

SECTION
OUTCOMES

- Describe quantitatively mechanical energy as the sum of kinetic and potential energies.
- Analyze quantitatively problems related to kinematics and dynamics using the mechanical energy concept.
- Analyze common energy transformation situations using the closed system work-energy theorem.

KEY
TERMS

- mechanical energy
- conservative force
- nonconservative force
- amplitude

In Chapter 6, you learned that by doing work on an object you can change its position or condition and thus give it energy. The three forms of energy that you studied — kinetic energy, gravitational potential energy, and elastic potential energy — are classified as **mechanical** energy. In this chapter, you will focus on what happens to the energy after you have done work on an object.



Figure 7.1 Although this large pendulum might oscillate for days, it will eventually slow down and stop.

A hockey stick does work on a puck, and the puck slides across the ice with kinetic energy. However, if the ice surface was long enough, the puck would eventually come to a stop. You place a book on the shelf, giving it gravitational potential energy. Then you bump into the shelf and the book falls off, hits the floor, and stops. At first, the gravitational potential energy was transformed into kinetic energy, but soon all of the energy seemed to disappear. You do work on a mass on the end of a spring and give the spring elastic potential energy. When you release the mass, it oscillates up and down for a while, but eventually comes to a stop. You do work on a pendulum by pulling the bob up and to the side. After you release it, the pendulum bob will swing back and forth for minutes, hours, or possibly even days, such as the pendulum in Figure 7.1. Eventually, however, it too will stop. In every case, the object lost the *mechanical energy* that you gave it. In some cases, however, the object keeps the mechanical energy for extremely long periods of time. If a system such as a pendulum or a mass on a spring was “perfect,” the mechanical energy of the system would remain constant. You could say that the mechanical energy was conserved. Is there any specific characteristic of a system or process that you can use to predict whether the mechanical energy

will be conserved over a long period of time? To answer this question, you need to learn about two classes of forces that do work on a system.

Conservative and Non-Conservative Forces

If you lift your book one metre above a table and release it, it will drop back onto the table, gaining kinetic energy as it falls. If you push your book across the table, will it automatically return to its original spot, gaining kinetic energy as it moves? Of course not. In the first case, you were doing work against the gravitational force. In the second case, you were doing work against a frictional force.

These two forces, gravity and friction, represent two important classes of forces. Before defining these classes of forces, consider another property of doing work against them. If you lift your book to a certain height, then carry it across the room, you have done the same amount of work that you would do if you simply lifted it straight up. However, if you push your book from side to side as you move it from one end of the table to the other, you have done more work than you would if you pushed it in a straight line. The amount of work you do against friction depends on the path through which you push the object.

A **conservative force** is one that does work on an object in such a way that the amount of work done is *independent* of the path taken. The force of gravity is an example of a conservative force because it takes the same amount of work to lift a mass to height Δh , regardless of the path. The work done, and therefore the gravitational potential energy, depend only on the height Δh .

Friction, on the other hand, is not a conservative force. The work done against a frictional force when a crate is pushed across a rough floor depends on whether the path is straight, or curved, or zigzagged. The work done *is* path-dependent, and therefore the force of friction is **nonconservative**.

Work done by conservative forces results in energy changes that are independent of the path and are therefore reversible. Work done by non-conservative forces results in energy changes that are dependent on the path, and therefore may not be reversed. The force of gravity and the force that perfectly elastic materials exert on an object are conservative forces. No real material is perfectly elastic, but some materials are nearly so. Therefore, in many situations they can be considered to be perfectly elastic.

Conservation of Mechanical Energy

When all of the work done throughout an energy transformation process is done by conservative forces, mechanical energy is conserved. This property of energy transformation processes can be stated as a law and expressed mathematically as shown in the box on the next page.

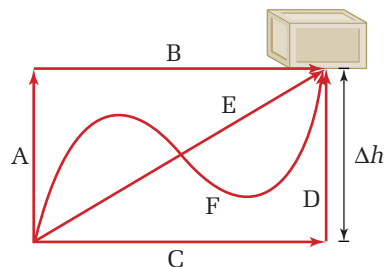


Figure 7.2 Regardless of the path taken, the work done to lift the box to height Δh is identical. Work done against a conservative force is *independent* of the path.

PROBEWARE

www.mcgrawhill.ca/links/atphysics

If your school has probeware equipment, visit the Internet site above and follow the links for several laboratory activities.

THE LAW OF CONSERVATION OF MECHANICAL ENERGY

When all of the work done throughout a process is done by conservative forces, the total mechanical energy of the system after the process is equal to the total mechanical energy of the system before the process.

$$E'_k + E'_g + E'_e = E_k + E_g + E_e$$

Quantity	Symbol	SI unit
kinetic energy after process	E'_k	J (joule)
gravitational potential energy after process	E'_g	J (joule)
elastic potential energy after process	E'_e	J (joule)
kinetic energy before process	E_k	J (joule)
gravitational potential energy before process	E_g	J (joule)
elastic potential energy before process	E_e	J (joule)

Unit Analysis

All of the units are joules.

Note: The symbol that looks like an apostrophe is called a prime. The prime is used to represent the condition of the system after the process has been completed.

Gravitational Potential Energy and Kinetic Energy

Concentrate first on energy transformation processes that involve only gravitational potential energy and kinetic energy.

• Conceptual Problem

- A marble oscillates back and forth in a U-shaped track, repeatedly transferring gravitational potential energy to kinetic energy, and back to gravitational potential energy. Does the law of conservation of mechanical energy apply? Defend your reasoning by discussing conservative and non-conservative forces.

The example of a falling rock in Figure 7.3 illustrates such a process. The rock, initially at rest, falls from a height, h , above the ground. The rock's gravitational potential energy is being transformed into kinetic energy as it falls farther and faster. Its mechanical energy, however, is being conserved until the instant before it strikes the ground.

