# Potential Energy and the Work-Energy Theorem 

When you do work on an object, will the object always gain kinetic energy? Are there situations where you do work on an object but leave the object at rest? The work done by the student in the photograph is a clear demonstration that an object may remain motionless after work is done on it. In this section, you will consider how doing work on an object can result in a change in potential energy, rather than in kinetic energy. Potential energy is sometimes described as the energy stored by an object due to its position or condition.


Figure 6.18 When you lift groceries onto a shelf, you have exerted a force on the groceries. However, when they are on the shelf, they have no kinetic energy. What form of energy have the groceries gained?

## Potential Energy

Consider the work you do on your physics textbook when you lift it from the floor and place it on the top shelf of your locker. You have exerted a force over a distance. Therefore, you have done work on the textbook and yet it is not speeding off out of sight. The work you did on your textbook is now stored in the book by virtue of its position. Your book has gained potential energy. By doing work against the force of gravity, you have given your book a special form of potential energy called gravitational potential energy.

Gravitational potential energy is only one of several forms of potential energy. For example, chemical potential energy is stored in the food you eat. Doing work on an elastic band by stretching it stores elastic potential energy in the elastic band. A battery contains both chemical and electrical potential energy.

SECTION

## OUTCOMES

- Define gravitational potential energy.
- Define elastic potential energy.
- Use algebraic deductions to develop the work-energy theorem.
- Investigate forced stretch relationships for springs.


## K E Y <br> TERMS

- gravitational potential energy
- elastic
- elastic potential energy
- Hooke's law
- restoring force
- spring constant

Figure 6.19 Gravitational potential energy is stored in the water. When the water begins to fall, it gains kinetic energy. As it falls, it turns the wheel, giving the wheel kinetic energy.

## PHYSICS FILE

The constant $g$, the acceleration due to gravity, affects objects even when they are not moving. When something is preventing an object from falling, such as your desk holding up your book, $g$ influences its weight. The weight of an object is its mass times $g$. If nothing were preventing it from falling, your book, or any other object, would accelerate at $9.81 \mathrm{~m} / \mathrm{s}^{2}$, the value of $g$. The value of $g$ varies with the size and mass of the planet, moon, or star. On the Moon, for example, your book would weigh less and, if falling, would accelerate at a lower rate ( $1.62 \mathrm{~m} / \mathrm{s}^{2}$ ).

## Gravitational Potential Energy

For hundreds of years, people have been using the gravitational potential energy stored in water. Many years ago, people built water wheels like the one shown in Figure 6.19. Today, we create huge reservoirs and dams that convert the potential energy of water into electricity.


To determine the factors that contribute to gravitational potential energy, try another "thought experiment." Ask yourself the following questions.

- If a golf ball and a Ping Pong ${ }^{\text {TM }}$ ball were dropped from the same distance, which one might you try to catch and which one would you avoid?
- If one golf ball was dropped a distance of 10 cm and another a distance of 10 m , which one would hit the ground with a greater force?
- Which golf ball would hit the surface with the greatest force, one dropped a distance of 1.0 m on Earth or 1.0 m on the Moon?

Everyday experience tells you that mass and height (vertical distance between the two positions) contribute to an object's gravitational potential energy. Your knowledge of gravity also helps you to understand that $g$, the acceleration due to gravity, affects gravitational potential energy as well.

An important characteristic of all forms of potential energy is that there is no absolute zero position or condition. We measure only changes in potential energy, not absolute potential energy. Physicists must always assign a reference position and compare the potential energy of an object to that position. Gravitational potential energy depends on the difference in height between two positions. Therefore, the zero or reference level can be assigned to any convenient position. We typically choose the reference position as the solid surface toward which an object is falling or might fall.

## GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy is the product of mass, the acceleration due to gravity, and the change in height.

$$
E_{g}=m g \Delta h
$$

## Quantity

gravitational potential
energy
mass
acceleration due to gravity
change in height
(from reference position)

## Unit Analysis

(mass)(acceleration)(height) $=\mathrm{kg} \frac{\mathrm{m}}{\mathrm{s}^{2}} \mathrm{~m}=\mathrm{N} \cdot \mathrm{m}=\mathrm{J}$

The equation for gravitational potential energy in the box on the left is an example of what physicists call a "special case." The numerical value of $g$, $9.81 \mathrm{~m} / \mathrm{s}^{2}$, applies only to cases near Earth's surface. The value would be different out in space or on a different planet. Therefore, because the equation contains $g$, the equation itself applies only to cases close to Earth's surface. For example, you could not use the equation to find the gravitational potential energy of an astronaut in the International Space Station.

## MODEL PROBLEM

## Calculating Gravitational Potential Energy

You are about to drop a 3.0 kg rock onto a tent peg. Calculate the gravitational potential energy of the rock after you lift it to a height of 0.68 m above the tent peg.

## Frame the Problem

Make a sketch of the situation.

- You do work against gravity when you lift the rock.
- All of the work gives gravitational potential energy to the rock.
- The expression for gravitational potential energy applies.


B

## Identify the Goal

The gravitational potential energy, $E_{\mathrm{g}}$, of the rock

## Variables and Constants

## Known

$m=3.0 \mathrm{~kg}$

## Implied

$g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$

Unknown
Eg
$\Delta h=0.68 \mathrm{~m}$

## Strategy

Use the formula for gravitational potential energy.

Substitute.
Multiply.
$1 \mathrm{~N} \cdot \mathrm{~m}$ is equivalent to 1 J .

## Calculations

$E_{\mathrm{g}}=m g \Delta h$
$E_{\mathrm{g}}=(3.0 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(0.68 \mathrm{~m})$
$E_{\mathrm{g}}=2.0 \times 10^{1} \mathrm{~kg} \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \mathrm{~m}$
$E_{\mathrm{g}}=2.0 \times 10^{1} \mathrm{~N} \cdot \mathrm{~m}$
$E_{\mathrm{g}}=2.0 \times 10^{1} \mathrm{~J}$

The rock has $2.0 \times 10^{1} \mathrm{~J}$ of gravitational potential energy.

## Validate

Doing work on the rock resulted in a change of position of the rock relative to the tent peg. The work done is now stored by the rock as gravitational potential energy.

## PRACTICE PROBLEMS

27. A framed picture that is to be hung on the wall is lifted vertically through a distance of 2.0 m . If the picture has a mass of 4.45 kg , calculate its gravitational potential energy with respect to the ground.
28. The water level in a reservoir is 250 m above the water in front of the dam. What is the potential energy of each cubic metre of surface water behind the dam? (Take the density of water to be $1.00 \mathrm{~kg} / \mathrm{L}$.)
29. How high would you have to raise a 0.300 kg baseball in order to give it 12.0 J of gravitational potential energy?


## Work and Gravitational Potential Energy

To develop a mathematical relationship between work and gravitational potential energy, start with the equation for work.

- Work is the product of the force that is parallel to the direction of the motion and the displacement that the force caused the object to move.

$$
W=F_{\|} \Delta d
$$

- Recall from Chapter 4 that the force of gravity on a mass near Earth's surface is given by $\vec{F}=m \vec{g}$, where $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$. Since the force of gravity and the acceleration due to gravity are always downward, and since work is a scalar quantity, we will omit vector notations.

$$
F=m g
$$

- Substitute mg for $F$ into the expression for work.

$$
W=m g \Delta d
$$

- Substitute $\Delta h$ for height in place of $\Delta d$ to emphasize that the displacement vector is vertical.

$$
W=m g \Delta h
$$

- This is the equation for work done to lift an object to height $\Delta h$, relative to its original position.

$$
W=m g \Delta h
$$

- The work, $W$, done on the object has become gravitational potential energy stored in the object by virtue of its position. $\quad E_{\mathrm{g}}=m g \Delta h$

Depending on your choice of a reference level, an object may have some gravitational potential energy before you do work on it. For example, choose the floor as your reference. If your book was on the desk, it would have an amount of gravitational potential energy, $m g \Delta h_{1}$, in relation to the floor, where $\Delta h_{1}$ is the height of the desk. Then you do work against gravity to lift it to the shelf, where it has gravitational potential energy $m g \Delta h_{2}$, where $\Delta h_{2}$ is the height of the shelf. The work you did changed the book's gravitational potential. You can describe this change mathematically as

$$
\begin{aligned}
& W=m g \Delta h_{2}-m g \Delta h_{1} \\
& W=E_{\mathrm{g} 2}-E_{\mathrm{g} 1} \\
& W=\Delta E_{\mathrm{g}}
\end{aligned}
$$

The mathematical expression above is a representation of the work-energy theorem in terms of gravitational potential energy.


