6.4 Power and Efficiency

SECTION OUTCOMES

- Analyze quantitatively the relationships among work, time, and power.
- Determine the percentage efficiency of energy transformation.

KEY TERMS

- power
- efficiency

A 400 t passenger jet, loaded with over 450 people and their luggage, waits at the end of a runway. In less than 60 s, the jet will speed down a 2 km long runway and lift into the air. Meanwhile, halfway around the world, in the Andes Mountains, an old converted school bus slowly bounces along a gravel road. Both the jet and the bus burn fuel to obtain energy for motion. The difference between these vehicles is, of course, the rate at which they convert the chemical energy into motion.

Power

The engine in an old school bus could, over a long period of time, do as much work as jet engines do when a jet takes off. However, the school bus engine could not begin to do enough work fast enough to make a jet lift off. In this and many other applications, the rate at which work is done is more critical than the amount of work done. **Power** is the rate at which work is done. Since work is defined as a transfer of energy, power can also be defined as the rate at which energy is transferred.

Figure 6.23 Jet engines do an incredible amount of work in a very short interval of time. They generate a large amount of power.

DEFINITION OF POWER

Power is the quotient of work and time interval.

$$P = \frac{W}{\Delta t} \quad \text{or} \quad P = \frac{E}{\Delta t}$$

Quantity	Symbol	SI unit
power	P	W (watt)
energy transferred	E	J (joule)
work done	W	J (joule)
time interval	Δt	s (seconds)

Note: A watt is equivalent to a joule per second: $\text{W} = \frac{\text{J}}{\text{s}}$

Any machine that does mechanical work or any device that transfers energy via heat can be described by its power rating; that is, the rate at which it can transfer energy. The SI unit of power, the watt, can be used to quantify the power of motors, rockets, or even dynamite, but it is most familiar as a power rating for a light bulb. A 60 W bulb transforms 60 J of electric energy into thermal energy and light in 1 s, as compared to a 100 W bulb that transforms 100 J of electric energy into light and thermal energy in 1 s.

The language of power is subtle and different from that of work. Recall that work is done *on* an object and results in a *transfer* of energy to that object. The *rate* of this energy transfer, or power, is often referred to as the power that is *generated* in doing the work. The term “power” not only applies to the rate at which energy is transferred from one object to another or transformed from one form to another, but also to the rate at which energy is transported from one location to another. For example, electric *power lines* carry electric energy across vast stretches of land.

Figure 6.24 Light bulbs and electric appliances are often labelled with a power rating.

History Link

The unit, the watt, was named in honour of the Scottish engineer, James Watt, who made such great improvements in the steam engine that it hastened the Industrial Revolution. The ability to do work did not change, but the rate at which the work could be accomplished did. Watt did experiments with strong dray horses and determined that they could lift 550 pounds a distance of one foot in 1 s. He called this amount of power one horsepower (hp). Converting to SI units, 1 hp is equivalent to 746 W. What do you think your horsepower is? Design a simple experiment that you could use to determine your horsepower, and then read ahead in the Multi Lab in this section to see another way to determine your horsepower.

MODEL PROBLEMS

Calculating Power

1. A crane is capable of doing 1.50×10^5 J of work in 10.0 s. What is the power of the crane in watts?

Frame the Problem

- The crane did *work* in a specified *time interval*.
- *Power* is defined as work done per unit time.
- Simply apply the power definition.

PROBLEM TIP

Remember that a capital “*W*” is the variable that represents work done and “W” also represents the unit of power, the watt. Be careful not to confuse the two, as they are very different. To help distinguish the difference, in this text, symbols for quantities are in italics while units are in roman print.

continued ▶

continued from previous page

Identify the Goal

Power generated by the crane

Variables and Constants

Known	Unknown
$W = 1.50 \times 10^5$ J	P
$\Delta t = 10.0$ s	

Strategy / Calculations

Use the formula for power.

$$P = \frac{W}{\Delta t}$$

All needed variables are given, so substitute the variables into the formula.

$$P = \frac{1.50 \times 10^5 \text{ J}}{10.0 \text{ s}}$$

Divide.

$$P = 1.50 \times 10^4 \text{ J/s}$$
$$P = 1.50 \times 10^4 \text{ W}$$

The crane is able to generate 1.50×10^4 W of power.