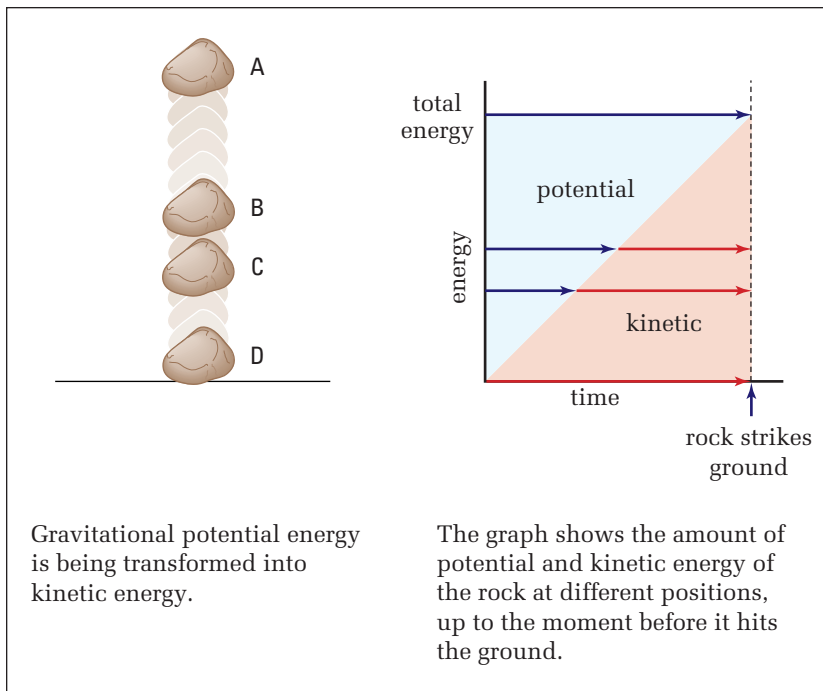
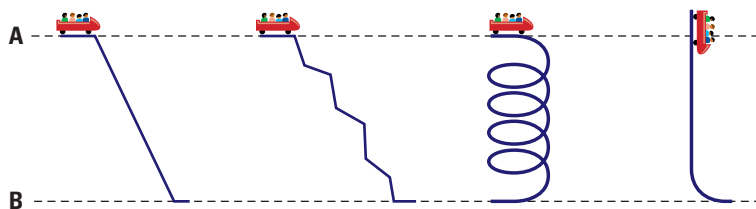


Figure 7.3 Although gravitational potential energy is being transformed into kinetic energy, the total mechanical energy of the rock remains the same as it falls.

• **Conceptual Problems**

- Imagine an amusement park that has rides like those illustrated here. The masses of all of the cars are identical and the same four people go on each ride. The wheels and track are effectively frictionless. Each car starts from rest at level A. What are the relative speeds of the cars when they reach level B?



- While playing catch by yourself, you throw a ball straight up as hard as you can throw it. Neglecting air friction, how does the speed of the ball, when it returns to your hand, compare to the speed with which it left your hand?

You can say, intuitively, that the kinetic energy of an object, the instant before it hits a solid surface, is the same as the gravitational potential energy at the point from which it fell from rest.

However, physicists try to show such relationships mathematically. Then, they can make calculations based on the principles. Start with the general expression for kinetic energy, $E_k = \frac{1}{2}mv^2$, and show how the kinetic energy at the end of a free fall relates to the gravitational potential energy, $E_g = mg\Delta h$, at a distance Δh above the surface.

The first step in deriving this relationship is to find the velocity of an object after it falls a distance, Δh .

- Recall from Chapter 3 the equation of motion for uniform acceleration. $v_2^2 = v_1^2 + 2a\Delta d$

- The object was initially at rest. $v_1 = 0$
 $v_2^2 = 0 + 2a\Delta d$

- Substitute Δh , representing height, for Δd . $\Delta d = \Delta h$
 $v_2^2 = 0 + 2a\Delta h$

- Since the only force acting on the object is gravity, the object's acceleration is g , the acceleration due to gravity. $a = g$

- This is the square of the velocity of the object, just before hitting a surface, after falling through height Δh . $v_2^2 = 2g\Delta h$

- Substitute the previous expression for v^2 into the general expression for kinetic energy. $E_k = \frac{1}{2}mv^2$

- The kinetic energy of the object just before it hits the surface becomes $E_k = \frac{1}{2}m(2g\Delta h)$

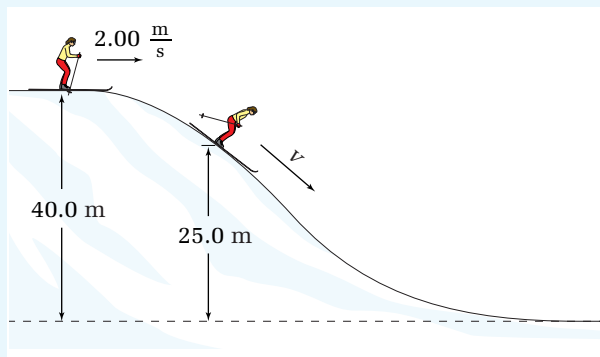
- Notice that the kinetic energy just before the object hits the solid surface is identical to $E_k = mg\Delta h$

- The equation for gravitational potential energy a distance Δh above the surface is $E_g = mg\Delta h$

The total energy of the object is not only the same at the top and at the bottom, but throughout the entire trip. The mathematical process used above could be applied to any point throughout the fall. You could show that, for any point within the fall, the gravitational potential energy that was lost up to that point has been transformed into kinetic energy. Therefore, the sum of the kinetic and potential energies remains constant at all times throughout the fall. Obviously, when an object hits a solid surface and stops, the mechanical energy is no longer conserved. Also, any effects of air friction were neglected in the discussion.

Conservation of Energy on the Ski Slopes

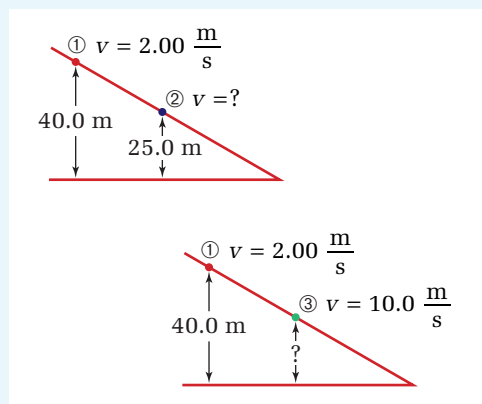
A skier is gliding along with a speed of 2.00 m/s at the top of a ski hill, 40.0 m high, as shown in the diagram. The skier then begins to slide down the icy (frictionless) hill.