## MODEL PROBLEM

## Applying the Work-Energy Theorem

A 65.0 kg rock climber did $1.60 \times 10^{4} \mathrm{~J}$ of work against gravity to reach a ledge. How high did the rock climber ascend?

## Frame the Problem

- The rock climber did work against gravity.
- Work done against gravity increased the rock climber's gravitational potential energy.
- The work-energy theorem that applies to potential energy is appropriate for this situation.



## Identify the Goal

The vertical height, $\Delta h$, that the climber ascended

## Variables and Constants

Known
$W=1.60 \times 10^{4} \mathrm{~J}$
$m=65.0 \mathrm{~kg}$

Implied
$g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$E_{\mathrm{g} 1}=0 \mathrm{~J}$

Unknown
$E_{\text {g2 }}$
$\Delta h$

## Strategy

## Calculations

Use the work-energy theorem to find the climber's gravitational potential energy from the amount of work done.

$$
W=E_{\mathrm{g} 2}-E_{\mathrm{g} 1}
$$

Choose the starting point as your reference for gravitational potential energy, so that $E_{\mathrm{g} 1}$ will be zero. Solve.

## Substitute first

$$
\begin{aligned}
& W=E_{\mathrm{g} 2}-E_{\mathrm{g} 1} \\
& 1.6 \times 10^{4} \mathrm{~J}=E_{\mathrm{g} 2}-0 \mathrm{~J} \\
& E_{\mathrm{g} 2}=1.6 \times 10^{4} \mathrm{~J}
\end{aligned}
$$

## Solve for $E_{\mathrm{g} 2}$ first

$$
W=E_{\mathrm{g} 2}-E_{\mathrm{g} 1}
$$

$$
W+E_{\mathrm{g} 1}=E_{\mathrm{g} 2}
$$

$$
1.6 \times 10^{4} \mathrm{~J}+0 \mathrm{~J}=E_{\mathrm{g} 2}
$$

$$
1.6 \times 10^{4} \mathrm{~J}=E_{\mathrm{g} 2}
$$

$$
\begin{aligned}
& E_{\mathrm{g} 2}=m g \Delta h \\
& 1.6 \times 10^{4} \mathrm{~J}=65 \mathrm{~kg} 9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \Delta h
\end{aligned}
$$

$$
\frac{1.6 \times 10^{4} \mathrm{~J}}{65 \mathrm{~kg} \mathrm{9.81} \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=\frac{65 \mathrm{~kg} 9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \Delta h}{65 \mathrm{~kg} 9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}
$$

$$
\frac{1.6 \times 10^{4} \mathrm{~J}}{65 \mathrm{~kg} \mathrm{9.81} \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=\Delta h
$$

Convert J to $\mathrm{kg} \frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}$, so that you can cancel units.
$\Delta h=25.09 \frac{\frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}}}{\mathrm{~kg} \frac{\frac{\mathrm{kx}}{8^{2}}}{}}$
$\Delta h=25.09 \mathrm{~m}$
$E_{\mathrm{g} 2}=m g \Delta h$
$\frac{E_{\mathrm{g} 2}}{m g}=\frac{m g \Delta h}{m g}$
$\Delta h=\frac{E_{\mathrm{g} 2}}{m g}$

$$
\Delta h=\frac{1.6 \times 10^{4} \mathrm{~J}}{65 \mathrm{~kg}}
$$

$\Delta h=25.09 \frac{\frac{\mathrm{~kg} \times \mathrm{m}^{2}}{\mathrm{~s}^{2}}}{\frac{\mathrm{~kg} \times m}{\mathrm{~s}^{2}}}$
$\Delta h=25.09 \mathrm{~m}$

The rock climber ascended 25.1 m .

