetic energy of the skydiver just before she lands on the ground.

(c) How much work did the nonconservative frictional force do?



Frame the Problem

- Make a sketch of the problem and label it.
- The hot air balloon can be considered to be at rest, so the vertical velocity of the skydiver initially is *zero*.
- As she starts to fall, the *force of air friction* does *negative* work on her, converting much of her *gravitational potential energy* into *thermal energy*.
- The rest of her *gravitational potential energy* was converted into *kinetic energy*.

Identify the Goal

Initial gravitational potential energy, E_g, of the skydiver relative to the ground
Final kinetic energy, E_k, of the skydiver just before touchdown
Work done by non-conservative force, W_{nc}

Variables and Constants

Known	Unknown
$m = 65.0$ kg	$E_{initial}$
$g = 9.81\ \frac{m}{s^2}$	E_g
$\Delta h = 5.00 \times 10^2$ m	W_{nc}
$v = 8.00\ \frac{m}{s^2}$	E_{final}
	E_k

Strategy

Use the equation for gravitational potential energy. All of the needed variables are known, so substitute into the equation.

Calculations

$$E_g = mg\Delta h$$

Multiply.

$$E_g = (65.0\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(5.00 \times 10^2\ \text{m})$$

$$E_g = 3.188 \times 10^5\ \text{kg}\ \frac{\text{m}}{\text{s}^2}\ \text{m}$$

$1\ \text{kg}\ \frac{\text{m}}{\text{s}^2}$ is equivalent to 1 N.

$$E_g = 3.188 \times 10^5\ \text{N} \cdot \text{m}$$

$1\ \text{N} \cdot \text{m}$ is equivalent to 1 J.

$$E_g = 3.188 \times 10^5\ \text{J}$$

(a) The skydiver's initial gravitational energy, before starting the descent, was 3.19×10^5 J.

Use the skydiver's velocity just before touchdown to find the kinetic energy before touchdown. All of the needed values are known, so substitute into the equation for kinetic energy, then multiply.

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2}(65.0\ \text{kg})\left(8.00\ \frac{\text{m}}{\text{s}}\right)^2$$

$$E_k = 2.080 \times 10^3\ \text{kg}\ \frac{\text{m}^2}{\text{s}^2}$$

$1\ \text{kg}\ \frac{\text{m}^2}{\text{s}^2}$ is equivalent to 1 J.

$$E_k = 2.080 \times 10^3\ \text{J}$$

(b) The skydiver's kinetic energy just before landing was 2.08×10^3 J.

Use the equation for the conservation of total energy to find the work done by air friction. The final mechanical energy is entirely kinetic energy and the initial mechanical energy was entirely gravitational potential energy. These values were just calculated.

$$W_{nc} = E_{final} - E_{initial}$$

$$W_{nc} = 2.080 \times 10^3\ \text{J} - 3.188 \times 10^5\ \text{J}$$

$$W_{nc} = -3.167 \times 10^5\ \text{J}$$

(c) Air friction did 3.17×10^5 J of negative work on the skydiver.

Validate

The 3.17×10^5 J of work done by air resistance is very large. This is fortunate for the skydiver, as it allows her to land softly on the ground, having lost 99.3 percent of her mechanical energy. This large loss of mechanical energy is what a parachute is designed to do.

How fast would the skydiver in the model problem be travelling just before hitting the ground if the work done by air resistance was ignored?

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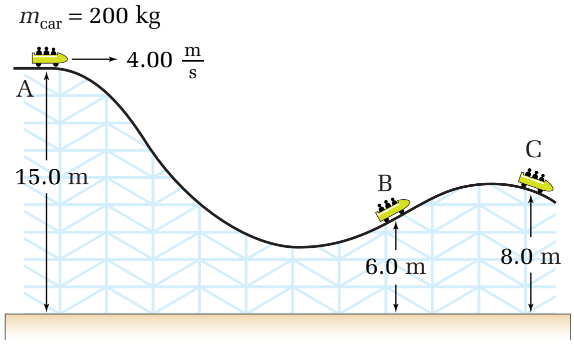
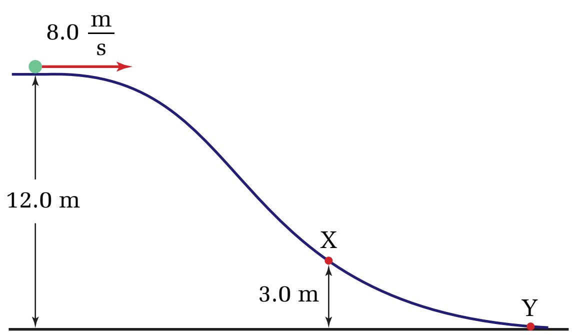
PRACTICE PROBLEMS

18. Determine the speed of the roller-coaster car in the sample problem at point C if point C is 8.0 m above the ground and another 4.00×10^2 J of heat energy are dissipated by friction between points B and C.



19. A sled at the top of a snowy hill is moving forward at 8.0 m/s, as shown in the diagram. The height of the hill is 12.0 m. The total mass of the sled and rider is 70.0 kg. Determine the speed of the sled at point X, which is 3.0 m above the base of the hill, if the sled does 1.22×10^3 J of work on the snow on the way to point X.