Validate

Work was given in joules and time in seconds.
Power is in J/s or W, which is correct.

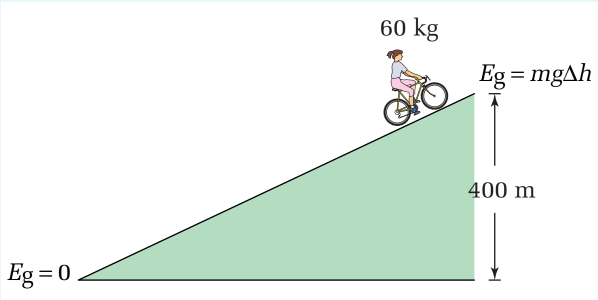
2. A cyclist and her mountain bike have a combined mass of 60.0 kg. She is able to cycle up a hill that changes her altitude by 4.00×10^2 m in 1.00 min. (Assume that friction is negligible.)

(a) How much work does she do against gravity in climbing the hill?

(b) How much power is she able to generate?

Frame the Problem

- The cyclist is *doing work* against gravity by cycling uphill, thus *changing her altitude*.
- She is therefore *changing* her *gravitational potential energy*.
- Her *work done* will be equal to her change in *gravitational potential energy*.
- The *time interval* is given, and you can calculate the work done. Therefore, you can use the equation for *power*.



Identify the Goal

(a) Her work done, W, in climbing the hill

(b) The power, P, that she generated

Variables and Constants

Known	Implied	Unknown
$m = 60.0$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	W
$d = 4.00 \times 10^2$ m [up]		E_g
$\Delta t = 1.00$ min		P

Strategy / Calculations

Work done is equal to change in gravitational potential energy.

$$W = E_g$$
$$E_g = mg\Delta h$$

Substitute known values.

$$E_g = (60.0\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(4.00 \times 10^2\text{ m})$$

Multiply.

$$E_g = 2.3544 \times 10^5 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}$$
$$E_g = 2.3544 \times 10^5\text{ N} \cdot \text{m}$$
$$E_g = 2.3544 \times 10^5\text{ J}$$
$$W = 2.3544 \times 10^5\text{ J}$$

(a) Therefore, the work done by the girl to cycle to the top of the 400 m hill is 2.35×10^5 J.

Use the equation for power.

$$P = \frac{W}{\Delta t}$$

Convert time to SI units.

$$t = (1.00\ \cancel{\text{min}})\left(\frac{60\text{ s}}{\cancel{\text{min}}}\right)$$
$$t = 60.0\text{ s}$$

The values are known, so substitute.
Divide.

$$P = \frac{2.3544 \times 10^5\text{ J}}{60.0\text{ s}}$$
$$P = 3.924 \times 10^3\ \frac{\text{J}}{\text{s}}$$
$$P = 3.924 \times 10^3\text{ W}$$

(b) The cyclist generated 3.92×10^3 W of power. That amount of power is equivalent to 5.25 hp or sixty-five 60 W light bulbs.

Validate

The power is in watts, which is correct.

continued ▶

continued from previous page

PRACTICE PROBLEMS

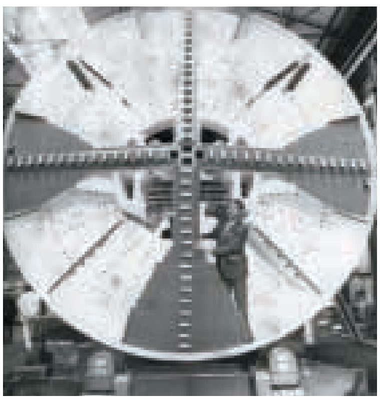
41. A mover pushes a 25.5 kg box with a force of 85 N down a 15 m corridor. If it takes him 8.30 s to reach the other end of the hallway, find the power generated by the mover, in watts.