## Validate

The climber did a large amount of work, so you would expect that the climb was quite high.
The units canceled to give m, which is correct for height.
30. A student lifts her 2.20 kg pile of textbooks into her locker from where they rest on the ground. She must do 25.0 J of work in order to lift the books. Calculate the height that the student must lift the books.
31. A 46.0 kg child cycles up a large hill to a point that is a vertical distance of 5.25 m above the starting position. Find
(a) the change in the child's gravitational potential energy
(b) the amount of work done by the child against gravity
32. A 2.50 kg pendulum is raised vertically 65.2 cm from its rest position. Find the gravitational potential energy of the pendulum.
33. A roller-coaster train lifts its passengers up vertically through a height of 39.4 m from its
starting position. Find the change in gravitational potential energy if the mass of the train and its passengers is $3.90 \times 10^{3} \mathrm{~kg}$.
34. The distance between the sixth and the eleventh floors of a building is 30.0 m . The combined mass of the elevator and its contents is $1.35 \times 10^{3} \mathrm{~kg}$.
(a) Find the gravitational potential energy of the elevator when it stops at the eighth floor, relative to the sixth floor.
(b) Find the gravitational potential energy of the elevator when it pauses at the eleventh floor, relative to the eighth floor.
(c) Find the gravitational potential energy of the elevator when it stops at the eleventh floor, relative to the sixth floor.

## Elastic Potential Energy

When a diver is standing on a diving board, it bends under the weight of the diver as seen in Figure 6.20. When the diver leaves the board, it returns to its original shape. Many objects can stretch, compress, bend, or change in shape in some way. If an object can return to its original condition, as does the diving board, it is said to be elastic. Since the object can undergo motion when the force is removed, there must have been energy stored in the object due to its condition. This form of stored energy is called elastic potential energy. In this section, you will examine elastic potential energy in the form of stretched and compressed springs.


Figure 6.20 Elastic potential energy is stored in this bent diving board.

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results


## Problem

What relationship exists between the force applied to a spring and its extension?


## Equipment

- retort stand and C-clamp
- weight hanger and accompanying set of masses
- coil spring
- ring clamp
- metre stick

CAUTION Wear protective eye goggles during this investigation.

## Procedure

1. Clamp the retort stand firmly to the desk.
2. Attach the ring clamp close to the top of the retort stand.
3. Hang the spring by one end from the ring clamp.
4. Prepare a data table with the headings: Mass on hanger, $m(\mathrm{~kg})$; Applied force, $F(\mathrm{~N})$; Height of hanger above desk, $h(\mathrm{~m})$; and Extension of spring, $x(m)$.
5. Attach the weight holder and measure its distance above the desktop. Record this value in the first row of the table. This value will be your equilibrium value, $h_{0}$, at which you will assign the value of zero to the extension of the spring, $x$. Put these values in the first line of your table.
6. To create an applied force, add a mass to the weight holder. Wait for the spring to come to rest and measure the height of the weight holder above the desk. Record these values in the table.
7. Complete the second row in the table by calculating the value of the applied force (weight of the mass) and the extension of the spring $\left(x=h_{0}-h\right)$.
8. Continue by adding more masses until you have at least five sets of data. Make sure that you do not overextend the spring.

## Analyze and Conclude

1. Draw a graph of the applied force versus the extension of the spring. Note: Normally, you would put the independent variable (in this case, the applied force) on the $x$-axis and the dependent variable (in this case, the extension of the spring) on the $y$-axis. However, the mathematics will be simplified in this case by reversing the position of the variables.
2. Draw a smooth curve through the data points.
3. Describe the curve and write the equation for the curve.
4. State the relationship between the applied force and the extension.
5. When the spring is at rest, what is the relationship between the applied force and the force that the spring exerts on the mass? This force is usually referred to as the "restoring force." Restate the force/extension relationship in terms of the restoring force of the spring.