- (a) What will be the skier's speed at a height of 25.0 m ?
- (b) At what height will the skier have a speed of 10.0 m/s ?

Frame the Problem

- Sketch the two parts of the problem separately. Label the initial conditions (top of the hill) "1." Label the position when $h = 25 \text{ m}$ as "2." Label the position at which the skier is travelling at 10.0 m/s as "3."
- Use subscripts 1 and 2 to indicate the initial and final conditions in part (a) and use subscripts 1 and 3 to indicate the initial and final conditions in part (b).
- Assume that the slope is frictionless.
- The law of *conservation of mechanical energy* can be applied.



Identify the Goal

- (a) the speed, v_2 , at a height of 25.0 m
- (b) the height, h_3 , at which the skier's speed will be 10.0 m/s

Variables and Constants

Known	Implied	Unknown
$v_1 = 2.00 \frac{\text{m}}{\text{s}}$	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	v_2
$h_1 = 40.0 \text{ m}$		h_3
$v_3 = 10.0 \frac{\text{m}}{\text{s}}$		
$h_2 = 25.0 \text{ m}$		

Strategy

State the law of conservation of mechanical energy without elastic potential energy.

Expand by replacing E with the expression that defines the type of energy.

Divide through by m .

Calculations

$$E'_k + E'_p = E_k + E_p$$

$$\frac{1}{2}mv_2^2 + mgh_2 = \frac{1}{2}mv_1^2 + mgh_1$$

$$\frac{1}{2}v_2^2 = \frac{1}{2}v_1^2 + gh_1 - gh_2$$

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Simplify and rearrange the equation to solve for v_2 .

$$v_2^2 = v_1^2 + 2g(h_1 - h_2)$$

$$v_2 = \sqrt{v_1^2 + 2g(h_1 - h_2)}$$

Substitute and solve.

$$v_2 = \sqrt{\left(2.00 \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(40.0 \text{ m} - 25.0 \text{ m})}$$

$$v_2 = \sqrt{298.3 \frac{\text{m}^2}{\text{s}^2}}$$

$$v_2 = \pm 17.271 \frac{\text{m}}{\text{s}}$$

$$v_2 \cong 17.3 \frac{\text{m}}{\text{s}}$$

Speed does not involve direction, so choose the positive root since speed can never be negative.

(a) The speed is 17.3 m/s at a height of 25.0 m.

Write the expanded version of the conservation of mechanical energy. Rearrange and solve for height.

$$\frac{1}{2}mv_3^2 + mgh_3 = \frac{1}{2}mv_1^2 + mgh_1$$

$$gh_3 = \frac{1}{2}v_1^2 + gh_1 - \frac{1}{2}v_3^2$$

$$h_3 = \frac{\frac{1}{2}v_1^2 + gh_1 - \frac{1}{2}v_3^2}{g}$$

Substitute numerical values and solve.

$$h_3 = \frac{\frac{1}{2}\left(2.00 \frac{\text{m}}{\text{s}}\right)^2 + \left(9.81 \frac{\text{m}}{\text{s}^2}\right)(40.0 \text{ m}) - \frac{1}{2}\left(10.0 \frac{\text{m}}{\text{s}}\right)^2}{9.81 \frac{\text{m}}{\text{s}^2}}$$

$$h_3 = 35.107 \text{ m}$$

$$h_3 \cong 35.1 \text{ m}$$

(b) The height must be 35.1 m when the speed is 10.0 m/s.

Validate

(a) The units for the right-hand side of the equation for v_2 are

$$\left[\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{m}}{\text{s}^2}\right)\text{m}\right]^{\frac{1}{2}} = \frac{\text{m}}{\text{s}}$$

These are the correct units for the speed. In addition, since the skier is going downhill, the final speed must be larger than the initial speed of 2.00 m/s.

(b) The units on the right-hand side of the equation for h_3 are

$$\frac{\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{m}}{\text{s}^2}\right)(\text{m}) - \left(\frac{\text{m}}{\text{s}}\right)^2}{\frac{\text{m}}{\text{s}^2}} = \text{m}$$

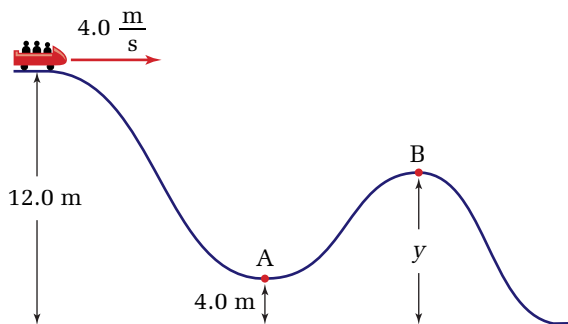
Since the speed in part (b) is less than the speed in part (a), the skier should be higher on the hill than in part (a).

PROBLEM TIP

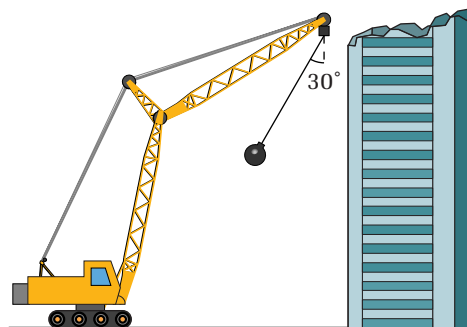
Students are often tempted to apply the equations for linear motion to the solution of these problems. However, the paths are not always linear, so the equations might not apply. One of the great advantages of using conservation of energy is that you generally do not need to know the exact path between two points or vector directions. You need to know the conditions at only those two points.

PRACTICE PROBLEMS

Use the following diagram for practice problems 1, 2, and 3.