42. A chair lift carries skiers uphill to the top of the ski run. If the lift is able to do 1.85×10^5 J of work in 12.0 s, what is the power of the chair lift in both watts and horsepower?

43. A 75.0 kg student runs up two flights of stairs in order to reach her next class. The total height of the stairs is 5.75 m from the ground level. If the student can generate 200 W of power and has 20.0 s to reach her classroom at the top of the stairs, will the student be on time for class?

Technology Link

The powerful machine shown here removed eighty million cubic metres of dirt from beneath the English Channel in less than six years. Two rail tunnels and a service tunnel, 52 km in length, now connect England to France. How important is the tunnel in stimulating trade between the United Kingdom and the European continent? What kinds of vehicles are allowed in the tunnel, and how are their exhaust gases controlled? You may find answers to these and other questions you would be interested in by investigating further.



MULTI LAB Investigating Power

Work, Power, and Gravity

You will need two marbles of different sizes, a golf ball, a stopwatch, and a board to act as a ramp. Set up the board so that it forms a ramp approximately 45° to the horizontal. Time the marbles and golf ball as they race to the bottom of the ramp. Verify that the race is fair by controlling necessary variables.

Analyze and Conclude

1. What did you notice about the time required for the different-sized balls to reach the bottom of the ramp?
2. What did you notice about the relative speed of each ball when it reached the bottom of the ramp?
3. What effect did mass have on the time or speed?
4. What ball required the most power to be generated?
5. If gravity is generating the power, what limits exist on the amount of power that could be generated?
6. How does society make use of the power generation of gravity?

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting
- Identifying variables
- Communicating results

What Is Your Horsepower?

How much horsepower can you develop? How important is speed? How important is mass? To determine your horsepower, all you need is your mass, a stopwatch, and a staircase of known height. Have a classmate record the time it takes you to climb a flight of stairs. Choose a relatively high flight of stairs and run more than one trial.

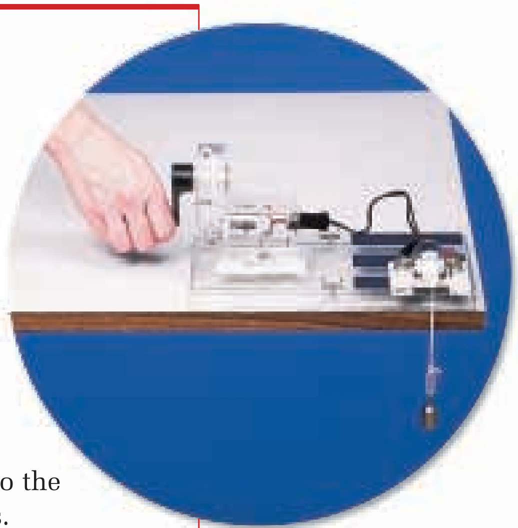
CAUTION If you have any respiratory or heart problems or any physical condition that could be compromised by running up stairs, do not actively participate in this investigation. Do the theory and calculations only.

Analyze and Conclude

1. Make an educated guess as to what your horsepower will be. Refer to the History Link on page 263 to help you with your guess.
2. Calculate the work you did against gravity.
3. Use the amount of work you did to calculate the power you generated.
4. Report your answer in watts (W) and in horsepower (hp). **Note:** 746 W = 1 hp.

Generating Power, Transforming Energy

Using a hand-held electric generator and electric winch assembly as shown, investigate what variables determine the amount of power generated. Try different masses and be very careful to observe the relative difficulty you experience when turning the hand crank.



Analyze and Conclude

1. What was generating the electric power?
2. What was generating the mechanical power?
3. What variables affect the amount of power generated?
4. Trace the energy path from your muscles all the way to the gravitational potential energy stored in the lifted mass.

Web Link

www.mcgrawhill.ca/links/atlphysics

Gravity is the force behind the generation of power in many examples in nature. Learn more about power generation in nature by going to the Internet site above and clicking on **Web Links**.