Figure 6.21 The relationship between the restoring force and the extension of a spring is linear.

## Hooke's Law

In Investigation 6-A, you examined the relationship between the extension of a spring and the force exerted on it. You observed that when a force causes the spring to stretch, the spring exerts a force in a direction that will return it to its original length. Many springs can be compressed as well as stretched and, in either case, will exert a force in the direction that will restore their original shape. This force exerted by the spring, called the restoring force, always acts in the direction opposite to the direction that the spring is stretched or compressed. This property of elastic objects such as springs is known as Hooke's law. A graphical illustration of this law for an extended spring is shown in Figure 6.21.

Since the data produce a straight line, the equation can be written in the form $y=m x+b$, where $m$ is the slope and $b$ is the $y$-intercept. The slope of the line describing the properties of a spring, called the spring constant, is symbolized by $k$ and has units of newtons/metre. Each spring has its own constant that describes the amount of force that is necessary to stretch (or compress) the spring a given amount. In your investigation, you were directed to assign the reference or zero position of your spring as the position of the spring with no applied force. As a result, $x$ was zero when $F$ was zero. This choice is the accepted convention for working with springs, and it makes the $y$-intercept equal to zero because the line on the graph passes through the origin. This relationship leads to the mathematical form of Hooke's law (which is summarized in the following box): $-F_{\mathrm{a}}=k x$, where $F_{\mathrm{a}}$ is the magnitude of the applied force, $x$ is the magnitude of the extension or compression, and $k$ is the spring constant.

## HOOKE'S LAW

The applied force is directly proportional to the extension or compression of a spring.

$$
F=-k x
$$

## Quantity

applied force
spring constant
amount of extension or compression of the spring $\quad x \quad m$ (metres)

## Unit Analysis

newtons $=\left(\frac{\text { newtons }}{\text { metre }}\right)$ (metre) $\quad \mathrm{N}=\frac{\mathrm{N}}{\mathrm{m}} \mathrm{m} I=\mathrm{N}$

## Hooke's Law in an Archery Bow

A typical compound archery bow requires a force of 133 N to hold an arrow at "full draw" (pulled back 71 cm ). Assuming that the bow obeys Hooke's law, what is its spring constant?

## Frame the Problem

- When an archer draws a bow, the applied force does work on the bow, giving it elastic potential energy.
- Hooke's law applies to this problem.


## Identify the Goal

The spring constant, $k$, of the bow

## Variables and Constants

## Known

$F_{\mathrm{a}}=133 \mathrm{~N}$
Unknown
$x=71 \mathrm{~cm}$

## Strategy

## Calculations

Use Hooke's law (applied force form).

$$
\begin{aligned}
F_{\mathrm{a}} & =k x \\
k & =\frac{F_{\mathrm{a}}}{X} \\
k & =\frac{133 \mathrm{~N}}{0.71 \mathrm{~m}} \\
k & =187.32 \frac{\mathrm{~N}}{\mathrm{~m}} \\
k & \cong 1.9 \times 10^{2} \frac{\mathrm{~N}}{\mathrm{~m}}
\end{aligned}
$$

Solve for the spring constant.
Substitute numerical values and solve.

The spring constant of the bow is about $1.9 \times 10^{2} \frac{\mathrm{~N}}{\mathrm{~m}}$.

## Validate

When units are carried through the calculation, the final quantity has units of $\mathrm{N} / \mathrm{m}$, which are correct for the spring constant.
35. A spring scale is marked from 0 to 50 N . The scale is 9.5 cm long. What is the spring constant of the spring in the scale?
36. A slingshot has an elastic cord tied to a Y-shaped frame. The cord has a spring constant of $1.10 \times 10^{3} \mathrm{~N} / \mathrm{m}$. A force of 455 N is applied to the cord.
(a) How far does the cord stretch?
(b) What is the restoring force from the spring?
37. The spring in a typical Hooke's law apparatus has a force constant of $1.50 \mathrm{~N} / \mathrm{m}$ and a maximum extension of 10.0 cm . What is the largest mass that can be placed on the spring without damaging it?