1. A car on a roller coaster is moving along a level section 12.0 m high at 4.0 m/s when it begins to roll down a slope, as shown in the diagram. Determine the speed of the car at point A.
2. What is the height of point B in the roller coaster track if the speed of the car at that point is 10.0 m/s?
3. What is the height of y if the speed of the roller coaster at B is 12.5 m/s?
4. You throw a ball directly upward, giving it an initial velocity of 10.0 m/s. Neglecting friction, what would be the maximum height of the ball? (Explain why you do not need to know the mass of the ball.)
5. A wrecking ball, with a mass of 315 kg, hangs from a crane on 10.0 m of cable. If the crane swings the wrecking ball so that the angle that the cable makes with the vertical is 30.0° , what is the potential energy of the wrecking ball in relation to its lowest position? What will be the kinetic energy of the wrecking ball when it falls back to the vertical position? What will be the speed of the wrecking ball?
6. A 2.5 kg lead ball and a 55 g piece of lead shot are both dropped from a height of 25 m. Neglecting air friction, what is the kinetic energy of each object just before it hits the ground? What is the velocity of each object just before it hits the ground?
7. A 32 kg crate slides down a frictionless ramp. Its initial velocity at the top of the ramp was 3.2 m/s. Its velocity when it reached the bottom of the ramp was 9.7 m/s. The ramp makes an angle of 25° with the horizontal. How long is the ramp?
8. A worker on the roof of a building that is under construction dropped a 0.125 kg wrench over the edge. Another workman on the eighth floor saw the wrench go by and determined that its speed at that level was 33.1 m/s. The first floor of the building is 12.0 m high and each successive floor is 8.00 m high. Neglecting air friction, how many floors does the building have? How fast was the wrench falling just before it hit the ground? What was its kinetic energy just before it hit the ground?



**ELECTRONIC
LEARNING PARTNER**



A video clip of a wrecking ball in action can be found on the Electronic Learning Partner.

Conservation of Energy of a Falling Mass

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

When scientists set out to test an hypothesis or challenge a law, they often use the hypothesis or law to make a prediction and then test it. In this investigation, you will make a prediction based on the law of conservation of mechanical energy.

Problem

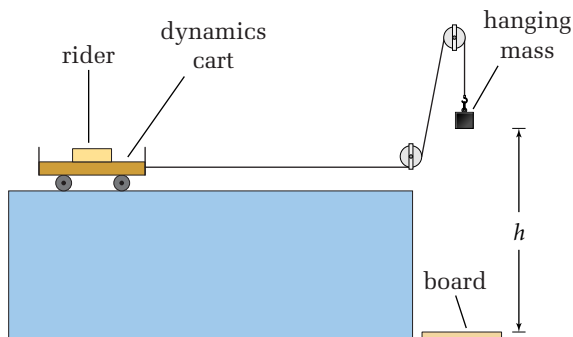
To perform a test of the law of conservation of mechanical energy

Equipment

- dynamics cart
- balance capable of measuring a mass of 1 to 2 kg
- 2 pulleys
- retort stand and clamps for the pulleys
- board (or similar material) to protect the floor from dropped masses
- metric measuring tape
- selection of masses, including several that can be suspended on a string
- stopwatch or photogate timers
- string about 4 m long

Procedure

1. Determine the mass of the dynamics cart.
2. Select a mass to be the rider and a mass to be the hanging mass. Decide on the height of the hanging mass above the board.
3. Set up the apparatus on a long desktop, as shown in the diagram.



4. Using the law of conservation of mechanical energy, calculate the expected speed of the cart and the hanging mass just before the mass strikes the board. If the cart is to be released from rest, determine the average speed of the system and then the predicted time interval for the hanging mass to drop to the board.
5. With the entire apparatus in place, hold the cart still. Release the cart and measure the time taken for the hanging mass to reach the board. Be sure to catch the cart before it hits the lower pulley.
6. Repeat the measurement several times and average the results.
7. Perform several trial runs with a different pair of masses for the cart and the hanging mass.

Analyze and Conclude

1. Prepare a table to show all of your data, as well as your calculations for the final speed, average speed, and the time interval.
2. What is the percent difference between the time as predicted by the law of conservation of energy and the measured average time?
3. Based on the precision of the timing devices, what range of experimental error would you expect in this investigation? How does this range compare with the percent difference determined in question 2?
4. Do the results of this investigation support the law of conservation of mechanical energy?

Apply and Extend

5. Which part of this investigation caused the greatest difficulty? Provide suggestions for overcoming this difficulty.
6. List some of the sources of error in this investigation and suggest how these errors might be reduced.

Energy Transformations of a Pendulum

TARGET SKILLS

- Initiating and planning
- Predicting
- Performing and recording
- Analyzing and interpreting

You have learned about mechanical energy and worked problems based on the conservation of energy. Now you will design an experiment that will test the law of conservation of mechanical energy. A well-designed pendulum has such a small amount of friction that it can be neglected.



Problem

How can you verify the conservation of mechanical energy for a system in which all of the work is done by conservative forces?

Prediction

Make predictions about the type of energy transformations that will occur and when, during the cycle, they will occur.

Equipment

- parts for assembling a pendulum
- motion sensor
- metre stick or other measuring device

Procedure

Write your own procedure for testing the law of conservation of energy using a pendulum. Clearly describe the required materials and the proposed procedure.

Verify, with the teacher, that the experiment is safe before proceeding.

Make detailed predictions identifying the

positions at which the mechanical energy of the pendulum will be (a) all gravitational potential, (b) all kinetic, and (c) half gravitational potential and half kinetic.

Analyze and Conclude

1. Which variables did you control in the experiment?
2. How might you improve your experiment?
3. How well did your results verify the law of conservation of energy? Was any discrepancy that you found within limits that are acceptable to you?
4. If mechanical energy was lost, explain how and why it might have been lost.

Apply and Extend

5. The transformation of energy from one form to another is never completely efficient. For example, predict what will happen to the chemical potential energy of a car's fuel as it is transferred to the kinetic energy of the motion of the car.