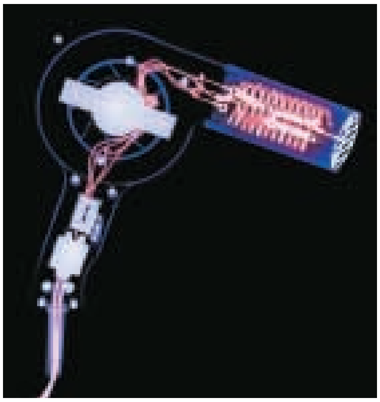


Figure 6.25 Even in a hair dryer that is designed to produce thermal energy, motion of the air is not 100% efficient. What other "wasted" forms of energy does it produce?

Efficiency

A light bulb is designed to convert electric energy into light energy. A car engine is designed to convert chemical potential energy stored in the fuel into kinetic energy for the car. However, both the light bulb and the car engine become extremely hot while they perform their designed function. Obviously, they have transformed much of the energy into thermal energy. While the light bulb and the car engine are transforming some of the potential energy into the desired form of energy, much energy is "lost."

Energy can be, and is, converted into forms that do no work or do not serve the intended purpose. Transforming energy from one form to another always involves some "loss" of useful energy. Often the lost energy is transformed into heat. The **efficiency** of a machine or device describes the extent to which it converts input energy or work into the intended type of output energy or work.

DEFINITION OF EFFICIENCY

Efficiency is the ratio of useful energy or work output to the total energy or work input.

$$\text{Efficiency} = \frac{E_o}{E_i} \times 100\%$$

or

$$\text{Efficiency} = \frac{W_o}{W_i} \times 100\%$$

Quantity	Symbol	SI unit
useful output energy	E_o	J (joule)
input energy	E_i	J (joule)
useful output work	W_o	J (joule)
input work	W_i	J (joule)
efficiency	(none)	none; efficiency is a ratio; units cancel in ratios

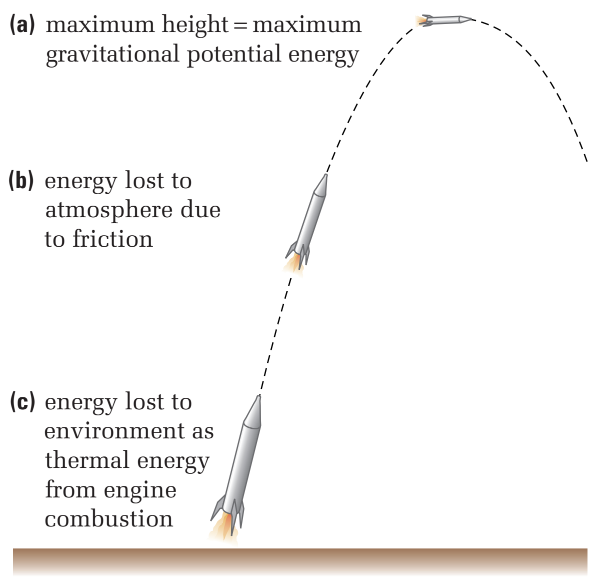
MODEL PROBLEM

Calculating Efficiency

A model rocket engine contains explosives storing 3.50×10^3 J of chemical potential energy. The stored chemical energy is transformed into gravitational potential energy at the top of the rocket's flight path. Calculate how efficiently the rocket transforms stored chemical energy into gravitational potential energy if the 0.500 kg rocket is propelled to a height of 1.00×10^2 m.

Frame the Problem

- The rocket engine transforms *chemical potential energy* first into kinetic energy that will propel the rocket *upward* against gravity.
- The kinetic energy is transformed into *gravitational potential energy*.
- If the energy transformation was 100% *efficient*, then the rocket would reach a height such that its *gravitational potential energy* would be equal to the chemical energy stored in the explosives.
- That will not happen, because a great deal of *energy is lost* as *thermal energy* created both by the combustion of the fuel and by friction with the rocket and the atmosphere.