20. If the sled in the previous question reaches the base of the hill with a speed of 15.6 m/s, how much work was done by the snow on the sled between points X and Y?

21. A 0.50 kg basketball falls from a 2.3 m shelf onto the floor, then bounces up to a height of 1.4 m before you catch it.

(a) Calculate the gravitational potential energy of the ball before it falls.

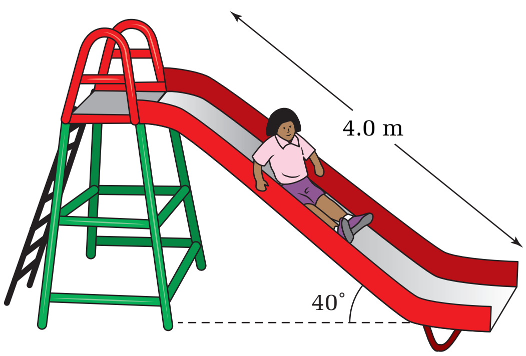
(b) Ignoring frictional effects, determine the speed of the ball as it strikes the floor, assuming that it fell from rest.

(c) How fast is the ball moving just before you catch it?

22. A 2.0 g bullet initially moving with a velocity of 87 m/s passes through a block of wood. On exiting the block of wood, the bullet's velocity is 12 m/s. How much work did the force of friction do on the bullet as it passed through the wood? If the wood block was 4.0 cm thick, what was the average force that the wood exerted on the block?

23. The Millennium Force, the tallest roller coaster in North America, is 94.5 m high at its highest point. What is the maximum possible speed of the roller coaster? The roller coaster's actual maximum speed is 41.1 m/s. What percentage of its total mechanical energy is lost to thermal energy due to friction?

24. A 15 kg child slides, from rest, down a playground slide that is 4.0 m long, as shown in the figure. The slide makes a 40° angle with the horizontal. The child's speed at the bottom is 3.2 m/s. What was the force of friction that the slide was exerting on the child?



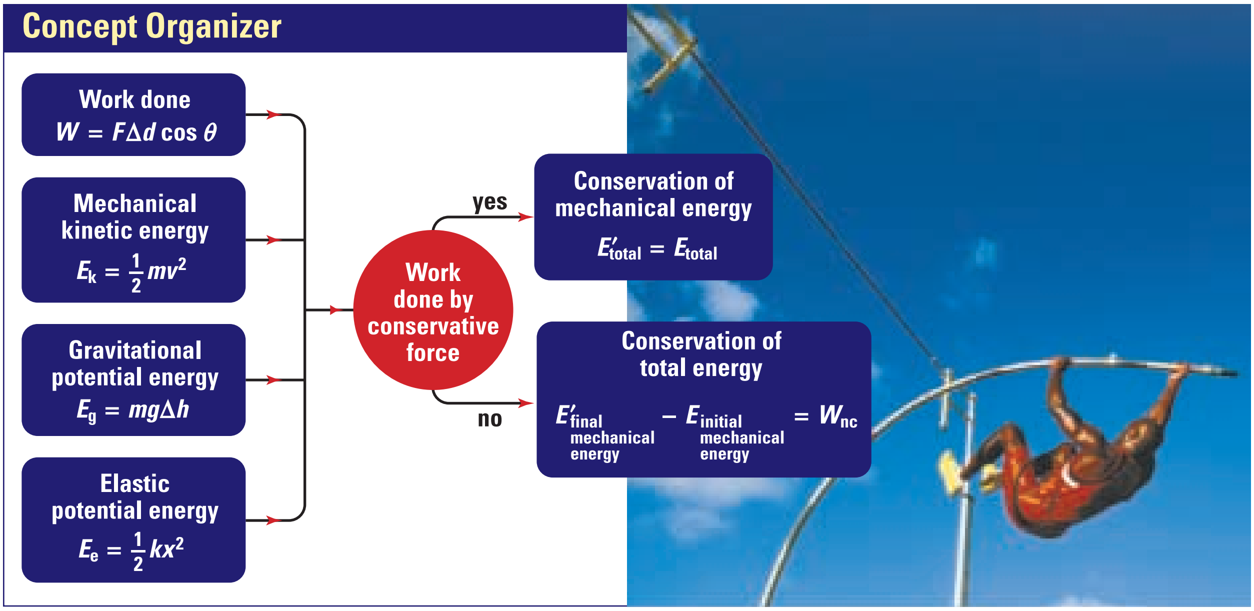


Figure 7.8 How many energy transformations are taking place in the photograph?

7.2 Section Review

1. K/U How are conservation of mechanical energy and conservation of energy different?

2. C How is the conservation of energy demonstrated if an in-line skater falls during a competition?

3. K/U A bullet is fired into a block of wood and penetrates to a distance Δd. An identical bullet is fired into a block of wood with a velocity three times that of the first. How much farther, in terms of Δd, does the second bullet penetrate the wood?

4. MC **(a)** In an amusement park there is a ride on which children sit in a simulated log while it slides rapidly down a water-covered slope. At the bottom, the log slams into a trough of water, which slows it down. Why did the ride designers not simply have the log slam into a large spring?

   (b) Steel or plastic barrels are located along highways to cushion the impact if a car skids into a bridge abutment. These barrels are often filled with energy-absorbing material. Why are these barrels used instead of large springs to bring the cars to a stop?