Elastic Potential Energy and Kinetic Energy

In Chapter 6, you learned that the amount of elastic potential energy stored in a stretched or compressed spring could be expressed as $E_e = \frac{1}{2}kx^2$ where x is the amount of stretch or compression of the spring. In this section, you will follow the energy transformation processes that occur when the elastic potential energy stored in a spring or other elastic object is transformed into kinetic energy.

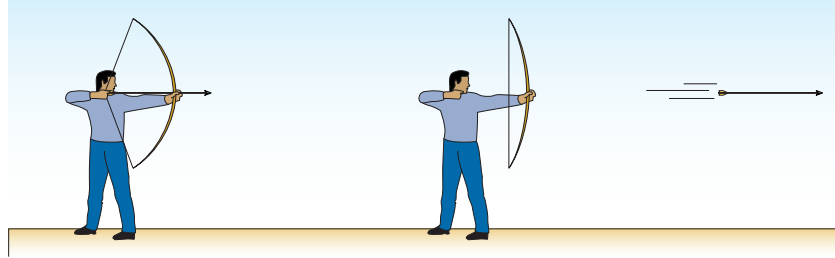


Figure 7.4 An archer does work on the bow by pulling the string and bending the bow. The bow stores elastic energy that can then be transformed into the kinetic energy of an arrow.

Hooke's law and the concept of elastic potential energy can apply to elastic objects other than springs. A practical example of the transformation of elastic potential energy into kinetic energy is shown in Figure 7.4 in which an archer is shooting an arrow. Some other examples of elastic devices are the elastic strips in sling shots that are sometimes used in paint ball games, and the bungee cords that are used in some current amusement park rides. In fact, to some extent, chemical bonds in molecules can act as springs with the atoms vibrating on the ends of the “springs.” You can apply the principles that you learn using springs to all of these devices and more.

A mass oscillating on the end of a spring is an excellent model from which to develop energy transformation principles. Start with an ideal spring and mass sliding on a horizontal, frictionless surface to eliminate the effects of both gravitational potential energy and of nonconservative forces. As shown in Figure 7.5, work is done on the mass to stretch it to a distance $x = A$. The symbol A is used to represent the maximum extension of the spring and is called the **amplitude**. When the mass is released, the energy stored in the spring will remain in the spring mass system as either elastic potential energy of the spring or kinetic energy of the mass. You can express this statement mathematically as shown.

$$E_T = E_e + E_k \quad \text{or} \quad E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

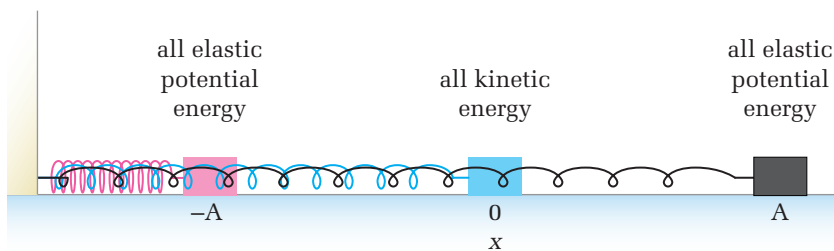


Figure 7.5 When the mass is released from point A , the mass will oscillate between points A and $-A$ indefinitely.

The point $x = 0$ is the equilibrium point at which the spring is neither stretched nor compressed and therefore has no elastic potential energy. The following points describe, in words and equations, the events that occur when the mass is released.

- When the mass is at point $x = A$, the total energy of the system is elastic potential energy.

$$E_T = \frac{1}{2}kA^2$$
- As the mass moves toward $x = 0$, elastic potential energy is transformed into kinetic energy and the total energy is a combination of both.

$$E_T = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$
- When the mass is at $x = 0$, there is no elastic potential energy and the total energy of the system is kinetic energy. At this point, the speed of the mass is at its maximum value, v_{\max} .

$$E_T = \frac{1}{2}mv_{\max}^2$$
- As the mass passes through $x = 0$, kinetic energy is transformed back into elastic potential energy of the spring because the spring is becoming compressed. The system contains both elastic potential energy and kinetic energy. Notice that the sign of position x and speed v does not affect the equation because both quantities are squared, making the values positive.

$$E_T = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$
- When the mass reaches $x = -A$, all of the kinetic energy has been transformed into elastic potential energy and the total energy is once again all elastic potential energy. Notice that $(-A)^2 = A^2$.

$$E_T = \frac{1}{2}kA^2$$

QUICK LAB

The Energy of a Spring and Mass System

TARGET SKILLS

- Identifying variables
- Analyzing and interpreting

This quick lab will help you visualize the distribution of the energy of a spring/mass system at every point in its oscillatory cycle. Imagine that you have a perfect spring and mass system such as the one shown in Figure 7.5. The object has a mass of 0.500 kg and the spring constant is 600.0 N/m. You pull the mass an amplitude of 20.0 cm from its rest position and release it.

Make a table with the headings Position, Elastic Potential Energy, and Kinetic Energy. In the column under “Position,” write the following values, one on each row: -20.0 cm, -15.0 cm, -10.0 cm, -5.0 cm, 0.0 cm, 5.0 cm, 10.0 cm, 15.0 cm, and 20.0 cm. For each position, calculate the elastic potential energy and place the value in the second column. Using $E_T = E_e + E_k$, calculate the kinetic energy for each position. On graph paper, make a plot of energy versus position. On the same graph, using different colours, plot elastic potential

energy versus position and kinetic energy versus position. Add a line representing total mechanical energy.

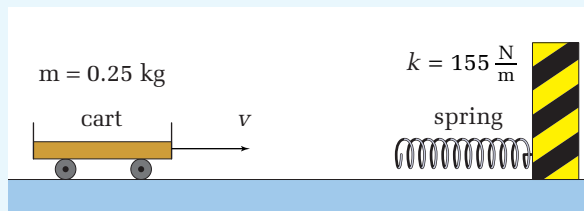
Analyze and Conclude

1. Describe the shape of the curves representing the different types of energy.
2. How does your graph display the law of conservation of mechanical energy?
3. Compare your graph with the graph of the distribution of energy for a falling stone in Figure 7.3. How does your graph of kinetic and elastic potential energies differ from the graph of kinetic and gravitational potential energies?
4. Examine the equations for gravitational potential and elastic potential energies and explain why the graphs differ.