Identify the Goal

Efficiency of the rocket and engine system in transforming chemical potential energy into gravitational potential energy

Variables and Constants

Known	Implied	Unknown
$m = 0.500$ kg	$g = 9.81\frac{\text{m}}{\text{s}^2}$	W_o
$\Delta h = 1.00 \times 10^2$ m		efficiency
$W_i = 3.50 \times 10^3$ J		

continued ►

continued from previous page

Strategy	Calculations
The useful work output is the gravitational potential energy.	$W_o = E_g$
Calculate the gravitational potential energy that the rocket has at the top of its flight.	$E_g = mg\Delta h$
Substitute in the variables and multiply.	$W_o = E_g = (0.500\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(100\ \text{m})$ $W_o = 490.5\frac{\text{kg m m}}{\text{s}^2}$ $W_o = 490.5\ \text{N} \cdot \text{m}$ $W_o = 490.5\ \text{J}$
All the variables needed to calculate the efficiency have been determined.	$\text{Efficiency} = \frac{W_o}{W_i} \times 100\%$
Calculate the efficiency.	$\text{Efficiency} = \frac{490.5\ \cancel{\text{J}}}{3500\ \cancel{\text{J}}} \times 100\% = 14\%$

The energy stored in the rocket engine is transformed into gravitational potential energy (height) of the rocket with an efficiency of 14%. Most of the "lost" energy is transferred to the surroundings as thermal energy.

Validate

The gravitational energy at the rocket's maximum height is correctly assumed to be the W_o.

Efficiency is given as a percentage, which is correct.

PRACTICE PROBLEMS

44. A portable stereo requires 265 J of energy to operate the CD player, yielding 200 J of sound energy.

(a) How efficiently does the stereo generate sound energy?

(b) Where does the "lost" energy go?

45. A 49.0 kg child sits on the top of a slide that is located 1.80 m above the ground. After her descent, the child reaches a velocity of 3.00 m/s at the bottom of the slide. Calculate how efficiently the potential energy is converted to kinetic energy.

46. A machine requires 580 J of energy to do 110 J of useful work. How efficient is the machine?

47. An incandescent light bulb transforms 120 J of electric energy to produce 5 J of light energy. A florescent bulb requires 60 J of electrical energy to produce the same amount of light.

(a) Calculate the efficiency of each type of bulb.

(b) Why is the fluorescent bulb more efficient than the incandescent bulb?

48. A microwave oven transforms 345 J of radiant energy into 301 J of thermal energy in some food. Calculate the efficiency of this energy transformation.

49. A 125 g ball is thrown with a force of 85.0 N that acts through a distance of 78.0 cm. The ball's velocity just before it is caught is 9.84 m/s.

(a) Calculate the work done on the ball.

(b) Calculate the kinetic energy of the ball just before it is caught.

(c) What fraction of the energy transferred to the ball was lost to the atmosphere during flight?

50. Rubbing your hands together requires 450 J of energy and results in a thermal energy increase in your palms of 153 J. Calculate how efficiently the kinetic energy is converted to thermal energy.

6.4 Section Review

1. K/U Describe the difference between work and power.

2. I Develop an algebraic relationship for power in terms of force, F, and constant velocity, v. (Hint: Begin with the power equation and make substitutions for work, W.)

3. MC Using Table 6.5, compare cycling efficiency to driving, flying, and using a snowmobile.

Table 6.5
Transportation Energy Requirements

Mode of transportation	Energy consumption (kJ/km)
bicycle	52
walking	170
city bus	360
car	674
jumbo jet	2252
snowmobile	6743
ocean liner	8117

continued ▶

INVESTIGATION 6-B

Muscle Efficiency and Energy Consumption

TARGET SKILLS

- Hypothesizing
- Performing and recording
- Analyzing and interpreting
- Communicating results

No real process is 100% efficient at transforming stored energy into either mechanical work or heat. Just as it is possible to calculate the efficiency of a light bulb or an automobile motor, with some basic assumptions, it is also possible to determine the approximate efficiency of your muscles.

Problem

You can demonstrate quantitatively that your muscles lose energy when they work. How can the results be put in terms of work done, muscle efficiency, and food energy equivalence?