## Work and Elastic Potential Energy

## PHYSICS FILE

A perfectly elastic material will return precisely to its original form after being deformed, such as stretching a spring. No real material is perfectly elastic. Each material has an elastic limit, and when stretched to that limit, will not return to its original shape. The graph below shows that when something reaches its elastic limit, the restoring force does not increase as rapidly as it did in its elastic range.


The graph of Hooke's law in Figure 6.21 not only gives information about the forces and extensions for a spring (or any elastic substance), you can also use it to determine the quantity of potential energy stored in the spring. As discussed previously, you can find the amount of work done or energy change by calculating the area under a force-versus-position graph. The Hooke's law graph is such a graph, since extension or compression is simply a displacement. The area under the graph, therefore, is equal to the amount of potential energy stored in the spring, as illustrated in Figure 6.22.


Figure 6.22 The triangular area under the Hooke's law graph gives you the amount of elastic potential energy stored in the spring at any amount of extension.

As you can see in Figure 6.22, the area under the curve of restoring force versus extension of a spring is a triangle. You can use the geometry of the graph to derive an equation for the elastic potential energy stored in a spring.

- Write the equation for the area of a triangle.

$$
\begin{aligned}
& A=\frac{1}{2} \text { (base) } \text { (height) } \\
& E_{\mathrm{e}}=A \\
& E_{\mathrm{e}}=\frac{1}{2} \text { (base)(height) }
\end{aligned}
$$

- The elastic potential energy stored in a spring is the area under the curve.
- The base of the triangle is the magnitude of extension or compression of the spring, $x_{1}$.
base $=x_{1}$
- The height of the triangle is the force at an extension of $x_{1}$.

$$
\begin{aligned}
\text { height } & =F\left(x_{1}\right) \\
F\left(x_{1}\right) & =k x_{1} \\
\text { height } & =k x_{1}
\end{aligned}
$$

- Substitute the values into the expression for elastic potential energy.

$$
\begin{aligned}
& E_{\mathrm{e}}=\frac{1}{2}\left(x_{1}\right)\left(k x_{1}\right) \\
& E_{\mathrm{e}}=\frac{1}{2} k x_{1}^{2} \\
& E_{\mathrm{e}}=\frac{1}{2} k x^{2}
\end{aligned}
$$

- The expression is valid for any value of $x$.

The equation you just derived applies to any perfectly elastic system and is summarized in the box below.

## ELASTIC POTENTIAL ENERGY

The elastic potential energy of a perfectly elastic material is one half the product of the spring constant and the square of the length of extension or compression.

$$
E_{\mathrm{e}}=\frac{1}{2} k x^{2}
$$

## Quantity

elastic potential energy
spring constant
length of extension or compression

## Unit Analysis

joule $=\frac{\text { newton }}{\text { metre }}$ metre $^{2}$

## Symbol SI unit

$E_{\mathrm{e}} \quad \mathrm{J}$ (joules)
$k \quad \frac{\mathrm{~N}}{\mathrm{~m}}$ (newtons per metre)
$x \quad m$ (metres)

Robert Hooke (1635-1703) was one of the most renowned scientists of his time. His studies in elasticity, which resulted in the law being named after him, allowed him to design better balance springs for watches. He also contributed to our understanding of optics and heat. In 1663, he was elected as a Fellow of the Royal Society in London. His studies ranged from the microscopic - he observed and named the cells in cork and investigated the crystal structure of snowflakes - to astronomy his diagrams of Mars allowed others to measure its rate of rotation. He also proposed the inverse square law for planetary motion. Newton used this relationship in his law of universal gravitation. Hooke felt that he had not been given sufficient credit by Newton for his contribution, and the two men remained antagonistic for the rest of Hooke's life.

## MODEL PROBLEM

## Elastic Potential Energy of a Spring

A spring with spring constant of $75 \mathrm{~N} / \mathrm{m}$ is resting on a table.
(a) If the spring is compressed a distance of 28 cm , what is the increase in its potential energy?
(b) What force must be applied to hold the spring in this position?