MODEL PROBLEMS

Horizontal Elastic Collisions

1. A low-friction cart with a mass of 0.25 kg travels along a horizontal track and collides head on with a spring that has a spring constant of 155 N/m. If the spring was compressed by 6.0 cm, how fast was the cart initially travelling?



Frame the Problem

- The cart is *moving* so it has *kinetic energy*.
- The spring does *negative work* on the cart, lowering its *kinetic energy*.
- The cart does *work* on the spring, giving it *elastic potential energy*.
- The *height* of the cart does *not change*, so there is no change in *gravitational potential energy*.

- The term *low friction* tells you to neglect the energy lost to work done by friction.
- The law of conservation of mechanical energy applies to this problem.

Identify the Goal

The initial speed, v , of the cart

Variables and Constants

Known

$$m = 0.25 \text{ kg} \quad x = 0.060 \text{ m}$$

$$k = 155 \frac{\text{N}}{\text{m}}$$

Unknown

$$v$$

Strategy

Write the law of conservation of mechanical energy, including the energy quantities associated with the interaction.

Initially, the spring was not compressed, so the initial elastic potential energy was zero.

After the interaction, the cart stopped, so the final kinetic energy was zero.

Substitute the values for energy listed above.

Expand by substituting the expressions for the energies.

Solve for the initial velocity.

Substitute numerical values and solve.

Calculations

$$E'_k + E'_e = E_k + E_e$$

$$E_e = 0 \text{ J}$$

$$E'_k = 0 \text{ J}$$

$$0 \text{ J} + E'_e = E_k + 0 \text{ J}$$

$$E_k = E'_e$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$v = \sqrt{\frac{kx^2}{m}}$$

$$v = \sqrt{\frac{(155 \frac{\text{N}}{\text{m}})(0.060 \text{ m})^2}{0.25 \text{ kg}}}$$

$$v = 1.493 \text{ 99 } \frac{\text{m}}{\text{s}}$$

$$v \cong 1.5 \frac{\text{m}}{\text{s}}$$

The cart was travelling at approximately 1.5 m/s before the collision.

Validate

Unit analysis of the equation $v = \sqrt{\frac{kx^2}{m}}$ shows that it is equivalent to m/s,

$$\text{the standard units for velocity. } \sqrt{\frac{\frac{\text{N}}{\text{m}} \text{ m}^2}{\text{kg}}} = \sqrt{\frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \text{ m}}{\text{kg}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}.$$

A velocity of 1.5 m/s is reasonable for a lab cart.

continued ►

2. A 225 g mass oscillates on a frictionless surface while attached to a spring that has a spring constant of 145 N/m. The amplitude of the oscillations is 80.0 cm.

- (a) What is the maximum velocity of the mass?
- (b) What is the position of the mass when its velocity is exactly half of its maximum velocity?

Frame the Problem

- The *total mechanical energy* of the mass is the same as the *elastic potential energy* when the mass is at its *maximum distance* from *equilibrium* position because its *speed* and therefore *kinetic energy* is zero.
- There is sufficient data to determine the *elastic potential energy* at A.
- The *speed* of the mass is at its maximum value when it is at its equilibrium position.
- The *total mechanical energy* of the mass is the same as the *kinetic energy* when the mass is at its *equilibrium* position.
- At any point in its motion, the total mechanical energy is the sum of the elastic potential energy and the kinetic energy.

Identify the Goal

- (a) The maximum speed, v_{\max} , of the mass
- (b) The position, x , at half maximum speed

Variables and Constants

Known

$m = 225 \text{ g}$
 $k = 145 \frac{\text{N}}{\text{m}}$
 $A = 80.0 \text{ cm}$

Unknown

v_{\max}
 $x(\text{at } \frac{1}{2}v_{\max})$
 E_T

Strategy

Write the equation for total mechanical energy at maximum amplitude.

All the values are known so substitute and solve.

Calculations

$$E_T = \frac{1}{2}kA^2$$

$$E_T = \frac{1}{2}\left(145 \frac{\text{N}}{\text{m}}\right)(0.80 \text{ m})^2$$

$$E_T = 46.4 \frac{\text{N}\cdot\text{m}^2}{\text{m}}$$

$$E_T = 46.4 \text{ J}$$

Write the equation for total mechanical energy at the equilibrium position.

$$E_T = \frac{1}{2}mv_{\max}^2$$

$$\frac{2E_T}{m} = v_{\max}^2$$

Solve for speed, v .

$$v_{\max} = \sqrt{\frac{2E_T}{m}}$$

$$v_{\max} = \sqrt{\frac{2(46.4 \text{ J})}{0.225 \text{ kg}}}$$

$$v_{\max} = \sqrt{412.444 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \frac{1}{\text{kg}}}$$

$$v_{\max} = 20.3087 \frac{\text{m}}{\text{s}}$$

$$v_{\max} \cong 20.3 \frac{\text{m}}{\text{s}}$$

(a) The maximum speed of the mass is 20.3 m/s.

Find half maximum speed.

$$v_{1/2(\max)} = \frac{1}{2} \left(20.3087 \frac{\text{m}}{\text{s}} \right)$$

$$v_{1/2(\max)} \cong 10.154 \frac{\text{m}}{\text{s}}$$

Write the equation for total mechanical energy at any position then solve for position, x .

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}kx^2 = E_T - \frac{1}{2}mv^2$$

$$x^2 = \frac{2}{k} \left(E_T - \frac{1}{2}mv^2 \right)$$

$$x = \sqrt{\frac{2}{k} \left(E_T - \frac{1}{2}mv^2 \right)}$$

$$x = \sqrt{\frac{2}{145 \frac{\text{N}}{\text{m}}} \left(46.4 \text{ J} - \frac{1}{2}(0.225 \text{ kg}) \left(10.154 \frac{\text{m}}{\text{s}} \right)^2 \right)}$$

$$x = \sqrt{0.0137931 \frac{\text{m}}{\text{N}} (46.4 \text{ J} - 11.5991 \text{ J})}$$

$$x = \sqrt{\left(0.0137931 \frac{\text{m}}{\text{N}} \right) (34.8008 \text{ N}\cdot\text{m})}$$

$$x = \sqrt{0.48001 \text{ m}^2}$$

$$x = \pm 0.69282 \text{ m}$$

$$x \cong \pm 0.693 \text{ m}$$

(b) The mass has half its maximum speed at 69.3 cm on either side of its equilibrium position.