Hypothesis

Form a hypothesis about the form the lost energy of the muscle will take. Predict the efficiency of your biceps muscle.

Equipment

- computer
- data collection interface
- temperature probe
- dumbbell

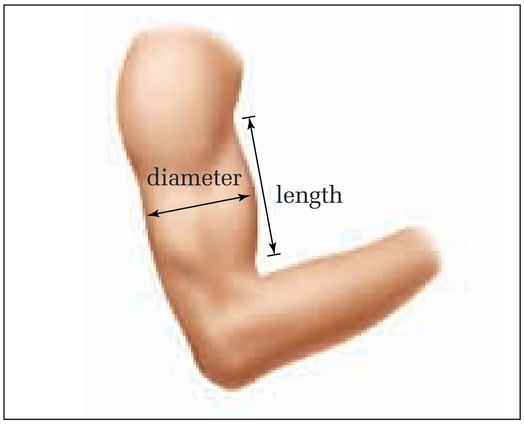
Procedure

The Workout

1. Select a dumbbell with which you will just be able to do 10 biceps curls with one arm.
2. Connect and activate the computer interface and temperature probe.
3. Obtain a base temperature reading for your muscle by holding the probe firmly against your rested biceps for 60 s.
4. With the probe still firmly pressed against your biceps, perform 10 biceps curls with the dumbbell, using only one arm.

Calculating Efficiency

5. Calculate the approximate volume of your biceps muscle. Assume that your arm is essentially a cylinder. Biceps curling involves the muscles on the front half of this cylinder. Therefore, you are able to approximate the volume of your biceps by measuring the length and circumference of your upper arm. Then, calculate the approximate volume of your biceps, using the method shown in the chart on the facing page.



Volume of a cylinder	$V_{\text{cylinder}} = \pi r^2 h$
Circumference of a circle	$C = 2\pi r$
Solve for r.	$r = \frac{C}{2\pi}$
Substitute into the volume formula.	$V_{\text{cylinder}} = \pi\left(\frac{C}{2\pi}\right)^2 h$
Expand.	$V_{\text{cylinder}} = \frac{\cancel{\pi} hC^2}{4\pi^{\cancel{2}}}$
Simplify.	$V_{\text{cylinder}} = \frac{hC^2}{4\pi}$
Divide by two for half of a cylinder.	$V_{\text{half cylinder}} = \frac{hC^2}{8\pi}$

6. Measure C and h. If you take measurements in centimetres, your volume will be in cubic centimetres.

7. To approximate the mass of your biceps muscle, recall that a large percentage of your muscle fibres are water. Convert your biceps volume to the equivalent mass of water that it could contain by using the following relationship: 1 mL of water = 1 cm^3 = 1 g.

8. Calculate the quantity of heat (Q) generated using the following equation.

   $Q = mc\Delta T$, where m = mass of biceps converted to kg

   c = 3500 J/°K kg (specific heat capacity for the human body)

   ΔT = change in temperature during workout

9. Determine the work done during the 10-repetition workout.

   Recall that $W = F_{\parallel}\Delta d$.

   In this case, the force is equal in magnitude but opposite in direction to the force of gravity acting on the mass: $F = mg$.

   The distance the mass is moved is equal to the length of the arm.

10. Calculate your biceps muscle's approximate efficiency.

    Recall: Efficiency = $\frac{W_o}{W_i} \times 100\%$,

    where W_o = useful work output
    W_i = total work output

    In the case of curling a mass, the total work input is the sum of both the work done on the mass and the heat generated in the biceps. The useful work output is only the work done on the mass.

Analyze and Conclude

1. How did the biceps curls affect the temperature of your working arm?

2. What caused the observed changes?

3. What is the approximate efficiency of your biceps muscle?

4. List all of the assumptions that you made in determining efficiency. How could your estimate be improved?

5. What biological function(s) generated the heat?

6. A single chocolate chip cookie contains 1.34×10^6 J of energy. Calculate how many times you would have to curl the mass you used in the experiment to consume all of the energy supplied by the cookie.