## Frame the Problem

- There is no change in the gravitational potential energy of the spring.
- The elastic potential energy of the spring increases as it is compressed.
- The applied force is equal in magnitude and opposite in direction to the restoring force.
- Hooke's law and the definition of elastic potential energy apply to this problem.


## Identify the Goal

The elastic potential energy, $E_{e}$, stored in the spring
The applied force, $F_{\mathrm{a}}$, required to compress the spring

## Variables and Constants

## Known

## Unknown

$k=75 \frac{\mathrm{~N}}{\mathrm{~m}}$
$E_{\text {e }}$
$x=0.28 \mathrm{~m}$
$F_{\mathrm{a}}$

## Strategy

## Calculations

Apply the equation for elastic potential energy.
Substitute and solve.

$$
\begin{aligned}
& E_{\mathrm{e}}=\frac{1}{2} k x^{2} \\
& E_{\mathrm{e}}=\frac{1}{2}\left(75 \frac{\mathrm{~N}}{\mathrm{~m}}\right)(0.28 \mathrm{~m})^{2} \\
& E_{\mathrm{e}}=2.94 \mathrm{~J} \\
& E_{\mathrm{e}} \cong 2.9 \mathrm{~J}
\end{aligned}
$$

(a) The potential energy of the spring increases by 2.9 J when it is compressed by 28 cm .

Use Hooke's law to calculate the

$$
\begin{aligned}
& F=-k x \\
& F=-\left(75 \frac{\mathrm{~N}}{\mathrm{~m}}\right)(0.28 \mathrm{~m}) \\
& F=-21 \mathrm{~N}
\end{aligned}
$$

(b) A force of 21 N is required to hold the spring in this position.

## Validate

Round the given information to 80 N and 0.3 m and do mental multiplication. The resulting estimated change in elastic potential energy is 3.6 J and the estimated applied force is 24 N . The exact answers are reasonably close to these estimated values. In addition, a unit analysis of the first part yields an answer in $\mathrm{N} \cdot \mathrm{m}$ or joules, while the second answer is in newtons.

## PRACTICE PROBLEMS

38. An object is hung from a vertical spring, extending it by 24 cm . If the spring constant is $35 \mathrm{~N} / \mathrm{m}$, what is the potential energy of the stretched spring?
39. An unruly student pulls an elastic band that has a spring constant of $48 \mathrm{~N} / \mathrm{m}$, producing a
2.2 J increase in its potential energy. How far did the student stretch the elastic band?
40. A force of 18 N compresses a spring by 15 cm . By how much does the spring's potential energy change?

### 6.3 Section Review

1. K/U Is gravitational potential energy always measured from one specific reference point? Explain.
2. K/O Define the term "potential" as it applies to "gravitational potential energy."
3. © Describe what happens to the gravitational potential energy of a stone dropped from a bridge into a river below. How has the amount of gravitational potential energy changed when the stone is (a) halfway down, (b) three quarters of the way down, and (c) all of the way down?
4. C Your physics textbook is sitting on a shelf above your desk. Explain what is wrong with the statement, "The gravitational potential energy of the book is 20 J. "
5. © The following is the derivation of the relationship between work and gravitational potential energy.

$$
\begin{aligned}
& W=F_{\|} \Delta d \\
& W=m g \Delta d \\
& W=m g \Delta h
\end{aligned}
$$

(a) Explain why mg could be substituted for force in this derivation but not in the derivation for the relationship between work and kinetic energy.
(b) Explain why $\Delta h$ was substituted for $\Delta d$.
6. K/U An amount of work, $W$, was done on one ball to raise it to a height $h$. In terms of $W$, how much work must you do on four balls, all identical to the first, to raise them to twice the height $h$ ?
7. K/U Explain how each of the following behave like a spring.
(a) a pole used in pole-vaulting
(b) the strings in a tennis racquet
(c) the string on a bow
8. (I) Prove that the expression for elastic potential energy has units equivalent to the joule.
9. MO In what way is a spring similar to a chemical bond?
10. (I) Describe an investigation to determine the force-extension characteristics of an archery bow.