Validate

Recalling that a J is equivalent to a N·m and to a kg·m²/s², the units cancel correctly to give m/s for speed and m for position. Also, by examining the curve for kinetic energy versus position (Quick Lab: The Energy of a Spring and Mass System) you see that the mass has travelled well over half the distance from equilibrium before the speed has dropped to half its maximum speed.

continued ►

PRACTICE PROBLEMS

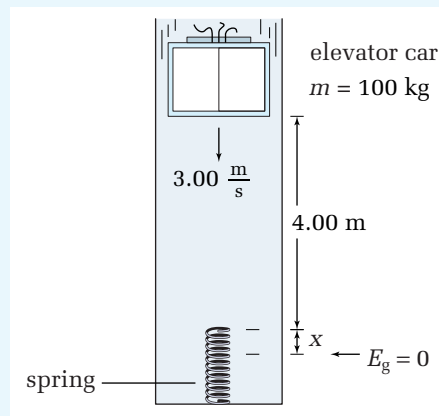
9. A 1.2 kg dynamics cart is rolling to the right along a horizontal lab desk at 3.6 m/s, when it collides head on with a spring bumper that has a spring constant of 2.00×10^2 N/m.
- Determine the maximum compression of the spring.
 - Determine the speed of the cart at the moment that the spring was compressed by 0.10 m.
 - Determine the acceleration of the cart at the moment that the spring was compressed 0.10 m.
10. A circus car with a clown has a total mass of 150 kg. It is coasting at 6.0 m/s, when it hits a large spring head on. If it is brought to a stop by the time the spring is compressed 2.0 m, what is the spring constant of the spring?
11. An archery bow has an effective spring constant of 485 N/m. The archer draws the bow a distance of 45.0 cm. If the arrow has a mass of 0.030 kg, what will be its speed at the moment it leaves the bow?
12. A 0.0250 kg mass on a frictionless surface is attached to a horizontal spring having a spring constant of 124 N/m. The spring is stretched to an amplitude of 9.00 cm and released. Find:
- the maximum speed of the mass
 - the speed of the mass at 3.00 cm on either side of its equilibrium position.
13. A 0.150 kg mass on a frictionless surface is attached to a spring having a spring constant of 215 N/m. A motion detector determines that the mass is travelling at 15.0 m/s when it passes the equilibrium position.
- What is the amplitude of the motion of the mass?
 - How much work was done to stretch the spring to its maximum amplitude?
14. An object on a frictionless surface is attached to a spring having a spring constant of 235 N/m. 50.0 J of work were done on the spring to stretch it to its maximum amplitude. As the mass passed through its equilibrium position, a motion detector determined that its speed was 14.6 m/s.
- What was the mass of the object?
 - What was its amplitude?
 - What was the object's position when its speed was 5.00 m/s?

Total Mechanical Energy

When solving problems involving gravitational and elastic potential energy and kinetic energy, the method is the same as you used for just two of the three forms of mechanical energy. However, you must take care in choosing your reference level for gravitational potential energy. Analyze the position that was chosen for zero potential energy in the following model problem until you understand the reasoning behind the choice.

Vertical Elastic Collisions

A freight elevator car with a total mass of 100.0 kg is moving downward at 3.00 m/s, when the cable snaps. The car falls 4.00 m onto a huge spring with a spring constant of 8.000×10^3 N/m. By how much will the spring be compressed when the car reaches zero velocity?



Frame the Problem

- Initially, the car is in *motion* and therefore has *kinetic energy*. It also has *gravitational potential energy*.
- As the car begins to *fall*, the *gravitational potential energy* transforms into *kinetic energy*. When the elevator hits the spring, the elevator *slows*, losing *kinetic energy*, and the spring compresses, gaining *elastic potential energy*.
- When the elevator comes to a complete *stop*, it has *no kinetic* or *gravitational potential energy*. All of the energy is now stored in the spring in the form of *elastic potential energy*.
- Since all of the motion is in a downward direction, define “down” as the positive direction for this problem.
- Choose the lowest level reached by the elevator as the reference level for gravitational potential energy

Identify the Goal

The compression of the spring, x , when the car comes to rest

Variables and Constants

Known	Implied	Unknown
$m_{\text{car}} = 100.0$ kg	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	x
$v = 3.00 \frac{\text{m}}{\text{s}}$ [down]		
$k = 8.000 \times 10^3 \frac{\text{N}}{\text{m}}$		
$h_{\text{(above spring)}} = 4.00$ m		

Strategy

Write the law of conservation of energy for the forms of energy involved in the problem.

The final gravitational potential energy of the car is zero.

The car comes to a rest at the lowest point.

Calculations

$$E'_g + E'_e + E'_k = E_g + E_e + E_k$$

$$E'_g = 0 \text{ J}$$

$$E'_k = 0 \text{ J}$$

continued ►

continued from previous page

Initially, the spring is not compressed.

$$E_e = 0 \text{ J}$$

Substitute these initial and final conditions into the equation for conservation of energy and simplify.

$$0 \text{ J} + E'_e + 0 \text{ J} = E_g + 0 \text{ J} + E_k$$

$$E'_e = E_g + E_k$$

Expand by substituting the expressions for the various forms of energy.

$$\frac{1}{2}kx^2 = mg\Delta h + \frac{1}{2}mv^2$$

The change in height for the gravitational potential is 4.00 m, plus the compression of the spring, x . Substitute this expression into the equation.

$$\frac{1}{2}kx^2 = mg(4.00 + x) + \frac{1}{2}mv^2$$

Since the equation yields a quadratic equation, you cannot solve for x .

$$\frac{1}{2}\left(8.00 \times 10^3 \frac{\text{N}}{\text{m}}\right)x^2 = (100.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(4.00 + x) \text{ m} \\ + \frac{1}{2}(100.0 \text{ kg})\left(3.00 \frac{\text{m}}{\text{s}}\right)^2$$

Substitute in the numerical values and rearrange so that the right-hand side is zero.

$$4.00 \times 10^3 x^2 - 981x - 3924 - 450 = 0$$

$$4.00 \times 10^3 x^2 - 981x - 4374 = 0$$

Use the quadratic formula to find the value of x .

$$x = \frac{981 \pm \sqrt{(-981)^2 - 4(4.00 \times 10^3)(-4374)}}{2(4.00 \times 10^3)}$$

$$x = 1.1756 \text{ m} \quad \text{or} \quad -0.93025 \text{ m}$$

$$x \cong 1.18 \text{ m}$$

Compression cannot be negative (or the spring would be stretching), so choose the positive value. The spring was compressed 1.18 m.

Validate

The units on the left-hand side of the final equation are $\frac{\text{N}}{\text{m}} \cdot \text{m}^2 = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}^2 = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$.

On the right-hand side of the equation, the units are $\text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$.

Both sides of the equation have the same units, so you can have confidence in the equation. The answer is also in a range that would be expected with actual springs.

Note: The negative root in this problem is interesting in that it does have meaning. If the car had somehow latched onto the spring during the collision, the negative value would represent the maximum extension of the spring if the car had bounced up from the bottom due to the upward push of the spring.

PRACTICE PROBLEMS

15. A 70.0 kg person steps through the window of a burning building and drops to a rescue net held 8.00 m below. If the surface of the net is 1.40 m above the ground, what must be

the value of the spring constant for the net so that the person just touches the ground when the net stretches downward?

16. A 6.0 kg block is falling toward a spring located 1.80 m below. If it has a speed of 4.0 m/s at that instant, what will be the maximum compression of the spring? The spring constant is 2.000×10^3 N/m.
17. In a “head dip” bungee jump from a bridge over a river, the bungee cord is fastened to the jumper’s ankles. The jumper then steps off and falls toward the river until the cord becomes taut. At that point, the cord begins

- to slow the jumper’s descent, until his head just touches the water. The bridge is 22.0 m above the river. The unstretched length of the cord is 12.2 m. The jumper is 1.80 m tall and has a mass of 60.0 kg. Determine the
- (a) required value of the spring constant for this jump to be successful
- (b) acceleration of the jumper at the bottom of the descent

PHYSICS & SOCIETY

Pole-Vaulting: The Art of Energy Transfer

Athlete Stacy Dragila won a gold medal for pole-vaulting at the 2000 Olympic Games in Sydney, the first games to feature women’s vaulting. During her winning vaults she demonstrated a fundamental principle of physics — the conservation of energy. Dragila has speed, power, and near-perfect technique. When she vaults, she uses and transfers energy as efficiently as possible.

Before a pole-vaulter begins to sprint toward the bar, she is storing chemical energy in her muscles. If the vaulter is well trained, she will be able to use the chemical energy as efficiently as possible. This chemical energy is converted into kinetic energy as she sprints toward the bar. Near the end of the sprint, the vaulter places her pole in a socket. Kinetic energy from the sprint bends the pole and the pole gains elastic potential energy. When the pole begins to extend from its bend, it converts the elastic energy into gravitational potential energy, lifting the vaulter into the air.

Ideally, a vaulter would be able to convert all of her kinetic energy into elastic energy. But not even Dragila has perfect technique. Even if she did, some of the energy would still be converted to forms that do not perform useful work. Some kinetic energy is converted to thermal and sound energy during the sprint, and some elastic energy is absorbed by the pole as thermal energy during the lift.

The important thing to remember is that the amount of energy involved in the pole vault remains constant. The type of energy and the efficiency with which the athlete uses the energy make the difference between champions such as Dragila and other pole-vaulters.

TARGET SKILLS

- Analyzing and interpreting



Olympic gold medallist Stacy Dragila in action

Analyze

1. Sporting events are sometimes held at high altitudes, such as the 1968 Olympic Games in Mexico City. The thinner air poses challenges for some athletes, but pole-vaulters, and also long-jumpers and high-jumpers, tend to benefit. Why?
2. Pole-vaulting began as an inventive way to get across rivers and other barriers. The first poles were made of bamboo. Today’s poles are much more sophisticated. What principles need to be considered when designing a pole?

7.1 Section Review

- K/U** Which types of energy are generally referred to as mechanical energy?
- C** Using examples not found in the textbook, describe and explain an example in which the forces are
 - conservative
 - non-conservative
- I** A student is sliding down a frictionless water slide at an amusement park.
 - Sketch a graph of gravitational potential energy against height for the descent. (No numbers are required on the axes.)
 - On the same axes, sketch a graph of the total energy of the student against height for the descent.
 - On the same axes, sketch a graph of the kinetic energy of the student against height for the descent.
- C** An extreme sport in-line skating competition often involves aerial tricks at the top of a curved ramp that has the shape of a cylinder cut in half. Describe how this activity demonstrates the concept of conservation of mechanical energy.
- K/U** A skier starts from rest at the top of a ski jump, skis down, and leaves the bottom of the jump with a velocity v . Assuming that the ski jump is frictionless, how many times higher would the jump have to be for the skier to jump with twice the velocity?
- MC** Some Olympic sports transform kinetic energy into gravitational potential energy of the athlete's body to aid performance. These sports include:
 - bob sledding
 - high diving off a diving board
 - pole vaulting
 - snow ski racing
 - snow ski jumping
 - trampoline jumping
 - In which of these sports can you increase your gravitational potential energy by using better sports equipment? (for example, a better diving board will allow you to jump higher).
 - Suppose you can use better materials to produce equipment with greater elastic potential energy. Which sports will be affected?



UNIT PROJECT PREP

Athletic activities from walking to skydiving involve work being done by conservative and non-conservative forces.

- Identify conservative and non-conservative forces involved with your project topic.
- How are non-conservative forces used in the design of sports equipment to increase safety?